PCFGs:
Parsing & Evaluation

LING 571 — Deep Processing Techniques for NLP

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Announcements

- HW2 due tonight at 11:59pm
  - readme.{txt|pdf}
    - Separate upload to Canvas
    - NOT in `hw2.tar.gz`
  - Run `check_hw2.sh` before submitting! (Also: `tar -tf hw2.tar.gz` to preview.)
    - Flat structure; just files, no directories, inside tar-ball
    - Include *only* the files we ask for, not more

- Start symbol: either "%start S" *or* first nonterminal
  - NB: needs to be readable by nltk's grammar loading methods
- Use **nltk.data.load**: best to use "**file:**path/to/grammar.cfg" as argument
  - Docs: https://www.nltk.org/api/nltk.data.html#nltk.data.load
- See hw2 slides as well on website for above points

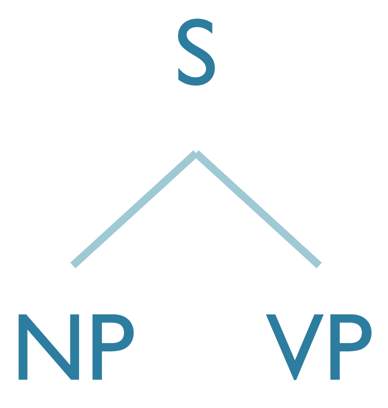
Roadmap

- CKY + back-pointers example
- PCFGs
- PCFG Parsing (PCKY)
- Inducing a PCFG
- Evaluation
- [Earley parsing]
- HW3 + collaboration

CKY + Back-pointers Example

```
cky_table[0,6][S] = {(NP, (0,1),
                      VP, (1,6))}
```

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



I *prefer* *a* *flight* *on* *TWA*

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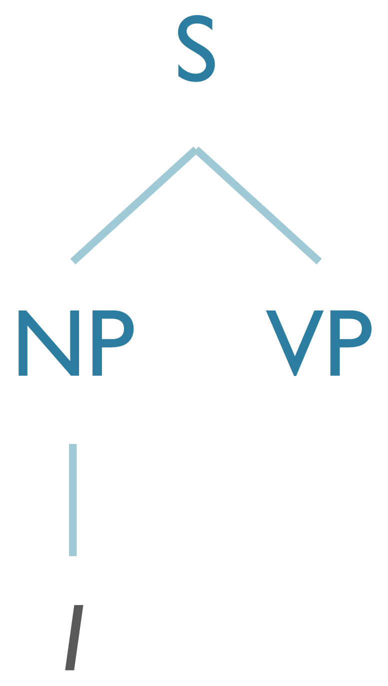

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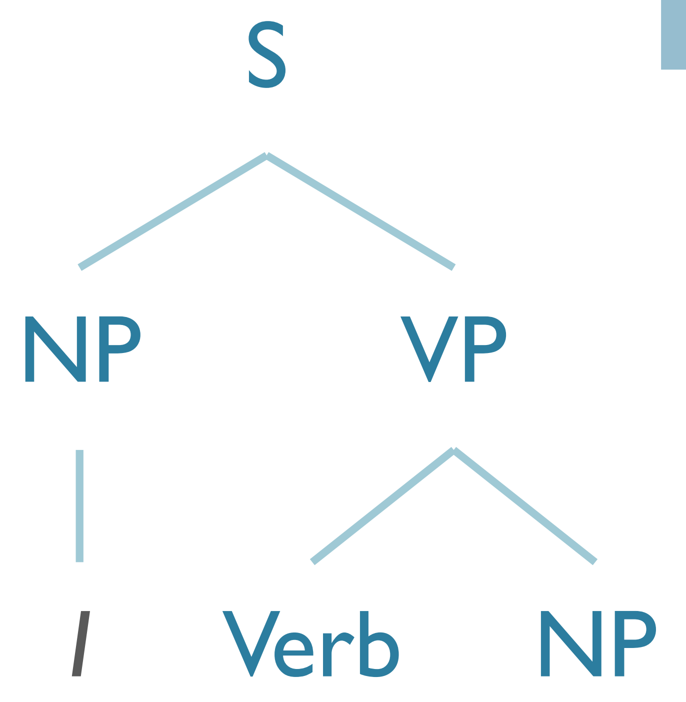

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cky_table[1,2][Verb] = {('prefer')}
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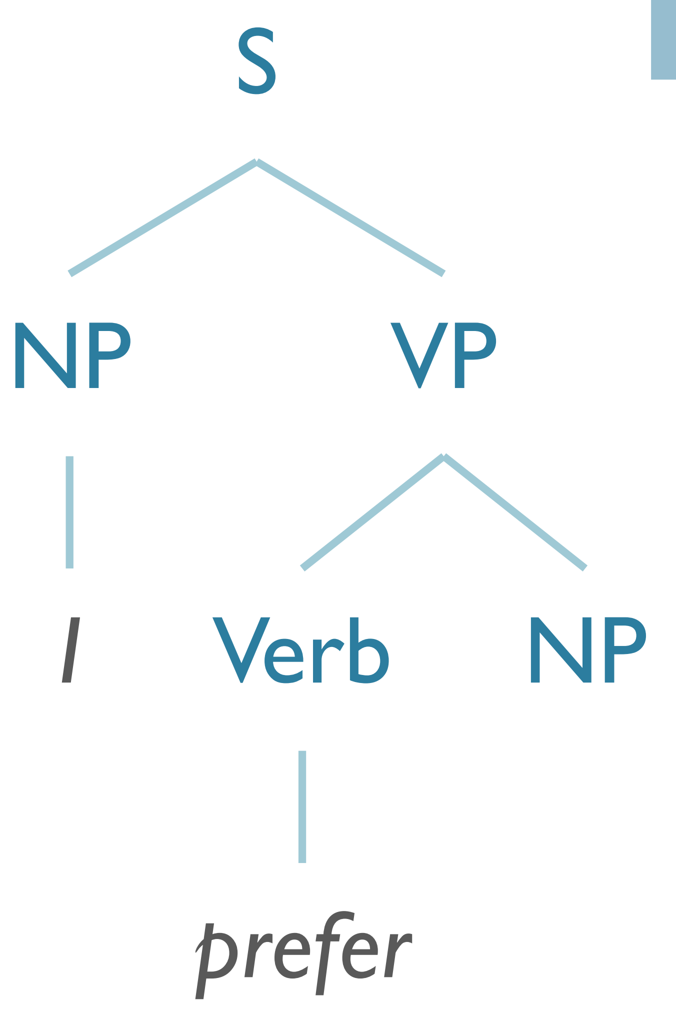

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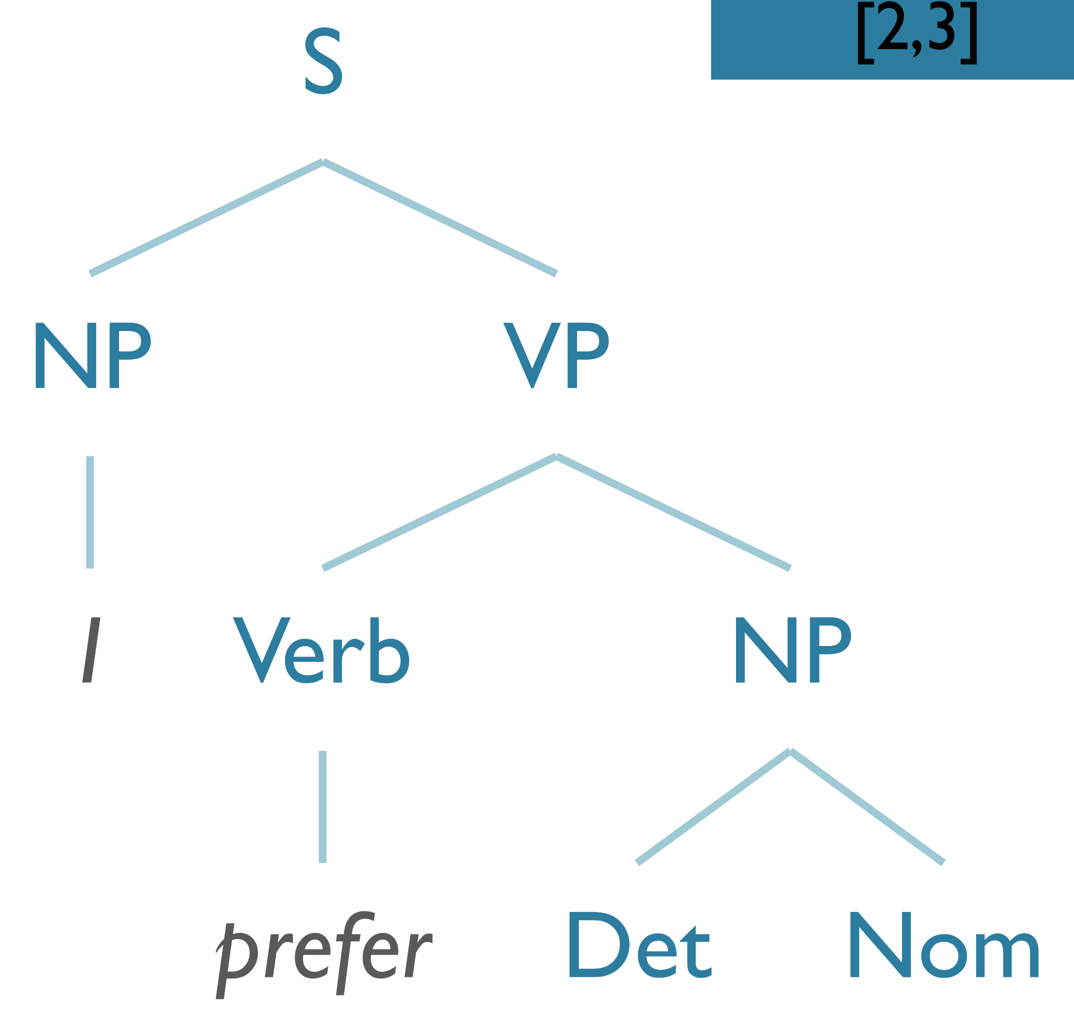

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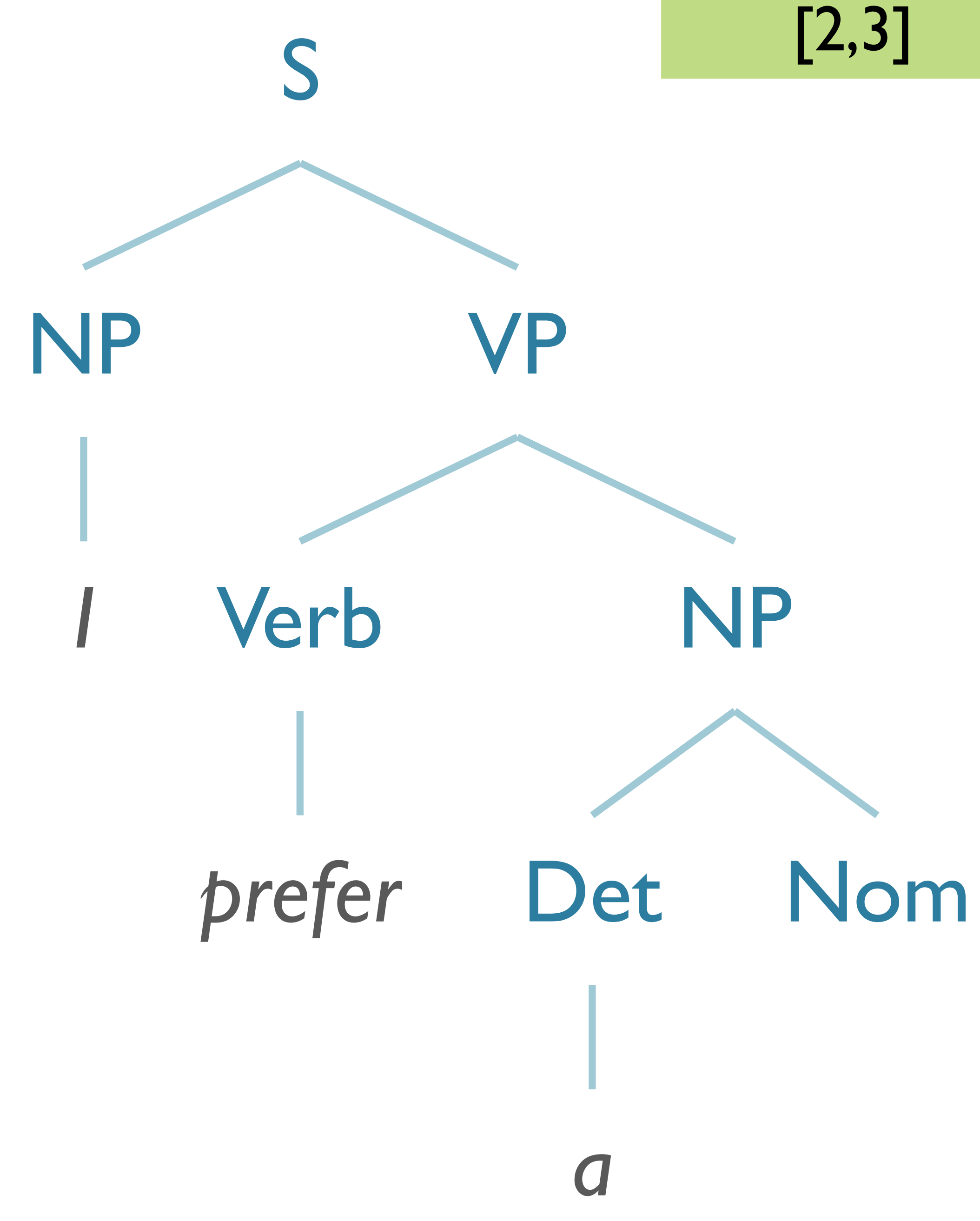

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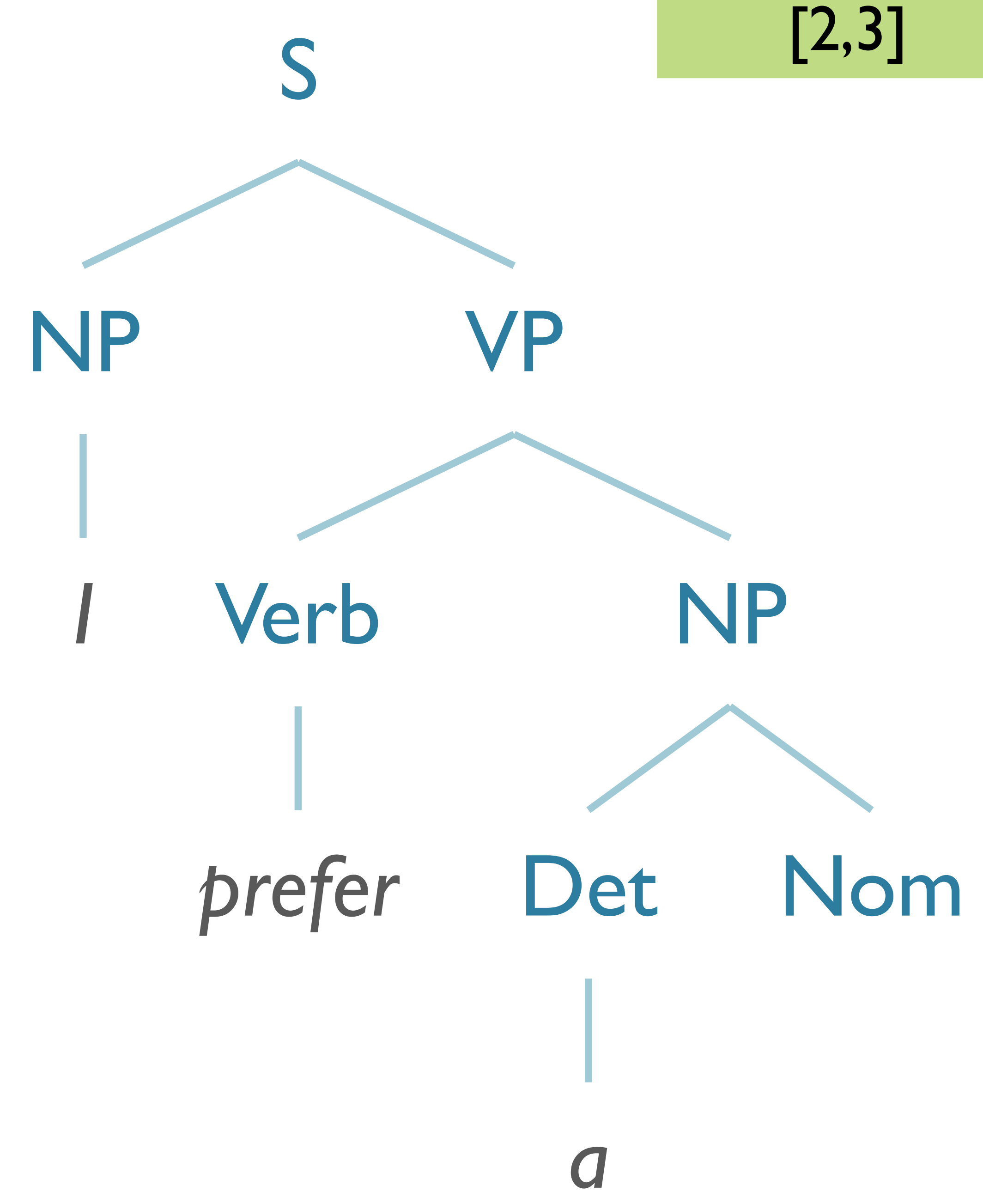

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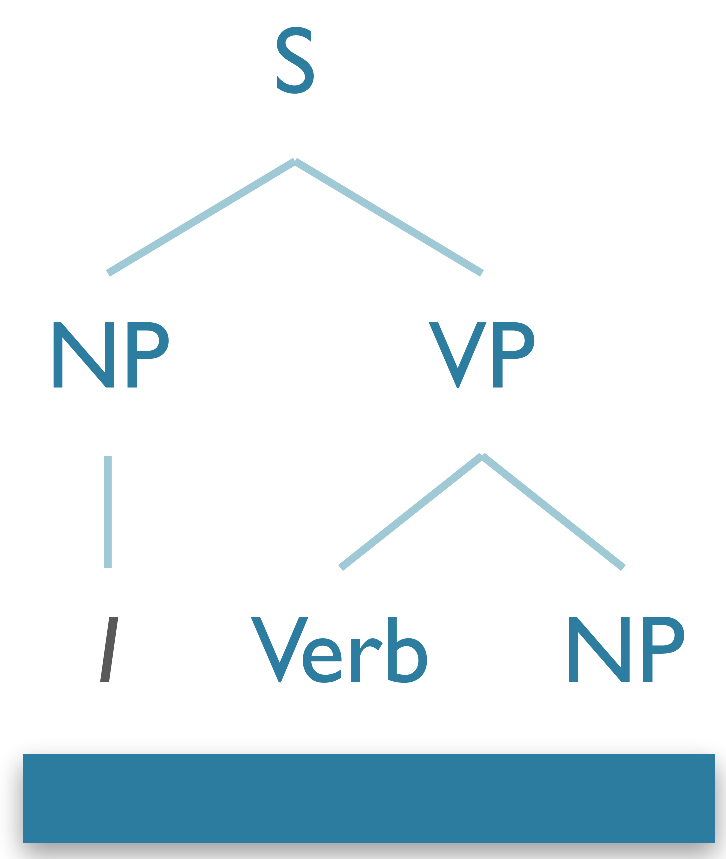

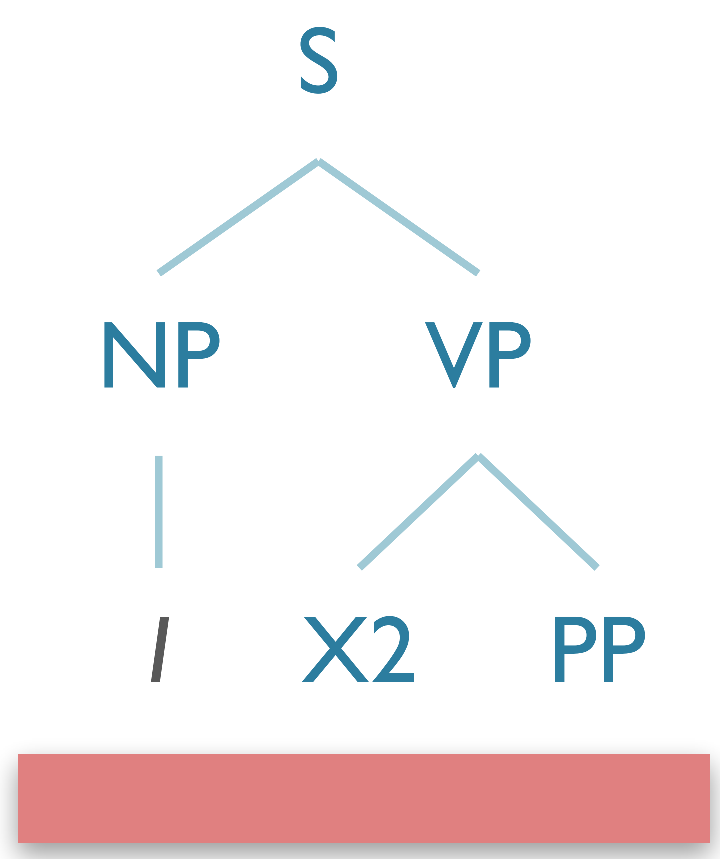

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Probabilistic Context-Free Grammars

Probabilistic Context-free Grammars: Roadmap

Motivation: Ambiguity

Approach:

- Definition
- Disambiguation
- Parsing
- Evaluation
- Enhancements

Motivation

What about ambiguity?

Current algorithm can ***represent*** it

…can't resolve it.

Probabilistic Parsing

- Provides strategy for solving disambiguation problem
  - Compute the probability of all analyses
  - Select the most probable

- Employed in language modeling for speech recognition
  - N-gram grammars predict words, constrain search
  - Also, constrain generation, translation

PCFGs: Formal Definition

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S	a designated **start symbol**

PCFGs

- Augment each production with probability that LHS will be expanded as RHS
  - $P(A \rightarrow \beta)$
  - $P(A \rightarrow \beta | A)$
  - $P(\beta | A)$
  - $P(RHS \mid LHS)$
- NB: the first is often used; but the latter are what's really meant.

PCFGs

- Sum over all possible expansions is **1**

$$\sum_{\beta} P(A \rightarrow \beta) = 1$$

- A PCFG is ***consistent*** if sum of probabilities of all sentences in language is **1**
  - Recursive rules often yield inconsistent grammars *([Booth & Thompson, 1973](#))*

Example PCFG: Augmented $\mathcal{L}_1$

Grammar		Lexicon
S → NP VP	[.80]	*Det → that* [.10] | *a* [.30] | *the* [.60]
S → Aux NP VP	[.15]	*Noun → book* [.10] | *flight* [.30] | *meal* [.15] | *money* [0.5] | *flights* [0.40] | *dinner* [.10]
S → VP	[.05]	*Verb → book* [.30] | *include* [.30] | *prefer* [.40]
NP → Pronoun	[.35]	*Pronoun → I* [.40] | *she* [.05] | *me* [.15] | *you* [.40]
NP → Proper-Noun	[.30]	*Proper-Noun → Houston* [.60] | *NWA* [.40]
NP → Det Nominal	[.20]	*Aux → does* [.60] | *can* [.40]
NP → Nominal	[.15]	*Preposition → from* [.30] | *to* [.30] | *on* [.20] | *near* [.15] | *through* [.05]
Nominal → Noun	[.75]	
Nominal → Nominal Noun	[.20]	
Nominal → Nominal PP	[.05]	
VP → Verb	[.35]	
VP → Verb NP	[.20]	
VP → Verb NP PP	[.10]	
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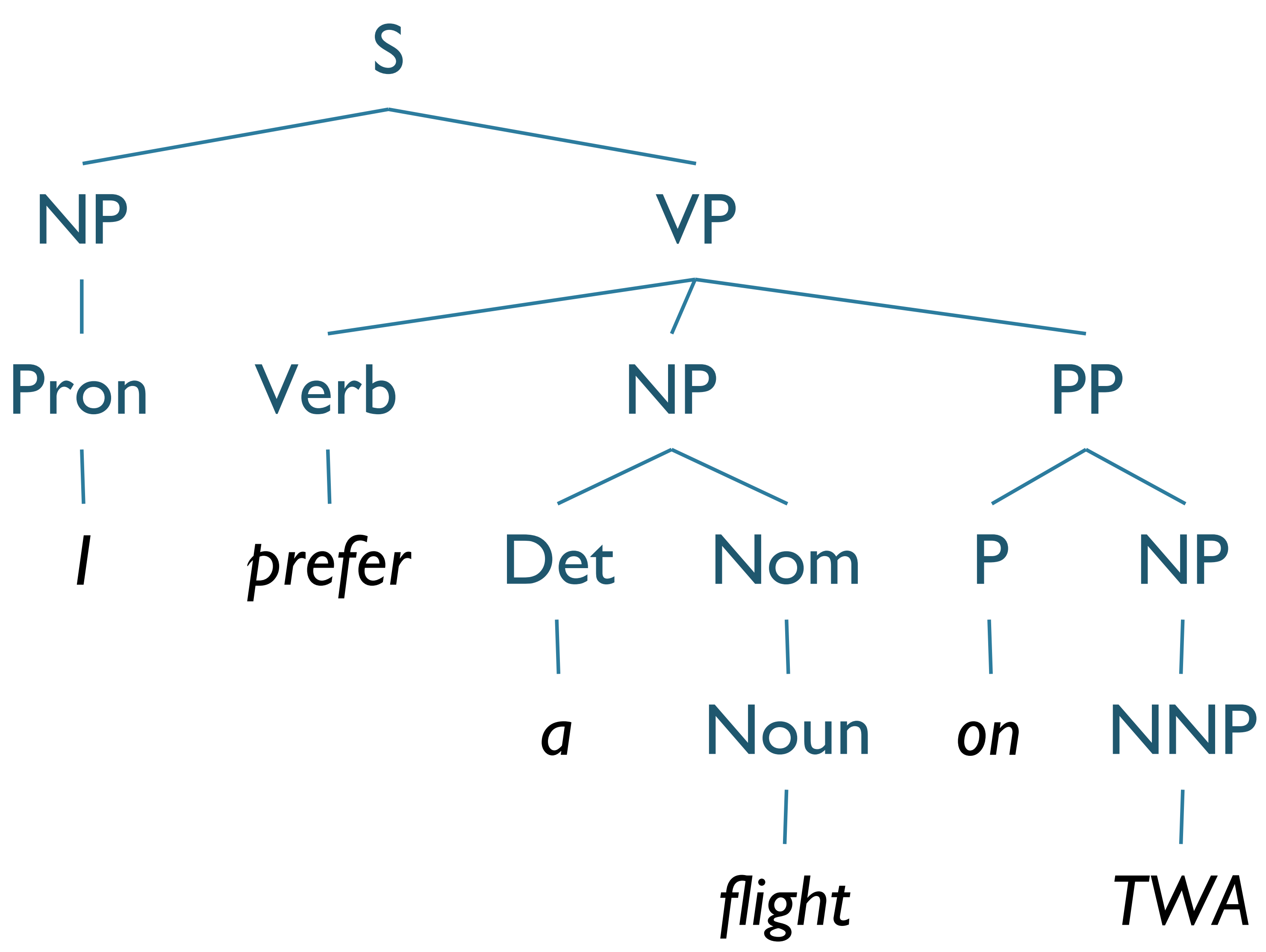
VP → *VP PP*	[.15]	
PP → *Preposition NP*	[1.0]	

Disambiguation

- A PCFG assigns probability to each parse tree *T* for input *S*
- Probability of *T*: product of all rules used to derive *T*

$$P(T, S) = \prod_{i=1}^{n} P(RHS_i \mid LHS_i)$$

$$P(T, S) = P(T)P(S \mid T) = P(T)$$



S → NP VP	[0.8]
NP → Pron	[0.35]
Pron → I	[0.4]
VP → V NP PP	[0.1]
V → prefer	[0.4]
NP → Det Nom	[0.2]
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Nom → N	[0.75]
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PP → P NP	[1.0]
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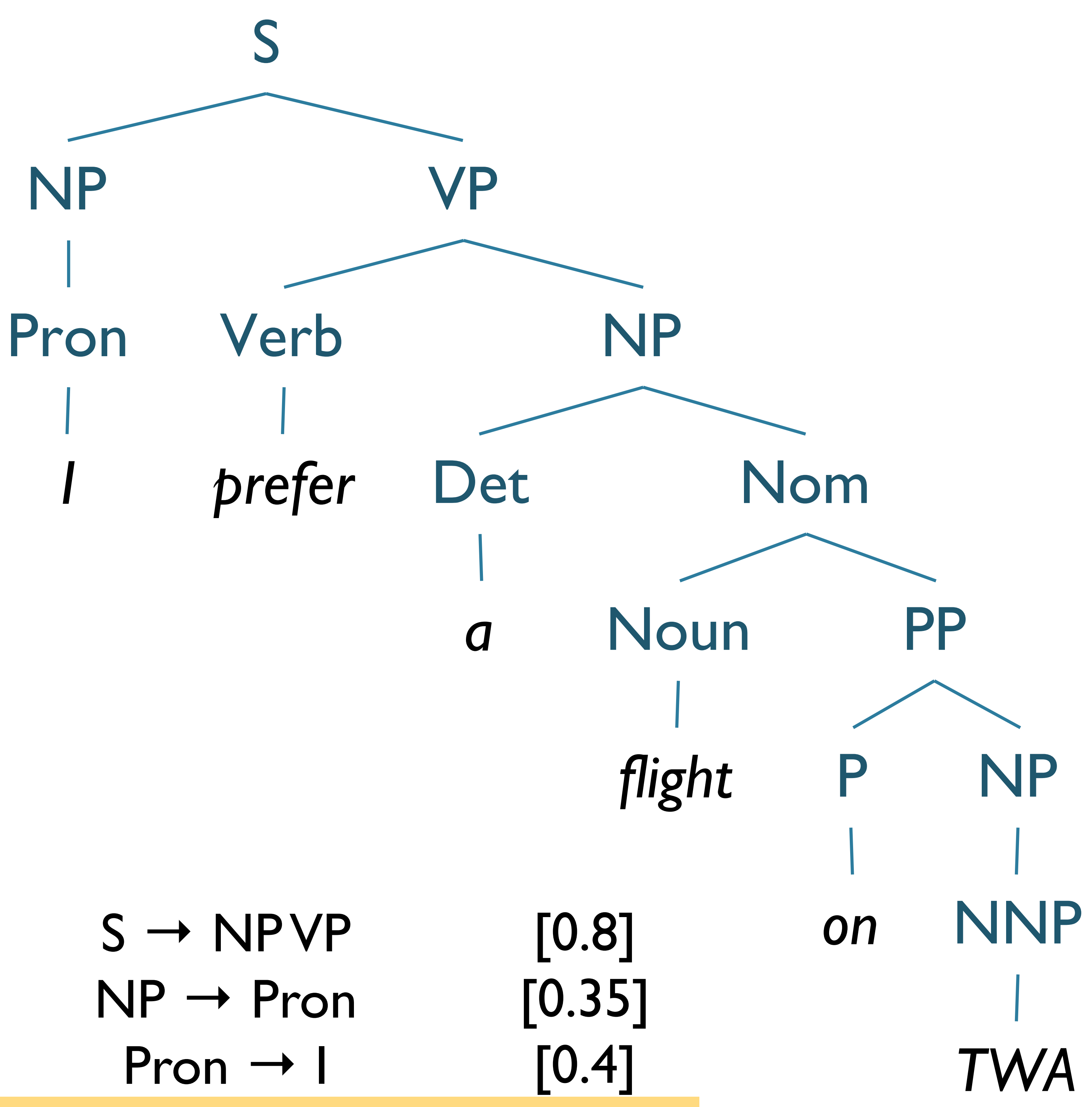

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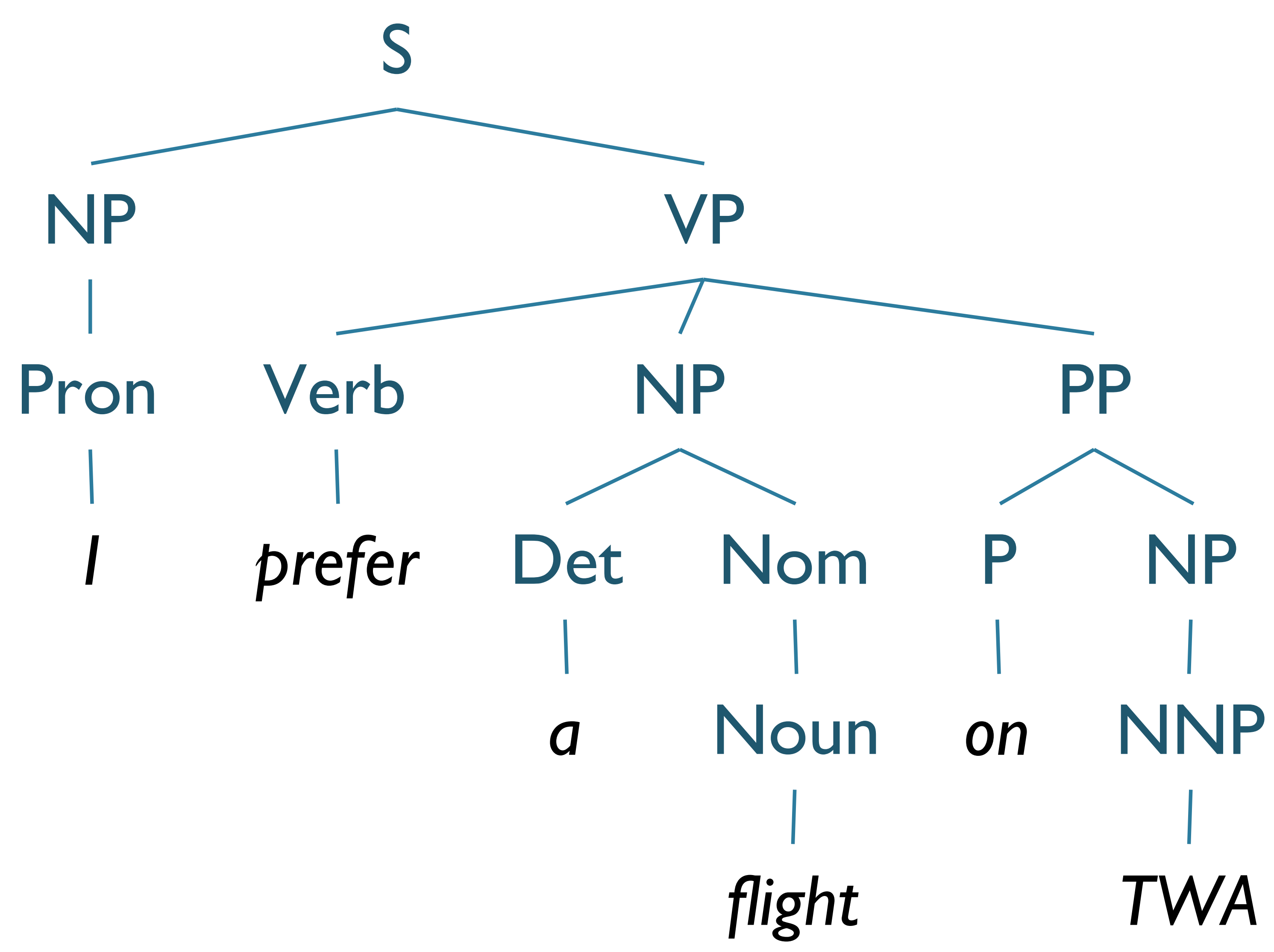

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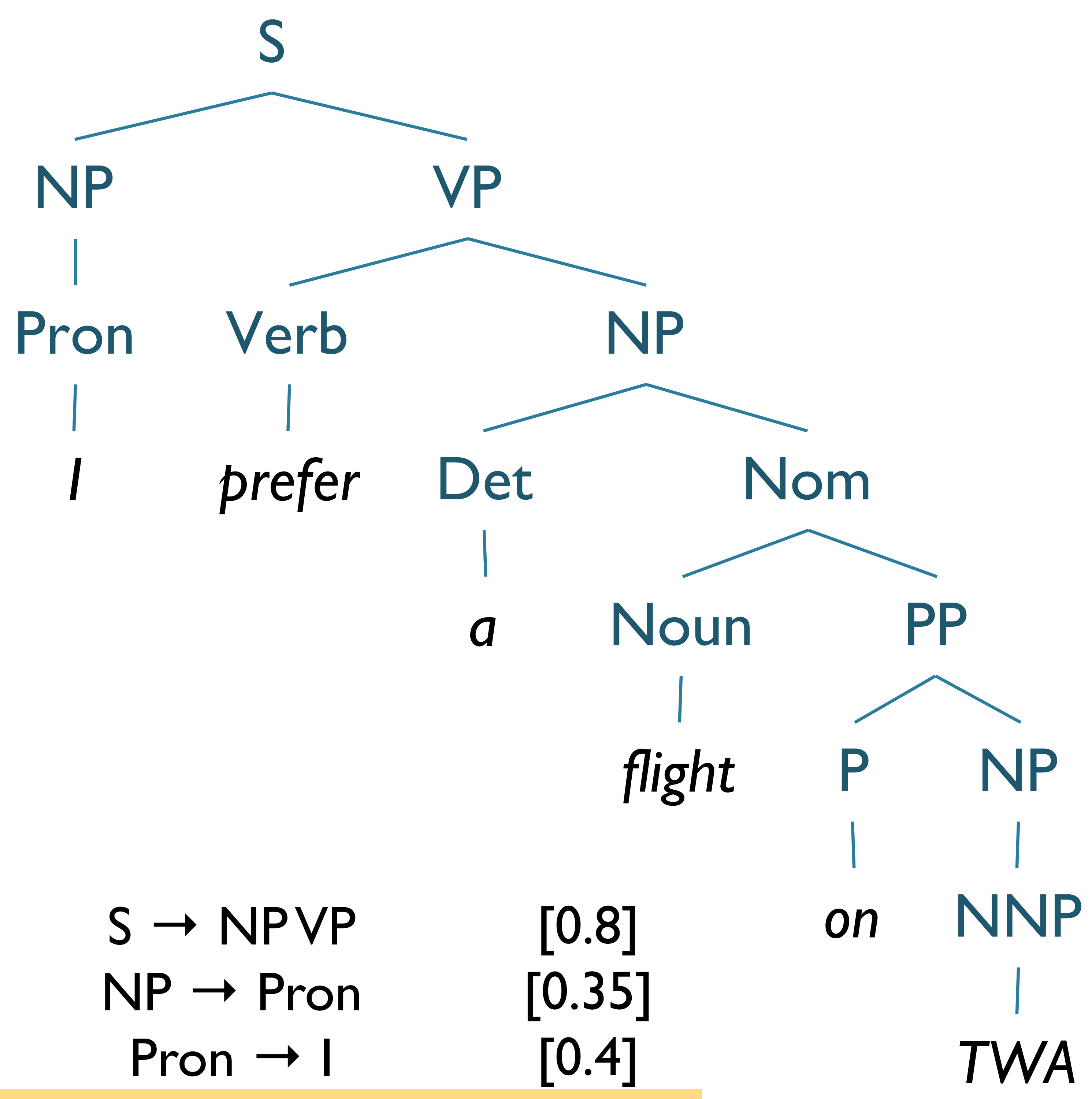
~1.452×10^{-6}



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P → on	[0.2]
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~1.452×10^{-7}

Parsing Problem for PCFGs

- Select ***T*** such that ***(s.t.)***

$$\hat{T}(S) = \underset{T\ s.t.\ S=yield(T)}{\operatorname{argmax}} P(T)$$

- String of words ***S*** is ***yield*** of parse tree
- Select the tree $\hat{T}$ that maximizes the probability of the parse

Application: Language Modeling

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- PCFGs are able to give probability of entire string without as bad sparsity
- Model probability of *syntactically valid* sentences
  - Not just probability of sequence of words

PCFGs: Parsing

Probabilistic CKY (PCKY)

- Like regular CKY
  - Assumes grammar in Chomsky Normal Form (CNF)
    - $A \rightarrow B\ C$
    - $A \rightarrow \mathrm{w}$
  - Represent input with indices b/t words:
    - $_0$ Book $_1$ that $_2$ flight $_3$ through $_4$ Houston $_5$

Probabilistic CKY (PCKY)

- For input string length *n* and non-terminals *V*
  - Cell [*i*, *j*, *A*] in (*n*+1) × (*n*+1) × *V* matrix
  - Contains probability that *A* spans *[i, j]*

PCKY Algorithm

function PROBABILISTIC-CKY-PARSE(*words*, *grammar*) **returns** most probable parse and its probability

for j ← **from** 1 to LENGTH(*words*) **do**

for all { *A* | *A* → *words[j]* ∈ *grammar* }

table[*j*–1, *j*, *A*] ← *P*(*A* → *words[j]*)

for *i* ← **from** *j*–2 **downto** 0 **do**

for *k* ← *i* + 1 **to** *j*–1 **do**

for all { *A* | *A* → *B C* ∈ *grammar*,

and *table*[*i*, *k*, *B*] > 0 **and** *table*[*k*, *j*, *C*] > 0 }

if (*table*[*i*, *j*, *A*] < *P*(*A* → *BC*)×*table*[*i*, *k*, *B*]×*table*[*k*,*j*,*C*]) **then**

table[*i*, *j*, *A*] ← *P*(*A* → *BC*)×*table*[*i*,*k*,*B*]×*table*[*k*,*j*,*C*]

back[*i*, *j*, *A*] ← { *k*, *B*, *C* }

return BUILD_TREE(*back*[1, LENGTH(*words*), *S*]), *table*[1,LENGTH(*words*), *S*]

PCKY Algorithm

```
function PROBABILISTIC-CKY-PARSE(words, grammar) returns most probable parse and its probability
for j ← from 1 to LENGTH(words) do
 for all { A | A → words[j] ∈ grammar }
      table[ j–1, j, A ] ← P(A → words[j])
  for i ← from j–2 downto 0 do
   for k ← i + 1 to j–1 do
   for all { A | A → B C ∈ grammar,
        and table[i, k, B] > 0 and table[ k, j, C ] > 0 }
   if (table[ i, j, A ] < P(A → BC )×table[ i, k, B ]×table[ k,j,C ]) then
      table[ i, j, A ] ← P(A → BC )×table[i,k,B]×table[k,j,C]
      back[ i, j, A ] ← { k, B, C }
   return BUILD_TREE(back[ 1, LENGTH(words), S ]), table[ 1,LENGTH(words), S ]
```

PCKY Algorithm

```
function PROBABILISTIC-CKY-PARSE(words, grammar) returns most probable parse and its probability
for j ← from 1 to LENGTH(words) do
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PCKY Algorithm

function PROBABILISTIC-CKY-PARSE(*words*, *grammar*) **returns** most probable parse and its probability

for j ← **from** 1 to LENGTH(*words*) **do**

for all { *A* | *A* → *words[j]* ∈ *grammar* }

table[*j*–1, *j*, *A*] ← *P*(*A* → *words[j]*)

for *i* ← **from** *j*–2 **downto** 0 **do**

for *k* ← *i* + 1 **to** *j*–1 **do**

for all { *A* | *A* → *B C* ∈ *grammar*,

and *table*[*i*, *k*, *B*] > 0 **and** *table*[*k*, *j*, *C*] > 0 }

if (*table*[*i*, *j*, *A*] < *P*(*A* → *BC*)×*table*[*i*, *k*, *B*]×*table*[*k*,*j*,*C*]) **then**

table[*i*, *j*, *A*] ← *P*(*A* → *BC*)×*table*[*i*,*k*,*B*]×*table*[*k*,*j*,*C*]

back[*i*, *j*, *A*] ← { *k*, *B*, *C* }

return BUILD_TREE(*back*[1, LENGTH(*words*), *S*]), *table*[1,LENGTH(*words*), *S*]

PCKY Grammar Segment

S → *NP VP*	[0.80]		*Det* → the	[0.40]
NP → *Det N*	[0.30]		*Det* → a	[0.40]
VP → *V NP*	[0.20]		*V* → includes	[0.05]
			N → meal	[0.01]
			N → flight	[0.02]

PCKY Matrix

$S \rightarrow NP\ VP$ [0.80]
$NP \rightarrow Det\ N$ [0.30]
$VP \rightarrow V\ NP$ [0.20]

$Det \rightarrow$ the [0.40]
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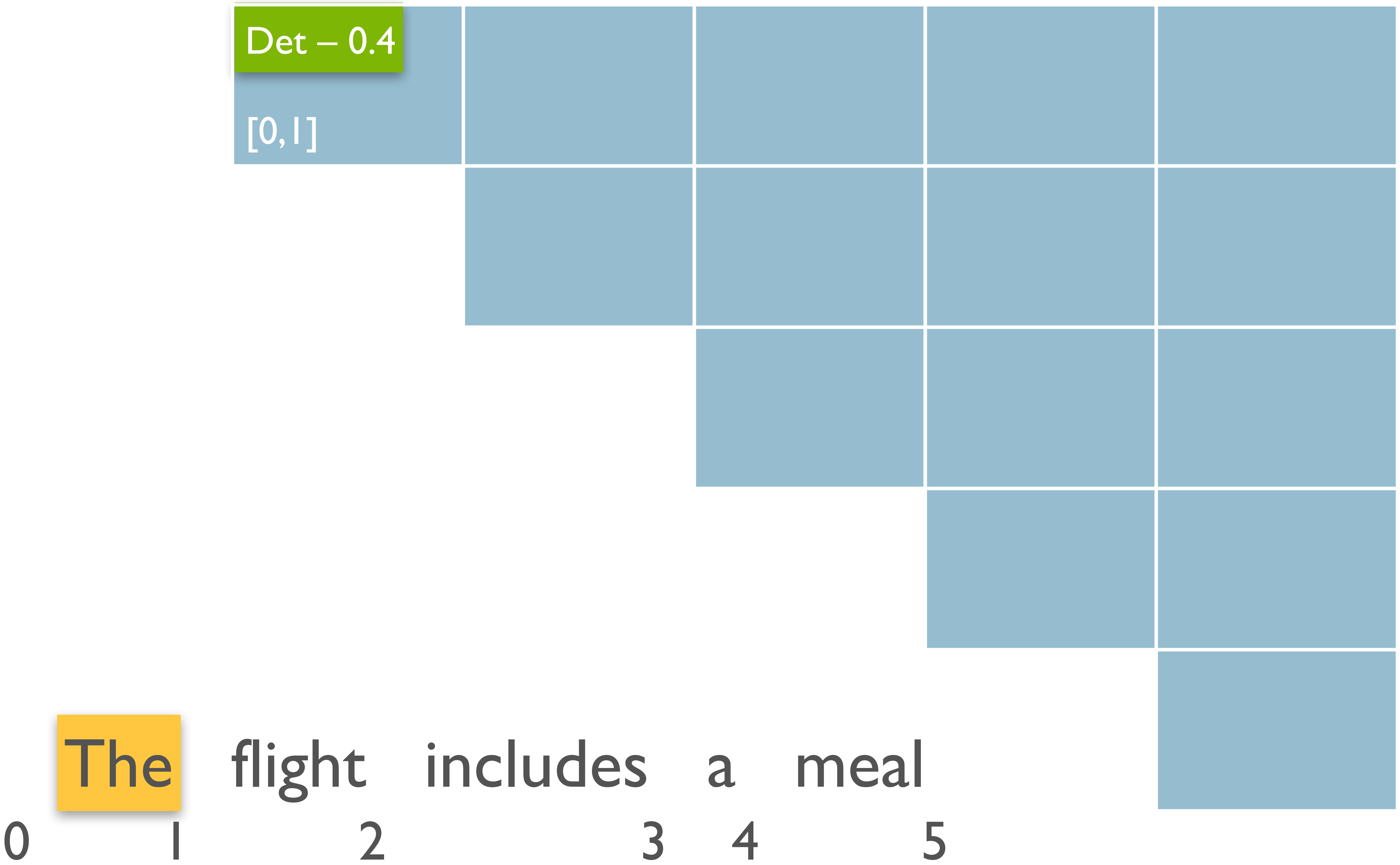

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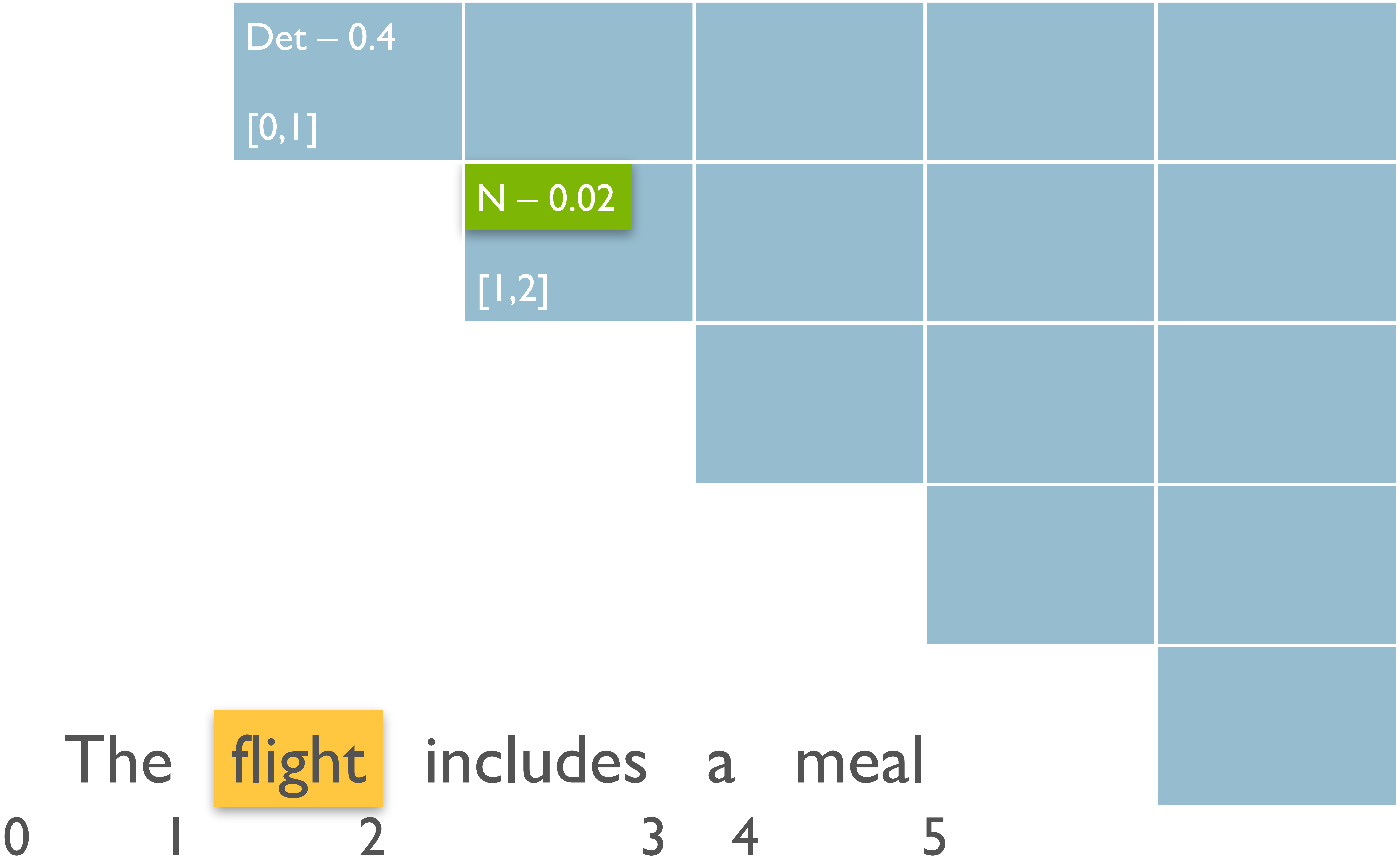

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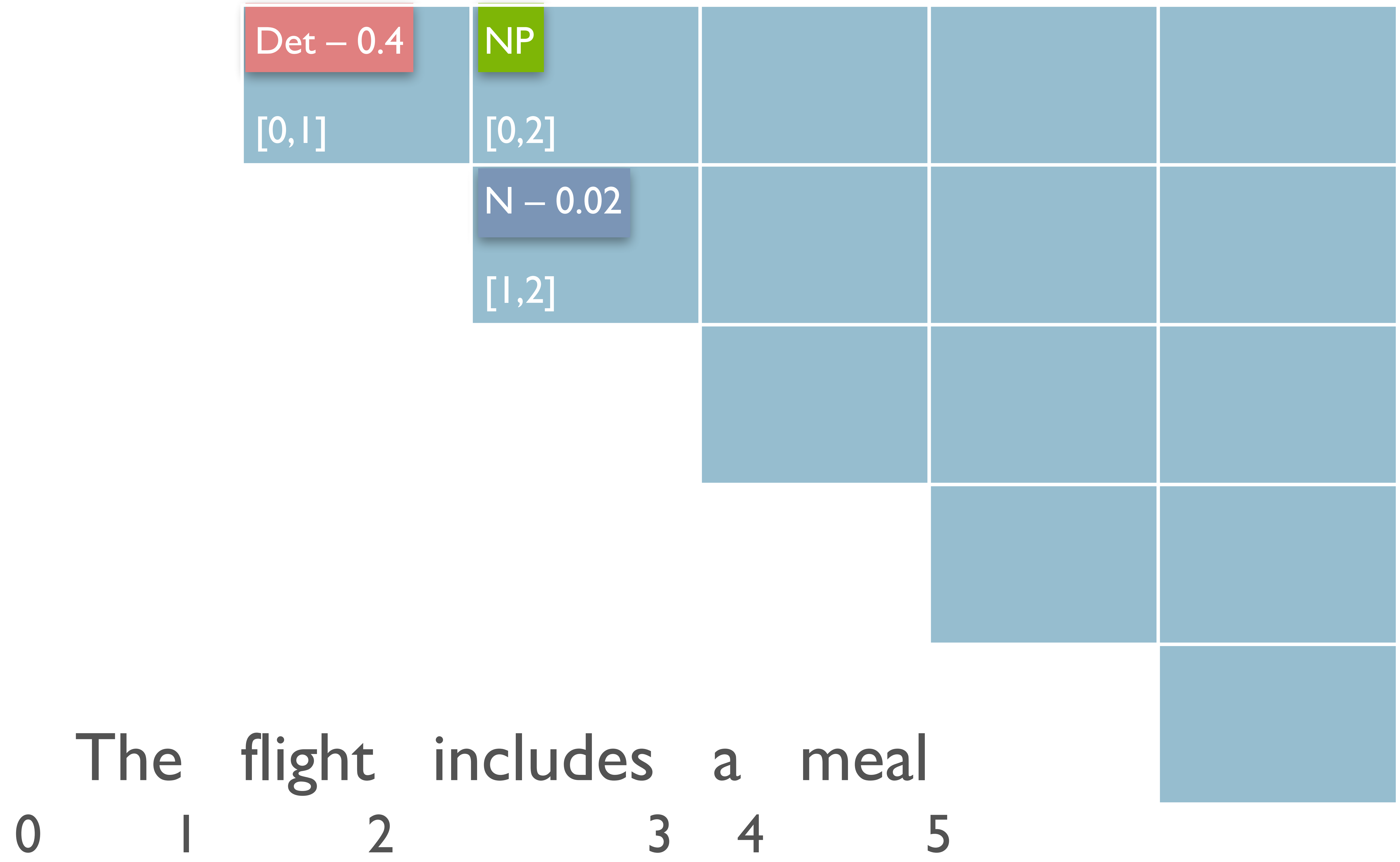

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$$P = P(NP \rightarrow Det\ N) \cdot P(Det \rightarrow \text{the}) \cdot P(N \rightarrow \text{flight})$$

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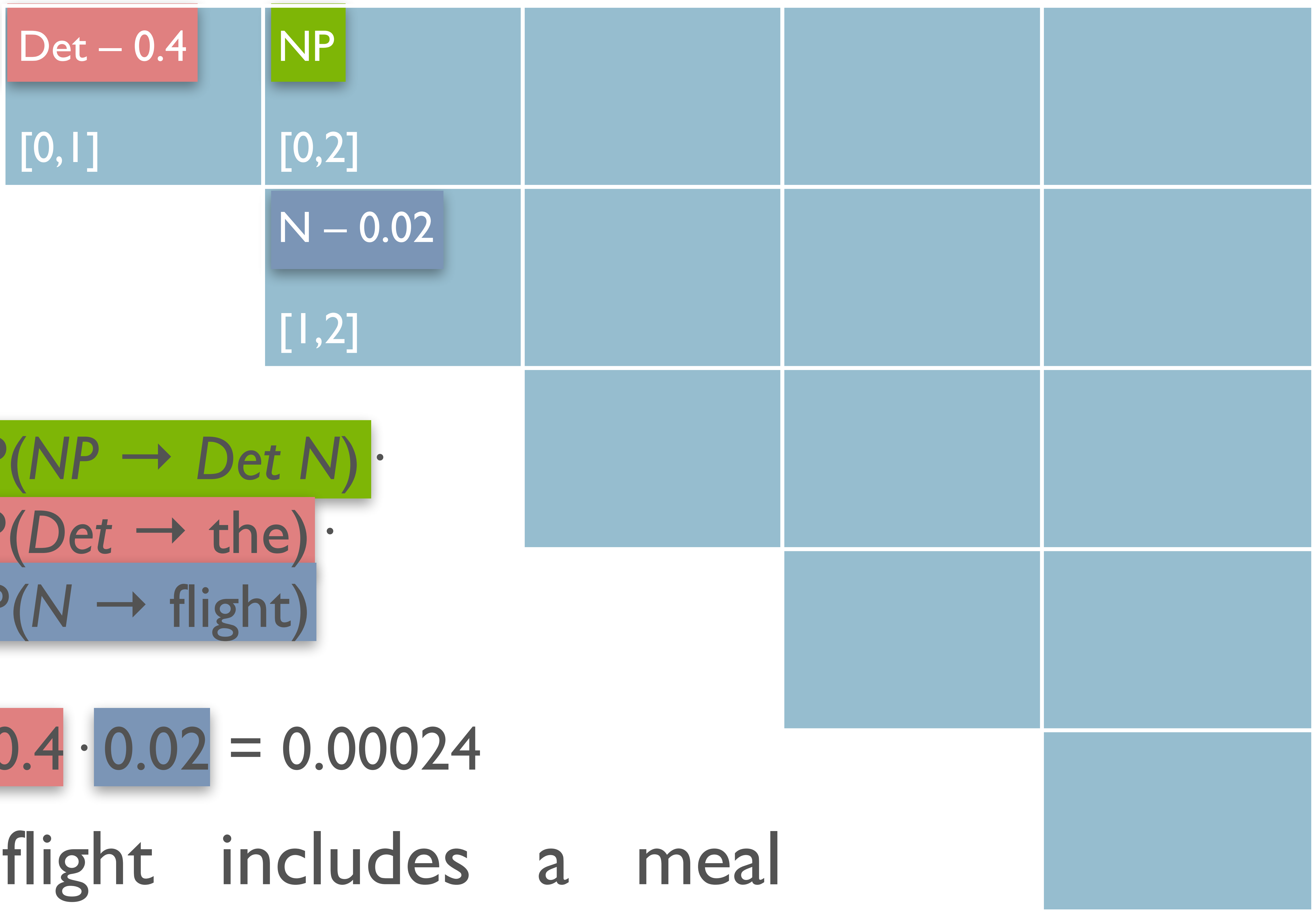

$$P = P(NP \rightarrow Det\ N) \cdot P(Det \rightarrow \text{the}) \cdot P(N \rightarrow \text{flight})$$

$$P = 0.3 \cdot 0.4 \cdot 0.02 = 0.00024$$

The flight includes a meal
0 1 2 3 4 5

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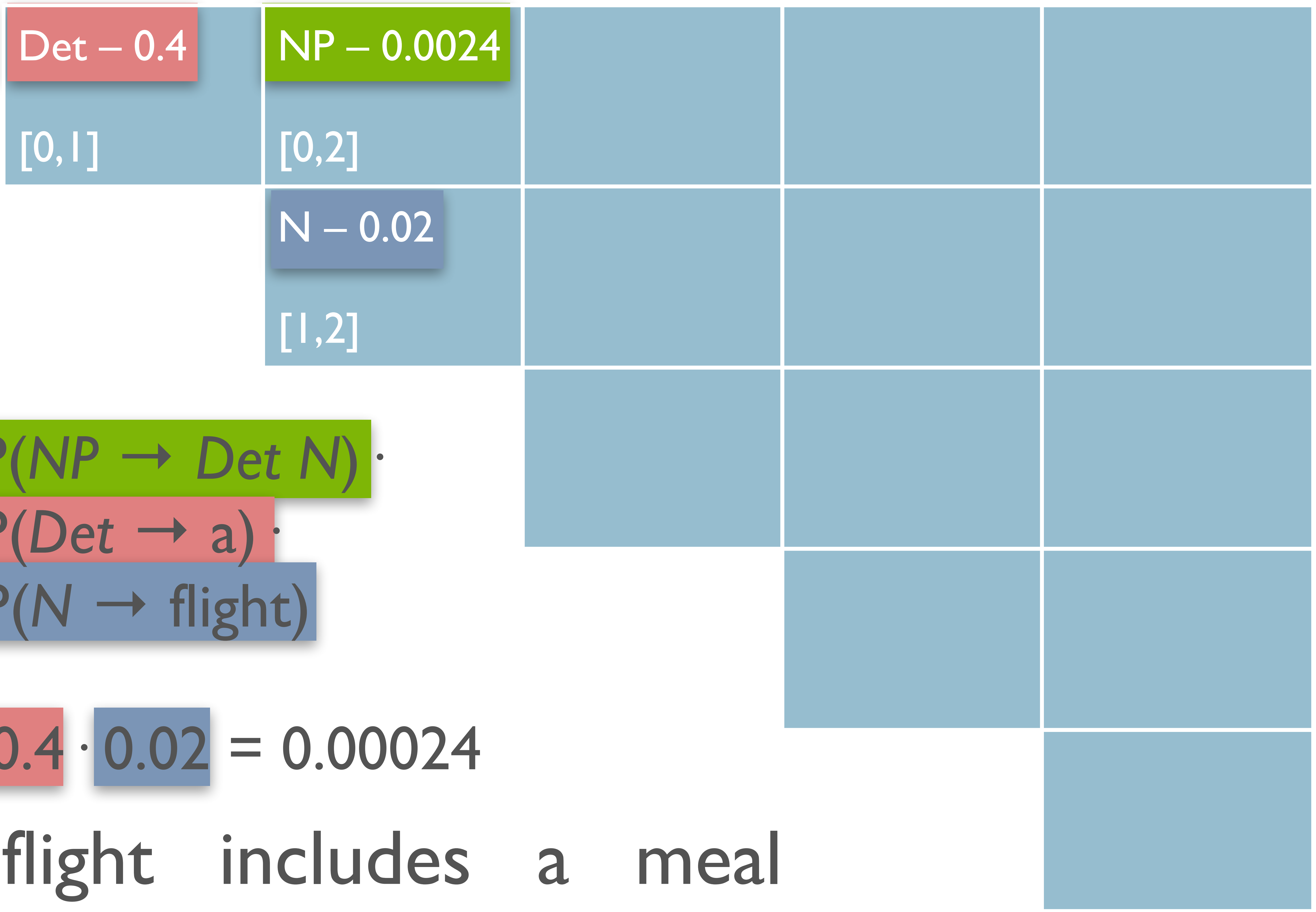

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Det → a [0.40]
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N → meal [0.01]
N → flight [0.02]

Det – 0.4 [0,1]	NP – 0.0024 [0,2]	[0,3]	[0,4]	S – 2.304×10^{-8} [0,5]
	N – 0.02 [1,2]	[1,3]	[1,4]	[1,5]
		V – 0.05 [2,3]	[2,4]	VP – 1.2×10^{-5} [2,5]
			Det – 0.4 [3,4]	NP – 0.0012 [3,5]
				N – 0.01 [4,5]

The flight includes a meal
0 1 2 3 4 5

Inducing a PCFG

Learning Probabilities

- Simplest way:
  - Use treebank of parsed sentences

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$$P(\alpha \rightarrow \beta \mid \alpha) = \frac{Count(\alpha \rightarrow \beta)}{\sum_\gamma Count(\alpha \rightarrow \gamma)} = \frac{Count(\alpha \rightarrow \beta)}{Count(\alpha)}$$

Learning Probabilities

- Simplest way:
  - Use treebank of parsed sentences
  - To compute probability of a rule, count:
    - Number of times a nonterminal is expanded: Σ_γ *Count*$(\alpha \rightarrow \gamma)$
    - Number of times a nonterminal is expanded by a given rule: *Count*$(\alpha \rightarrow \beta)$

$$P(\alpha \rightarrow \beta \mid \alpha) = \frac{Count(\alpha \rightarrow \beta)}{\sum_\gamma Count(\alpha \rightarrow \gamma)} = \frac{Count(\alpha \rightarrow \beta)}{Count(\alpha)}$$

- Alternative: Learn probabilities by re-estimating
  - (Later)

Probabilistic Parser Development Paradigm

	Train	**Dev**	**Test**
Size	Large (eg. WSJ 2–21, 39,830 sentences)	Small (e.g. WSJ 22)	Small/Med (e.g. WSJ, 23, 2,416 sentences)
Usage	Estimate rule probabilities	Tuning/Verification, Check for Overfit	Held Out, Final Evaluation

Parser Evaluation

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- Assume a ‘gold standard’ set of parses for test set

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    - Constituents in output match those in reference

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- How can we tell how good the parser is?
- How can we tell how good a parse is?
  - Maximally strict: identical to ‘gold standard’
  - Partial credit:
    - Constituents in output match those in reference
      - Same start point, end point, non-terminal symbol

Parseval

- How can we compute parse score from constituents?
- Multiple Measures:

$$\text{Labeled Recall (LR)} = \frac{\text{\# of \textbf{correct} constituents in \textbf{hypothetical} parse}}{\text{\# of \textbf{total} constituents in \textbf{reference} parse}}$$

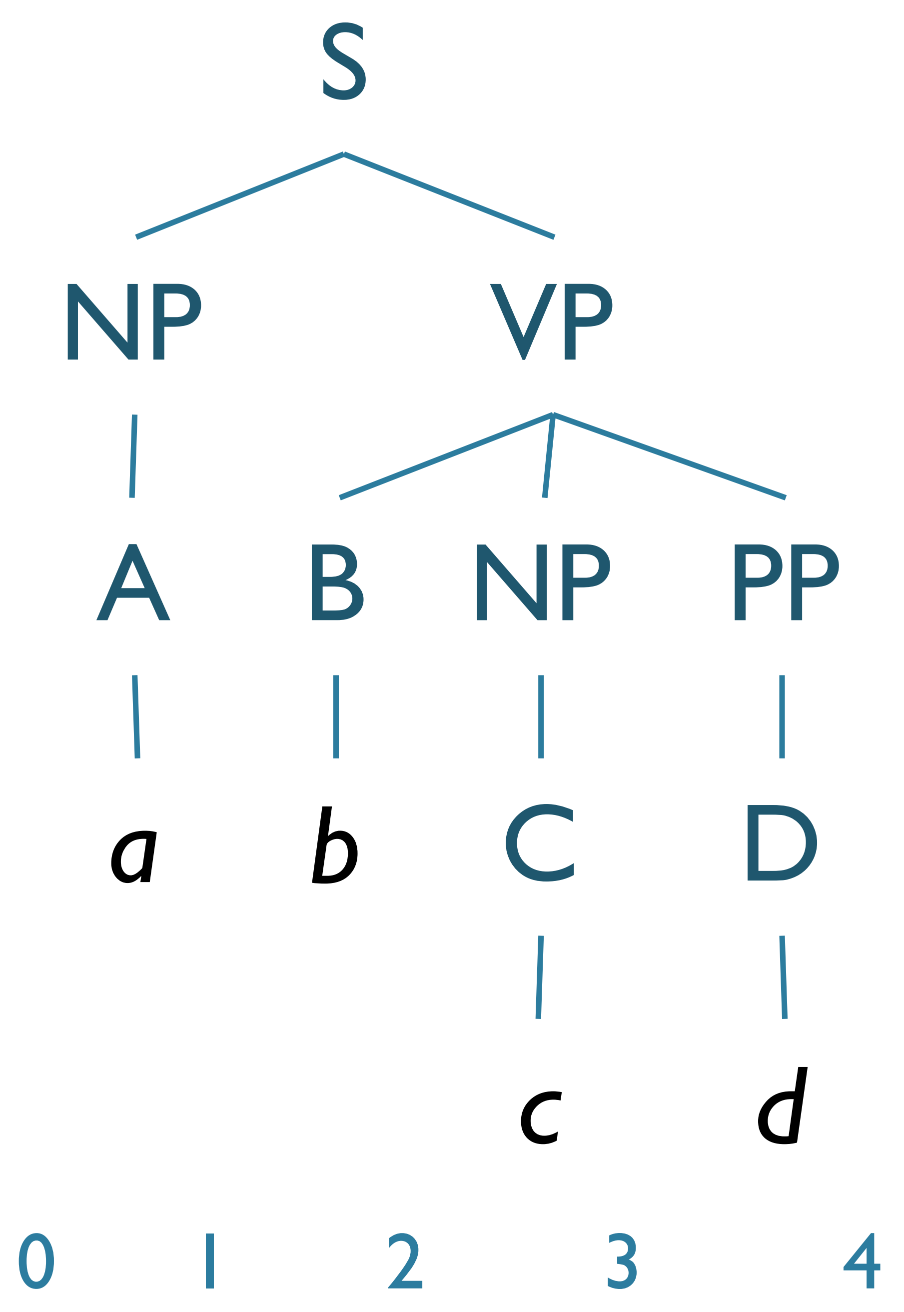
$$\text{Labeled Precision (LP)} = \frac{\text{\# of \textbf{correct} constituents in \textbf{hypothetical} parse}}{\text{\# of \textbf{total} consituents in \textbf{hypothetical} parse}}$$

Parseval

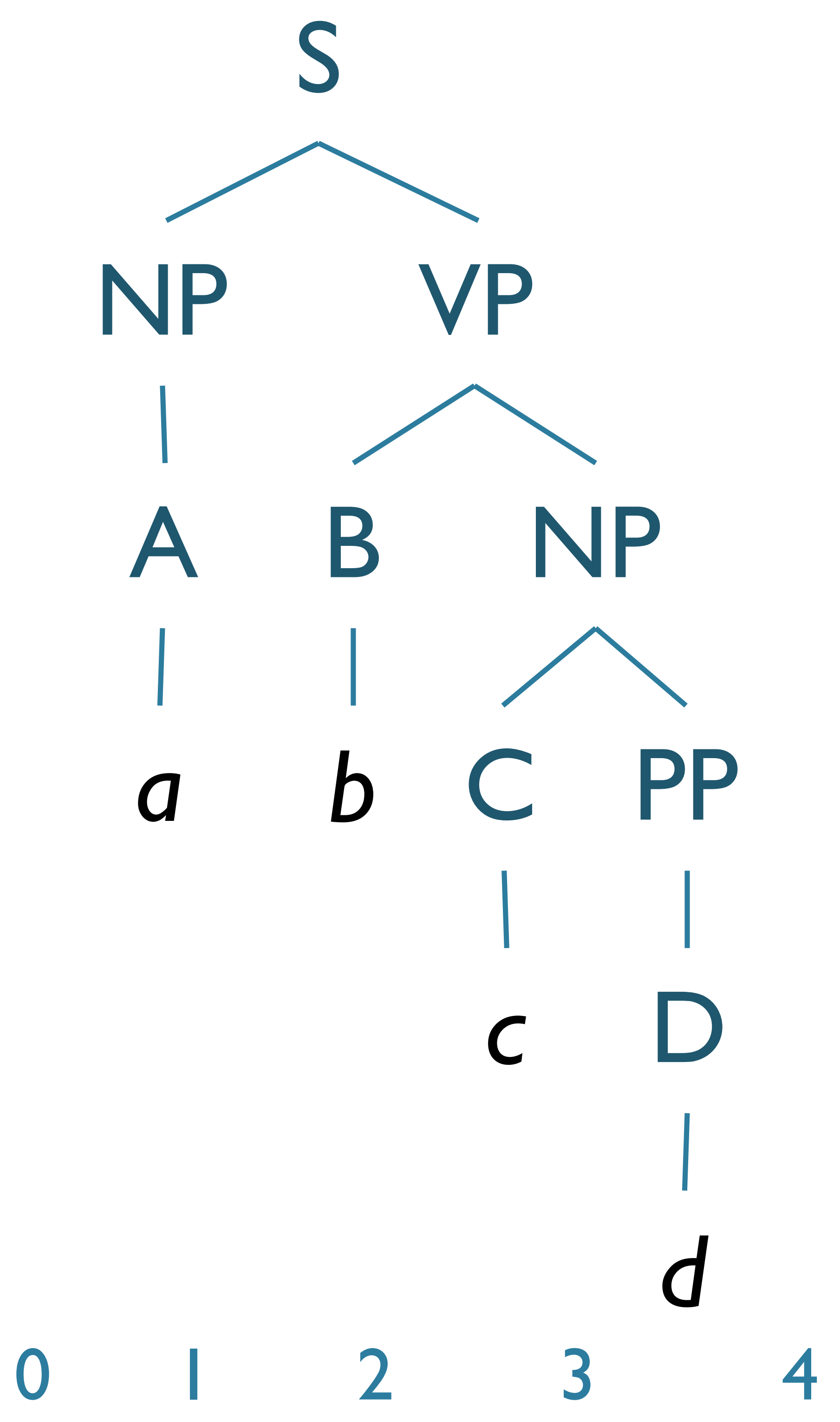
- **F-measure:**
  - Combines precision and recall
  - Let $\boldsymbol{\beta} \in \mathbb{R}$, $\boldsymbol{\beta} > 0$ that adjusts P vs. R s.t. $$\beta \propto \frac{R}{P}$$
  - F_β-measure is then: $$F_\beta = (1+\beta^2) \cdot \frac{P \cdot R}{\beta^2 \cdot P + R}$$
  - With F1-measure as $$F_1 = \frac{2PR}{P+R}$$

Evaluation: Example

Reference



Hypothesis



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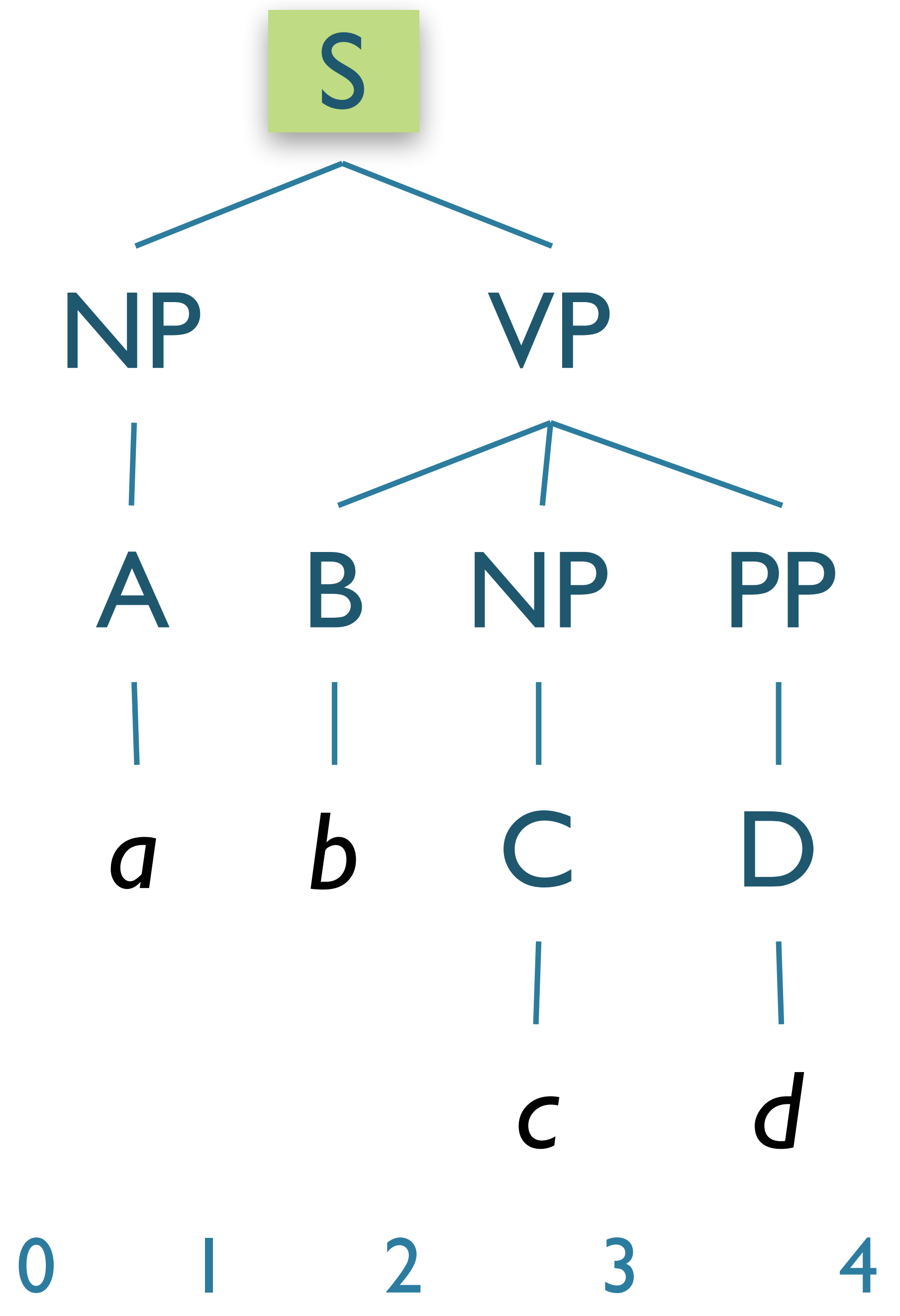
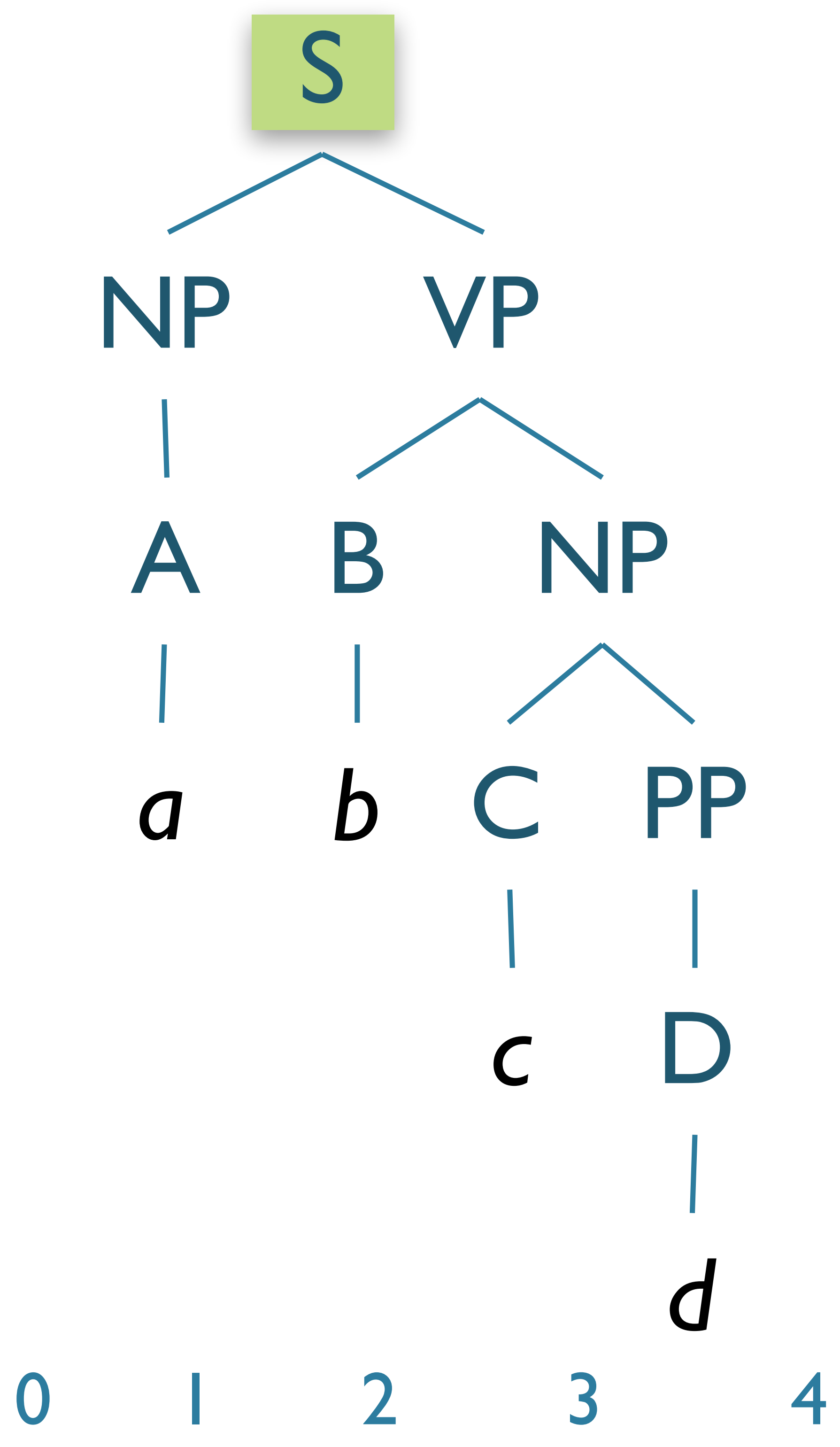


S(0,4)

Hypothesis



S(0,4)

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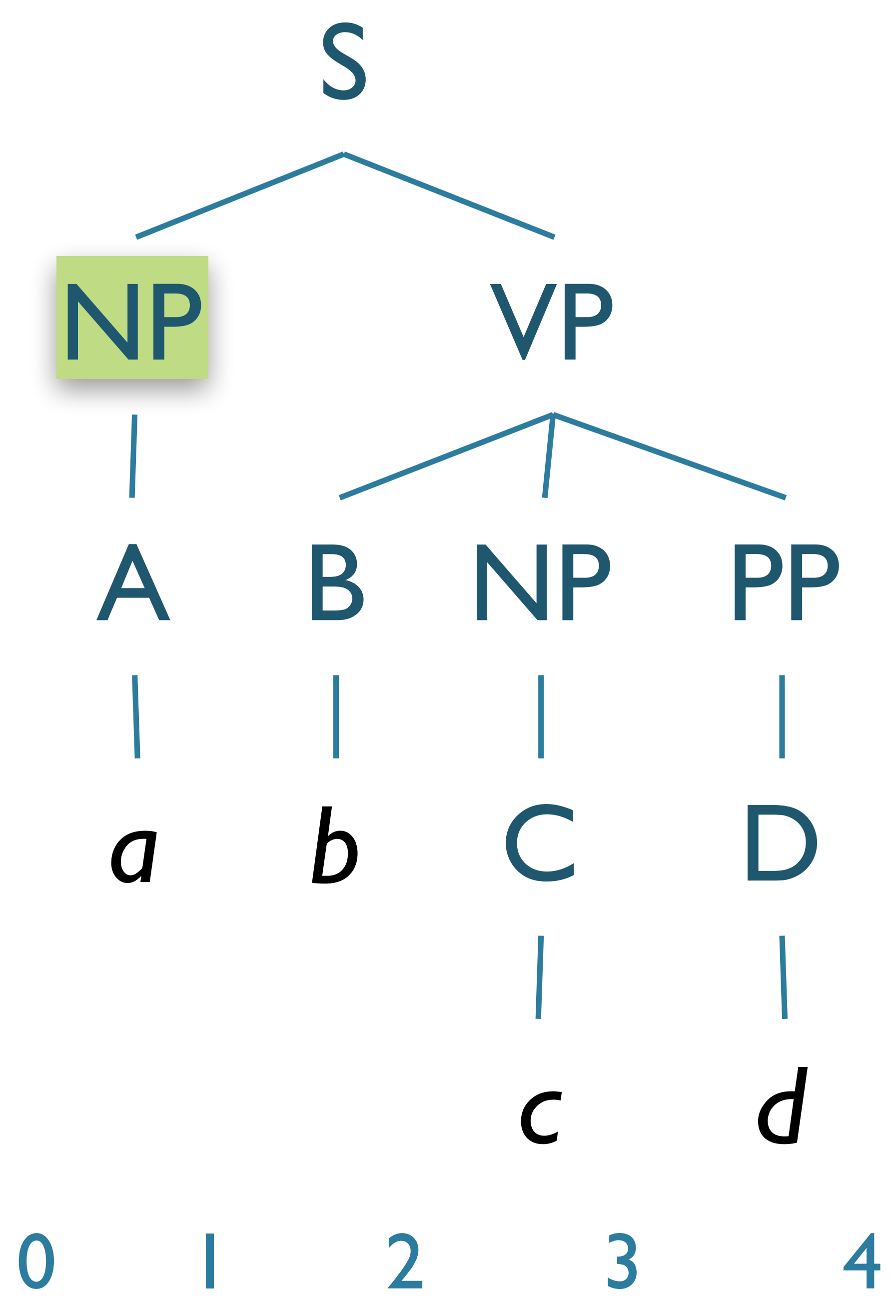

S(0,4)

NP(0,1)

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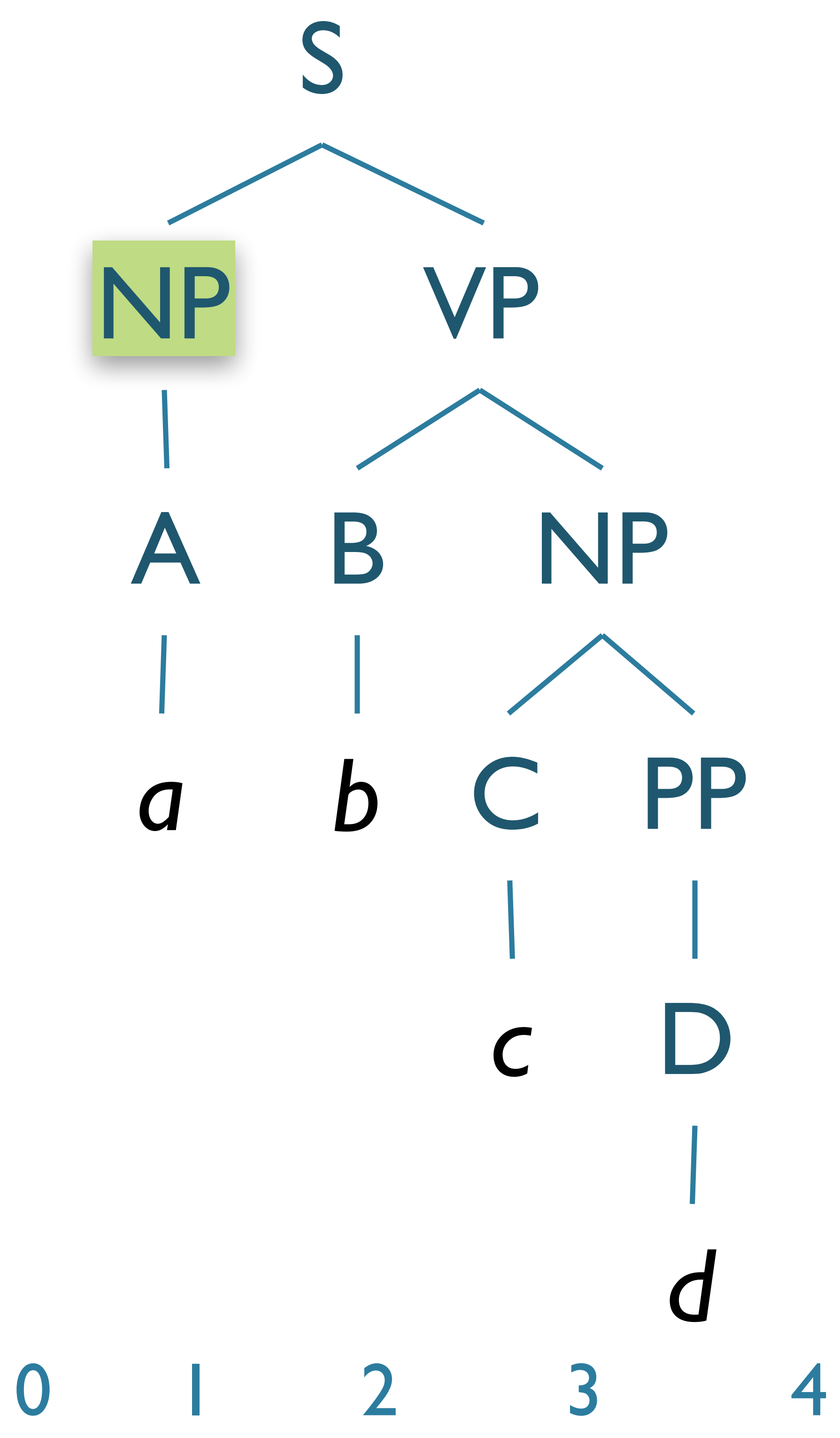

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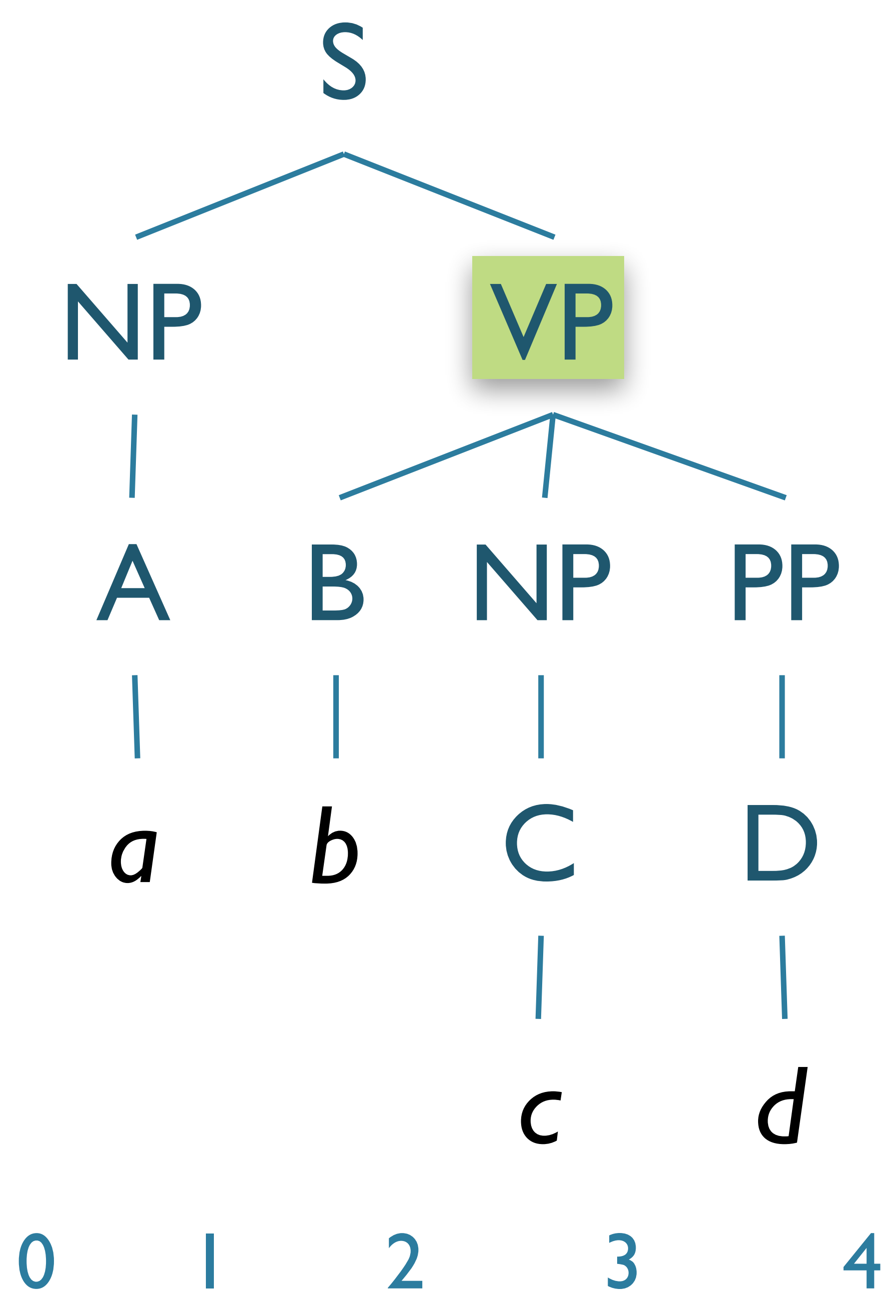
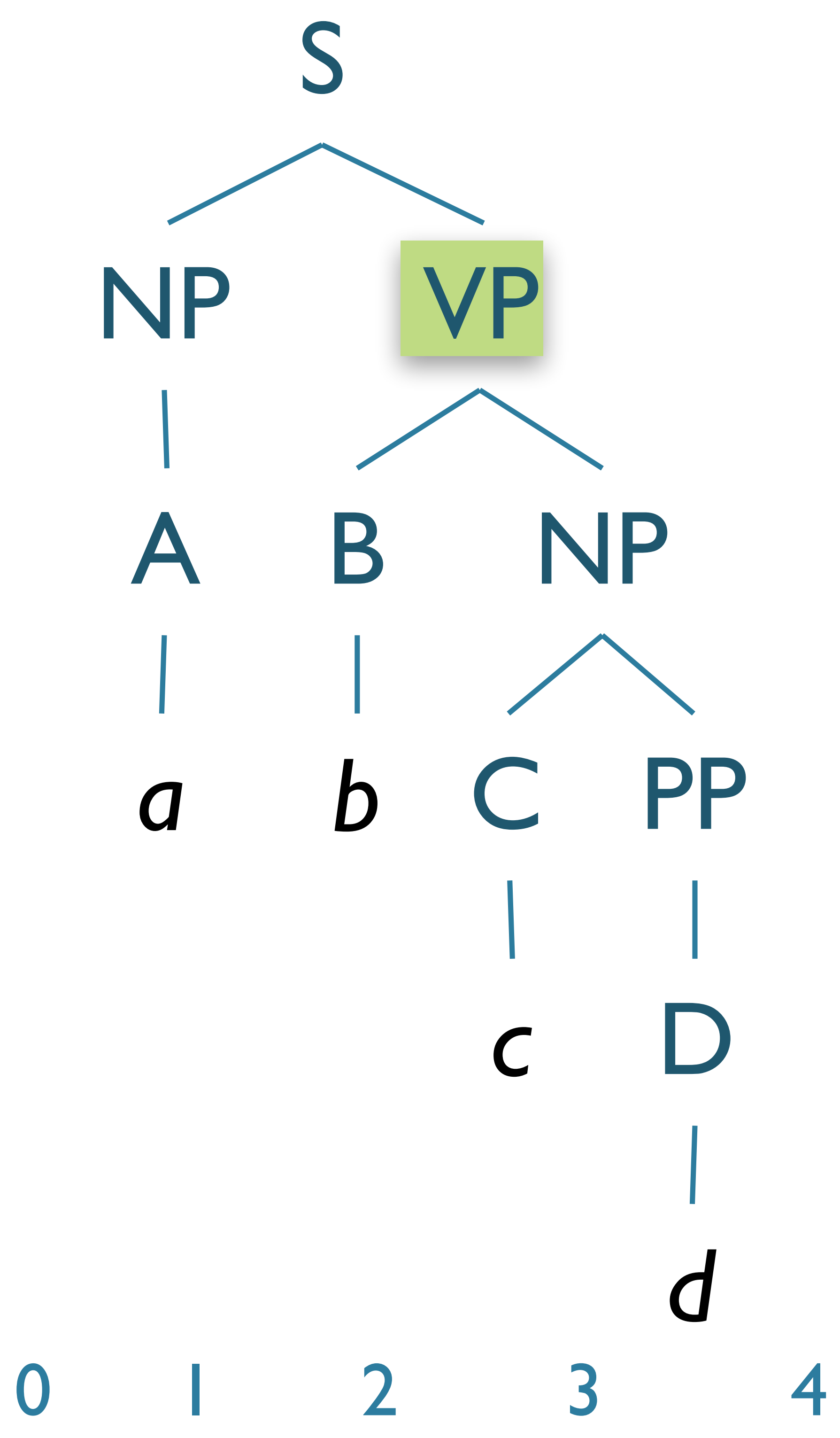


S(0,4)

NP(0,1)

VP(1,4)

Hypothesis



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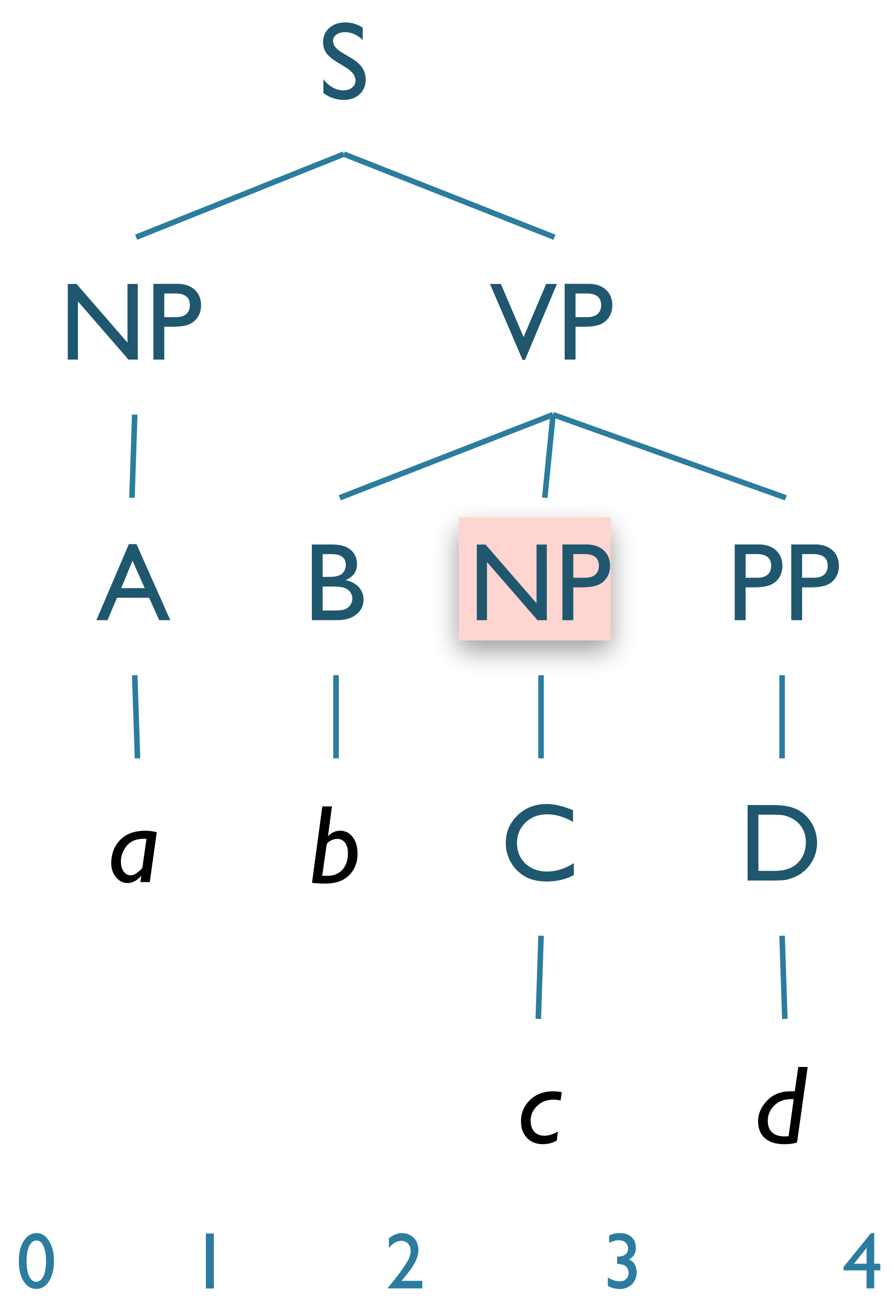

S(0,4)
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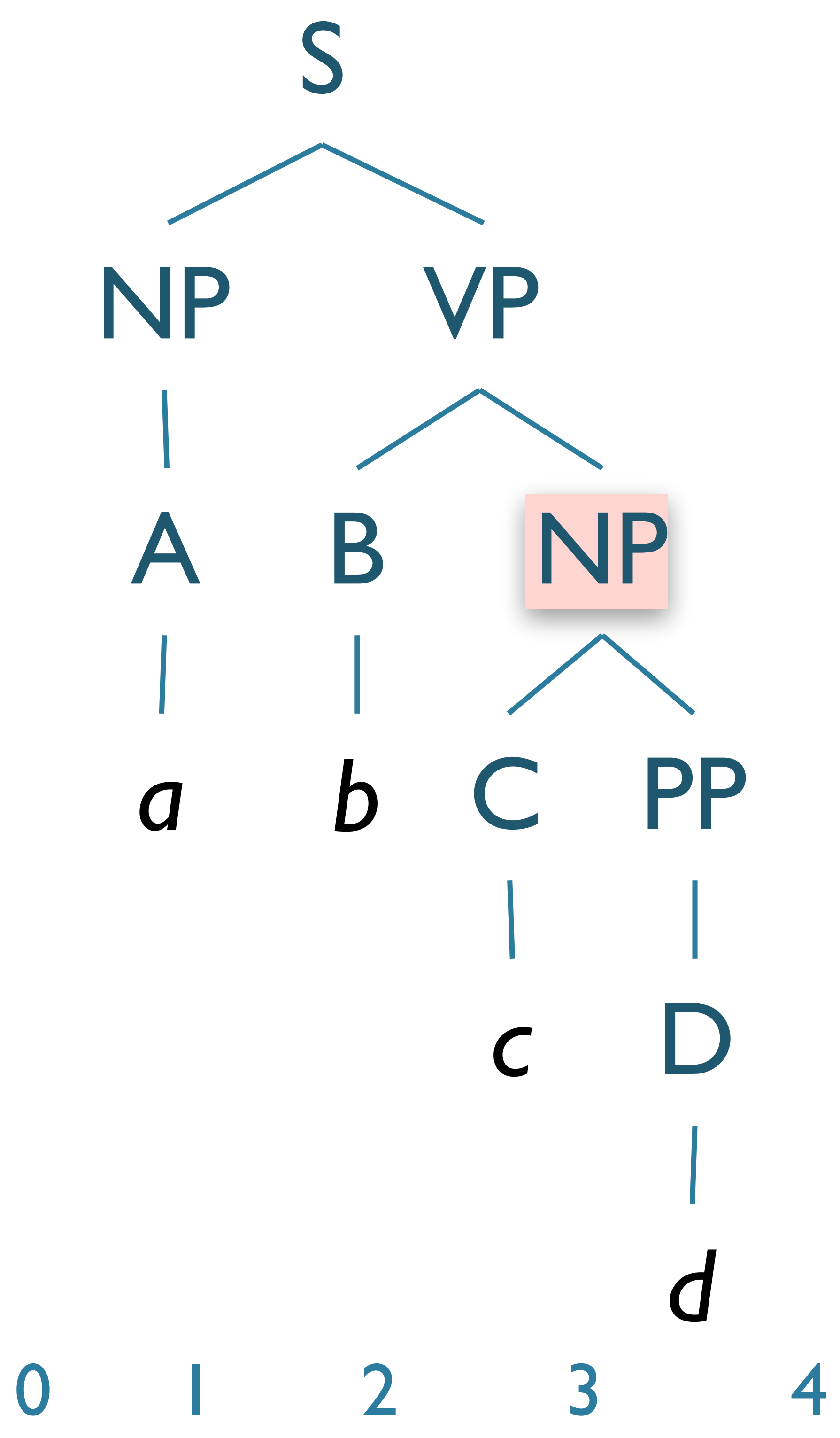
NP(2,3)

Hypothesis



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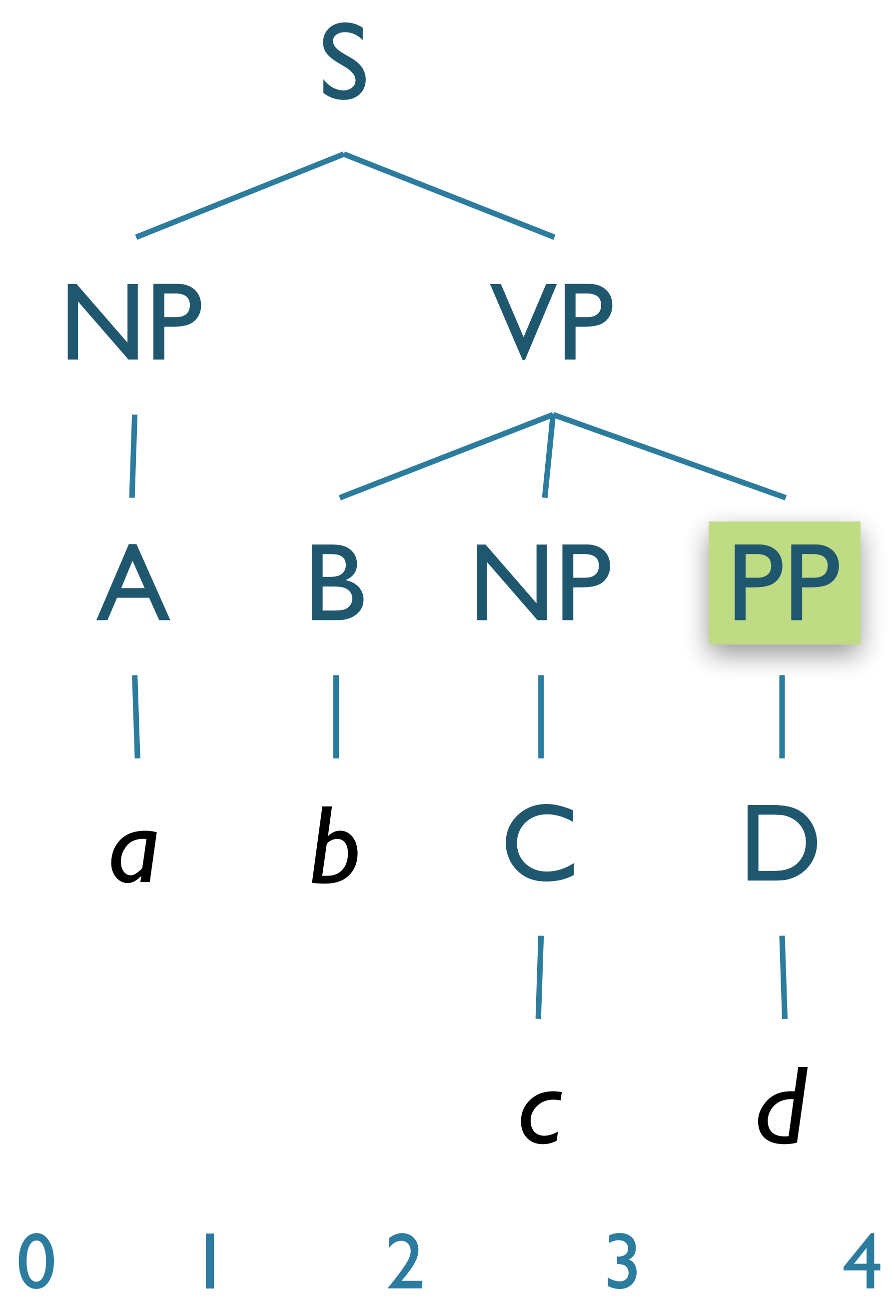

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PP(3,4)

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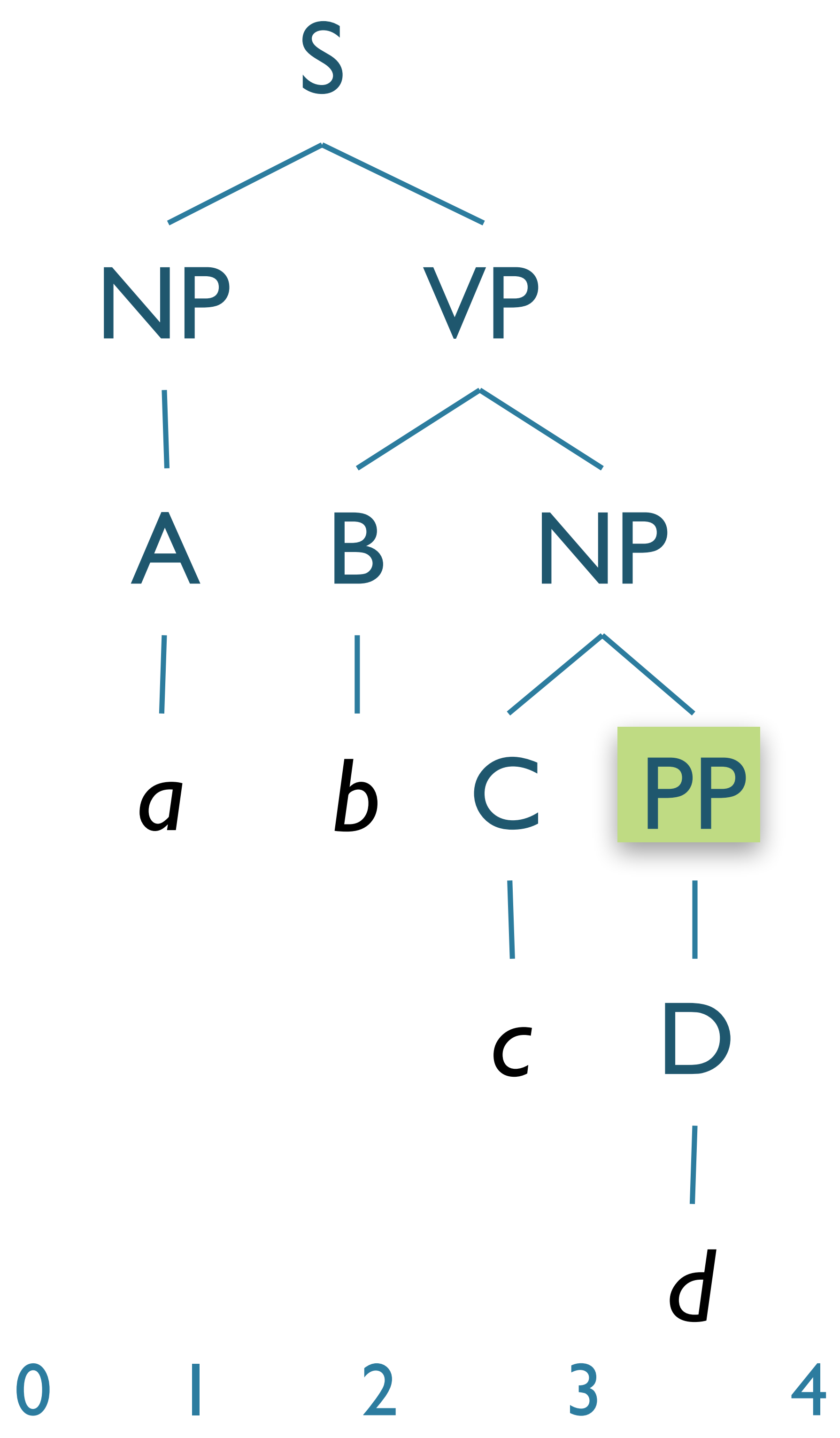

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Evaluation: Example

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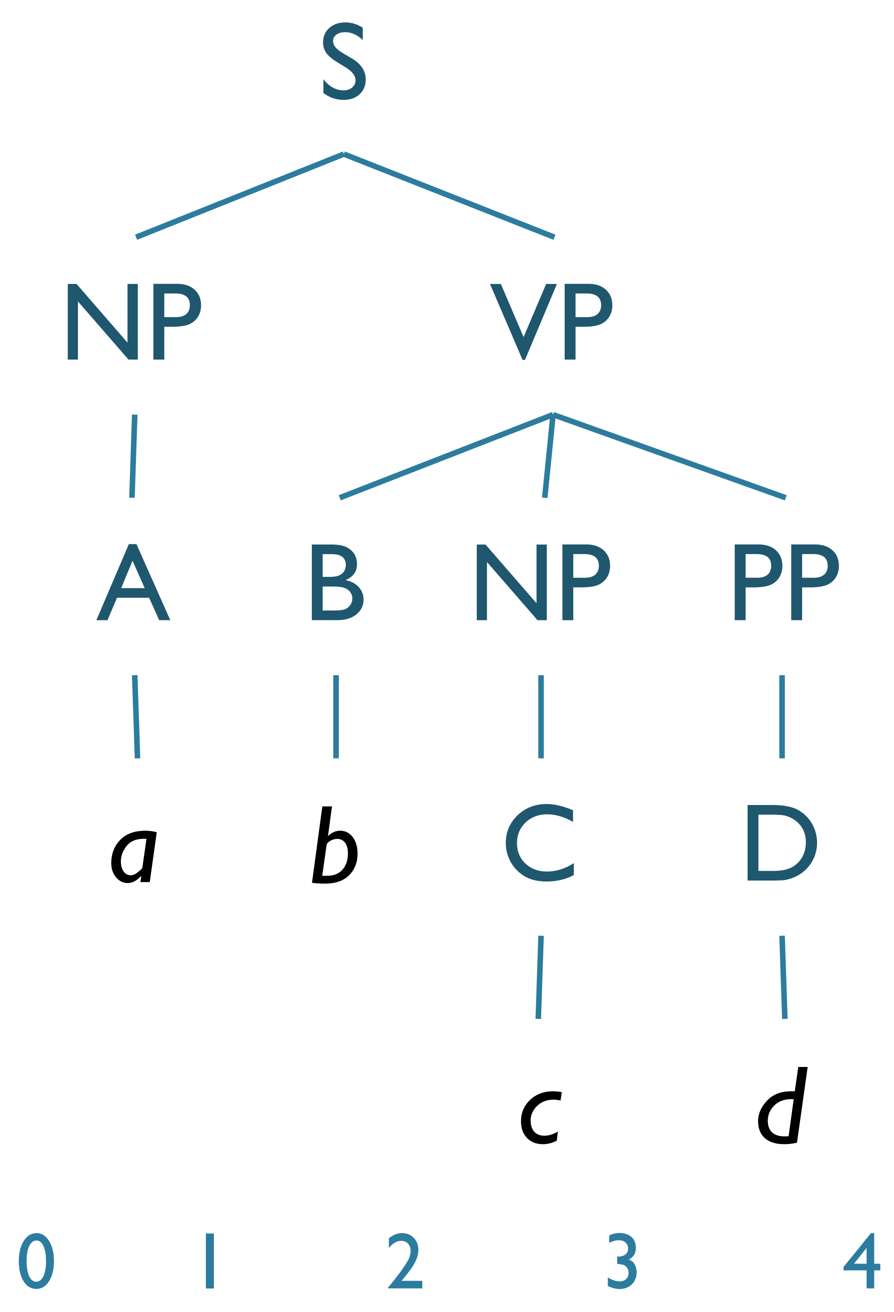

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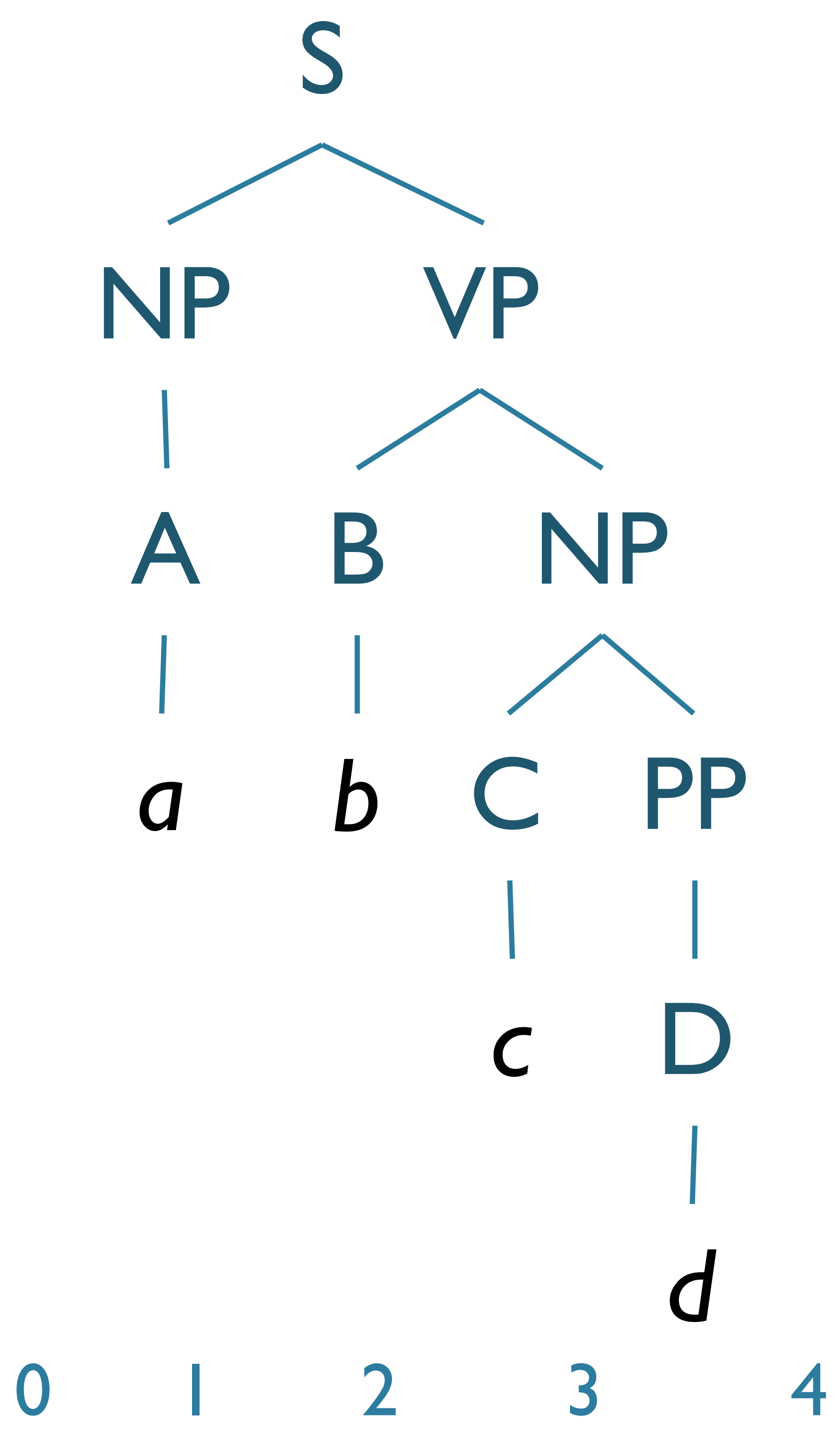

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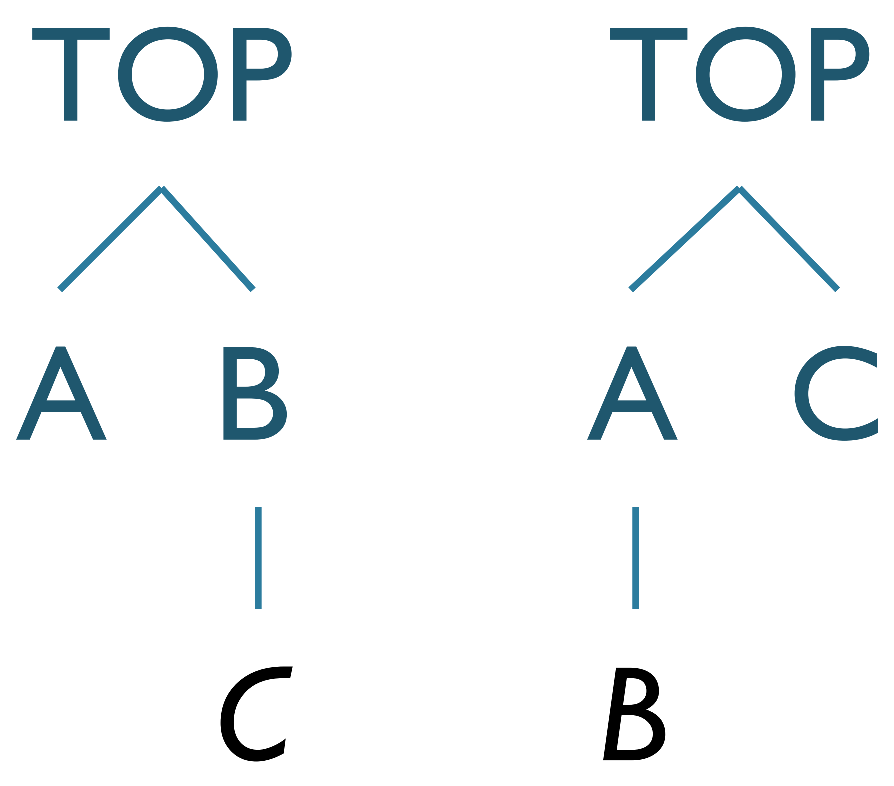
- NP(0,1)
- VP(1,4)
- NP(2,4)
- PP(3,4)

LP:	4/5
LR:	4/5
F_1:	4/5

Parser Evaluation

- Crossing Brackets:
  - # of constituents where produced parse has bracketings that overlap for the siblings:
  - ((A B) C) — { (0,2), (2,3) }
    and hyp. has
    (A (B C)) — { (0,1), (1, 3) }




```
/* crossing is counted based on the brackets */
/* in test rather than gold file (by Mike)   */
for(j=0;j<bn2;j++){
  for(i=0;i<bn1;i++){
    if(bracket1[i].result != 5 &&
       bracket2[j].result != 5 &&
       ((bracket1[i].start < bracket2[j].start &&
         bracket1[i].end   > bracket2[j].start &&
         bracket1[i].end   < bracket2[j].end) ||
        (bracket1[i].start > bracket2[j].start &&
         bracket1[i].start < bracket2[j].end &&
         bracket1[i].end   > bracket2[j].end))){
```

from evalb.c

State-of-the-Art Parsing

- Parsers trained/tested on Wall Street Journal PTB
  - LR: 94%+;
  - LP: 94%+;
  - Crossing brackets: 1%

- Standard implementation of Parseval:
  - **`evalb`**

Evaluation Issues

- Only evaluating constituency
- There are other grammar formalisms:
  - LFG (Constraint-based)
  - Dependency Structure
- **Extrinsic** evaluation
  - How well does getting the correct parse match the semantics, etc?

Earley Parsing

Earley vs. CKY

- CKY doesn't capture full original structure
  - Can back-convert binarization, terminal conversion
  - Unit non-terminals require change in CKY

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- CKY doesn't capture full original structure
  - Can back-convert binarization, terminal conversion
  - Unit non-terminals require change in CKY
- Earley algorithm
  - Supports parsing efficiently with arbitrary grammars
  - Top-down search
  - Dynamic programming
    - Tabulated partial solutions
  - Some bottom-up constraints

Earley Algorithm

- Another dynamic programming solution
  - Partial parses stored in “chart”
  - Compactly encodes ambiguity
  - $O(N^3)$
- Chart entries contain:
  - Subtree for a single grammar rule
  - Progress in completing subtree
  - Position of subtree w.r.t. input

Earley Algorithm

- First, left-to-right pass fills out a chart with *N+1* states
  - Chart entries — sit between words in the input string
  - Keep track of states of the parse at those positions
  - For each word position, chart contains set of states representing all partial parse trees generated so far
    - e.g. `chart[0]` contains all partial parse trees generated at the beginning of sentence

Chart Entries

- Three types of constituents:
  - Predicted constituents
  - In-progress constituents
  - Completed constituents

Parse Progress

- Represented by Dotted Rules
  - Position of • indicates type of constituent
- $_0$ Book $_1$ that $_2$ flight $_3$
  - *S → • VP* [0,0] (predicted)
  - *NP → Det • Nom* [1,2] (in progress)
  - *VP → V NP •* [0,3] (completed)
- [x,y] tells us what portion of the input is spanned so far by rule
- Each state s_i: *<dotted rule>*, [*<back pointer>*, *<current position>*]

$_0$ Book $_1$ that $_2$ flight $_3$

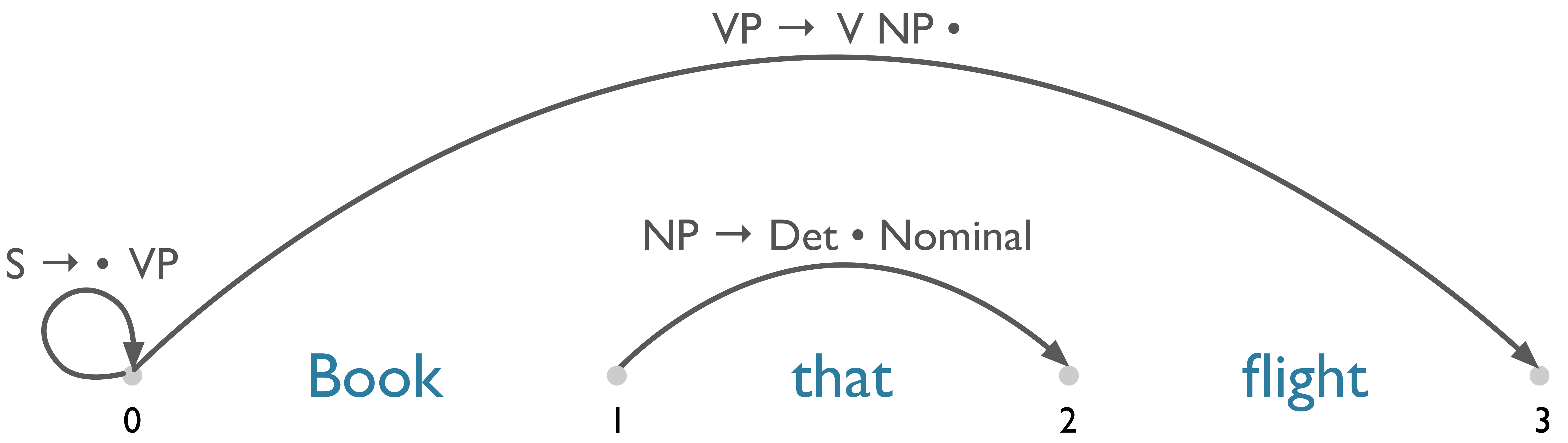
- $S \rightarrow \cdot\ VP$, [0,0]
  - First 0 means S constituent begins at the start of input
  - Second 0 means the dot is here too
  - So, this is a top-down prediction

$_0$ Book $_1$ that $_2$ flight $_3$

- *S* → • *VP*, [0,0]
  - First 0 means S constituent begins at the start of input
  - Second 0 means the dot is here too
  - So, this is a top-down prediction
- *NP* → *Det* • *Nom*, [1,2]
  - the NP begins at position 1
  - the dot is at position 2
  - so, Det has been successfully parsed
  - Nom predicted next

$_0$ Book $_1$ that $_2$ flight $_3$ (continued)

- *VP → V NP* • [0,3]
  - Successful VP parse of entire input



Successful Parse

- Final answer found by looking at last entry in chart
- If entry resembles $S \rightarrow \alpha \cdot [0,N]$ then input parsed successfully
- Chart will also contain record of all possible parses of input string, given the grammar

Parsing Procedure for the Earley Algorithm

- Move through each set of states in order, applying one of three operations:
  - **predictor**: add predictions to the chart
  - **scanner**: read input and add corresponding state to chart
  - **completer**: move dot to right when new constituent found
- Results (new states) added to current or next set of states in chart
- No backtracking and no states removed: keep complete history of parse

Earley Algorithm

```
function EARLEY-PARSE(words, grammar) returns chart
  ENQUEUE((γ→ • S, [0,0]), chart[0])
  for i ← from 0 to LENGTH(words) do
    for each state in chart[i] do
      if INCOMPLETE?(state) and
          NEXT-CAT(state) is not a part of speech then
        PREDICTOR(state)
      elseif INCOMPLETE?(state) and
          NEXT-CAT(state) is a part of speech then
        SCANNER(state)
      else
        COMPLETER(state)
      end
    end
  return(chart)
```

Earley Algorithm

procedure PREDICTOR(($A \to \alpha \bullet B\ \beta$, $[i,j]$))
  for each ($B \to \gamma$) **in** GRAMMAR-RULES-FOR(B,*grammar*) **do**
    ENQUEUE(($B \to \bullet\ \gamma$, $[j,j]$), *chart*[*j*])
  end

procedure SCANNER(($A \to \alpha \bullet B\ \beta$,$[i,j]$))
  if B ⊂ PARTS-OF-SPEECH(*word*[*j*]) **then**
    ENQUEUE((B → *word*[*j*] •, $[j,j+1]$), *chart*[*j*+1])

procedure COMPLETER(($B \to \gamma \bullet$, $[j,k]$))
 for each (***A* → α • *B* β, [*i*,*j*]**) **in** ***chart*[*j*]** **do**
    ENQUEUE(($A \to \alpha\ B \bullet\ \beta$, $[i,k]$), *chart*[*k*])
  end

3 Main Subroutines of Earley

- Predictor
  - Adds predictions into the chart
- Scanner
  - Reads the input words and enters states representing those words into the chart
- Completer
  - Moves the dot to the right when new constituents are found

Predictor

- Intuition:
  - Create new state for top-down prediction of new phrase
- Applied when non part-of-speech non-terminals are to the right of a dot:
  - $S \rightarrow \cdot\ VP$ [0,0]
- Adds new states to *current* chart
  - One new state for each expansion of the non-terminal in the grammar
    $VP \rightarrow \cdot\ V$ [0,0]
    $VP \rightarrow \cdot\ V\ NP$ [0,0]

Chart[0]

S0	*γ → • S*	[0,0]	Dummy start state
S1	*S → • NP VP*	[0.0]	Predictor
S2	*S → • Aux NP VP*	[0,0]	Predictor
S3	*S → • VP*	[0,0]	Predictor
S4	*NP → • Pronoun*	[0,0]	Predictor
S5	*NP → • Proper-Noun*	[0,0]	Predictor
S6	*NP → • Det Nominal*	[0,0]	Predictor
S7	*VP → • Verb*	[0,0]	Predictor
S8	*VP → • Verb NP*	[0,0]	Predictor
S9	*VP → • Verb NP PP*	[0,0]	Predictor
S10	*VP → • Verb PP*	[0,0]	Predictor
S11	*VP → • VP PP*	[0,0]	Predictor

Chart[1]

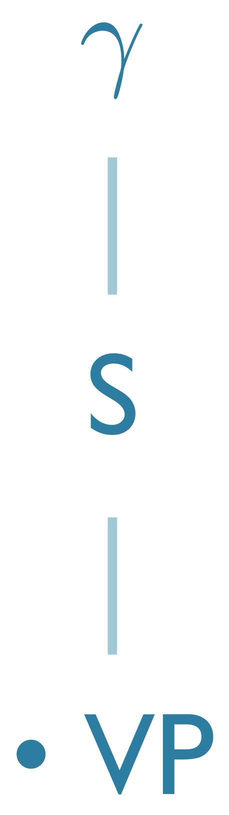
S12	*Verb → book •*	[0,1]	Scanner
S13	*VP → Verb •*	[0,1]	Completer
S14	*VP → Verb • NP*	[0,1]	Completer
S15	*VP → Verb • NP PP*	[0,1]	Completer
S16	*VP → Verb • PP*	[0,1]	Completer
S17	*S → VP •*	[0,1]	Completer
S18	*VP → VP • PP*	[0,1]	Completer
S19	*NP → • Pronoun*	[1,1]	Predictor
S20	*NP → • Proper-Noun*	[1,1]	Predictor
S21	*NP → • Det Nominal*	[1,1]	Predictor
S22	*PP → • Prep NP*	[1,1]	Predictor

Book that flight

S0: $\gamma \rightarrow \bullet\, S$ [0,0]

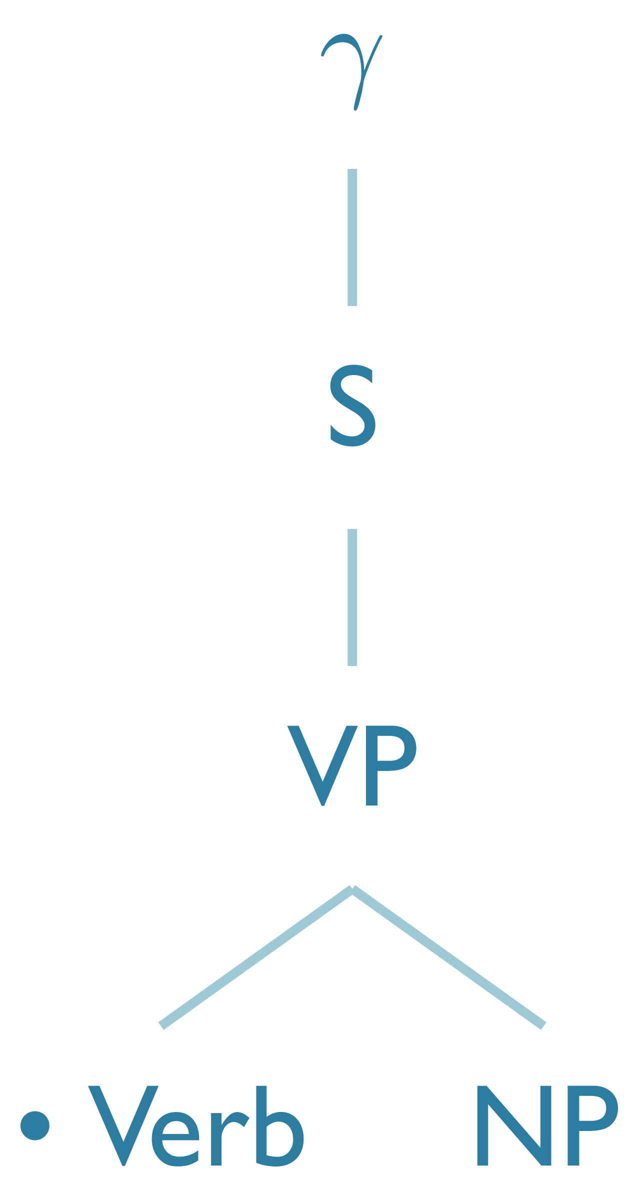
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]



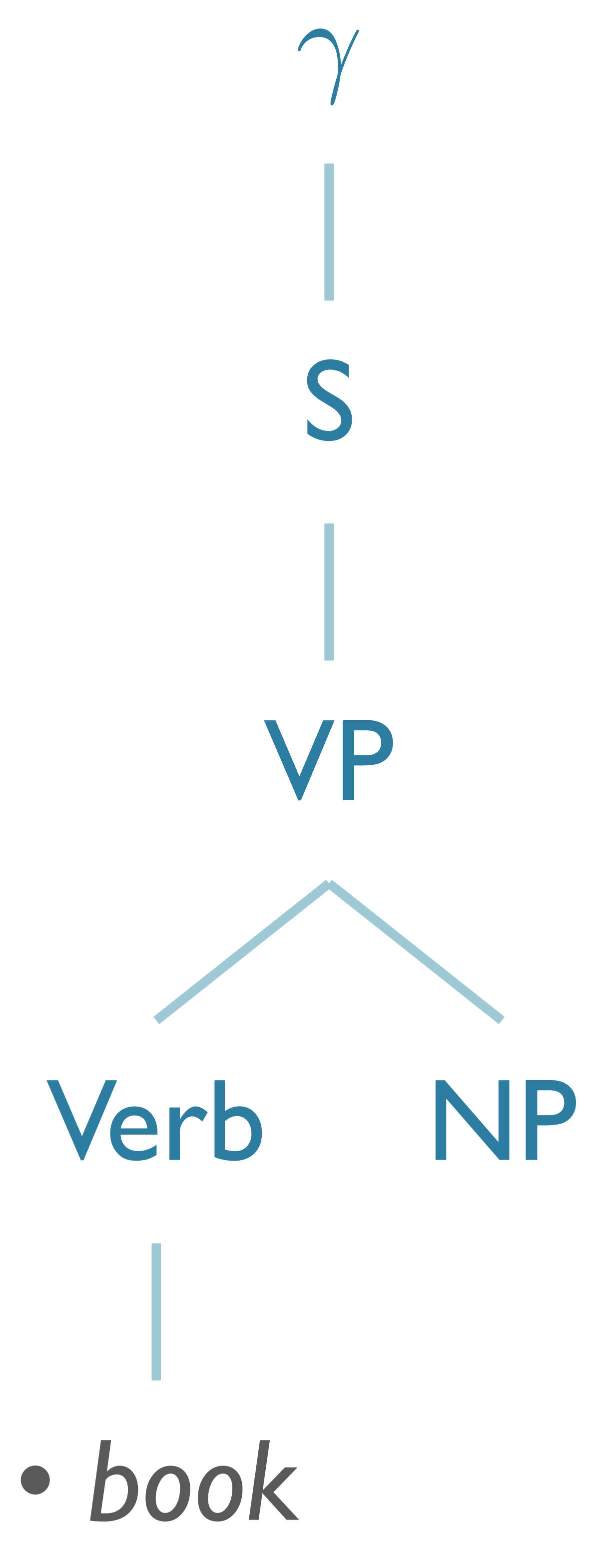
Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow \bullet VP$ [0,0]
S8: $VP \rightarrow \bullet Verb\ NP$ [0,0]



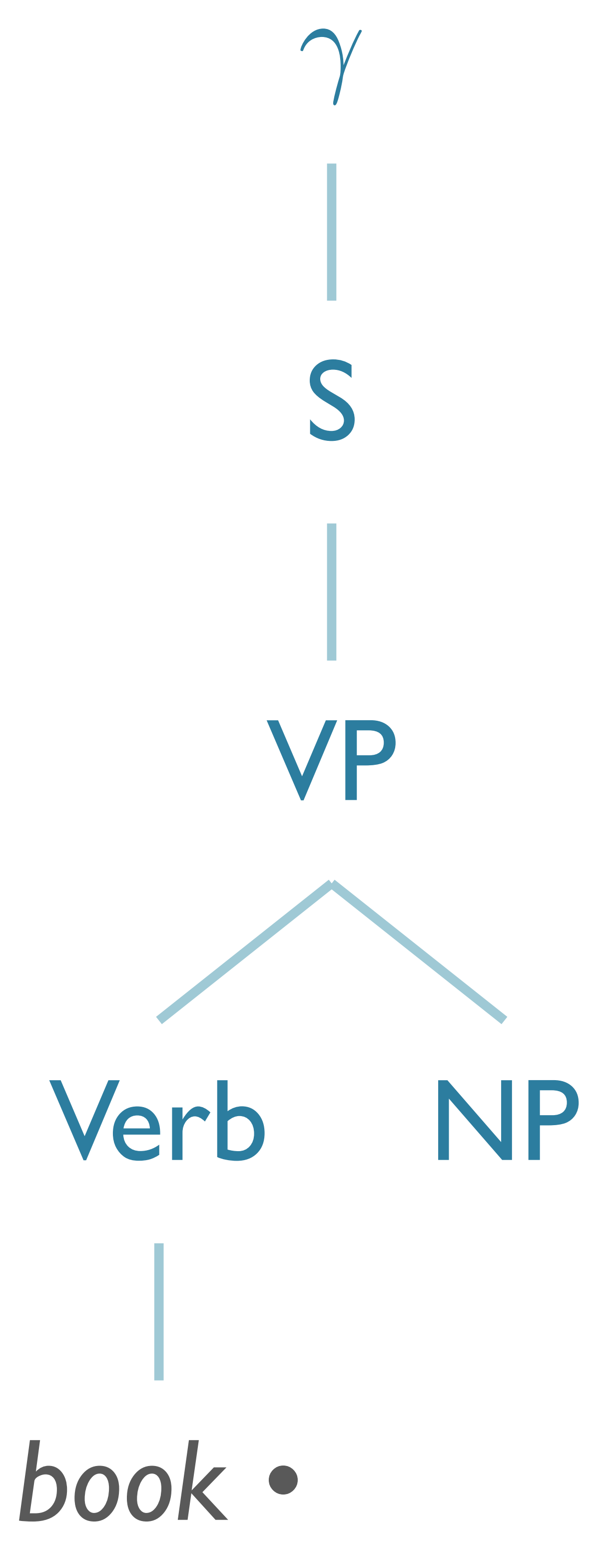
Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow \bullet VP$ [0,0]
S8: $VP \rightarrow \bullet Verb\ NP$ [0,0]
S12: $Verb \rightarrow \bullet book$ [0,0]



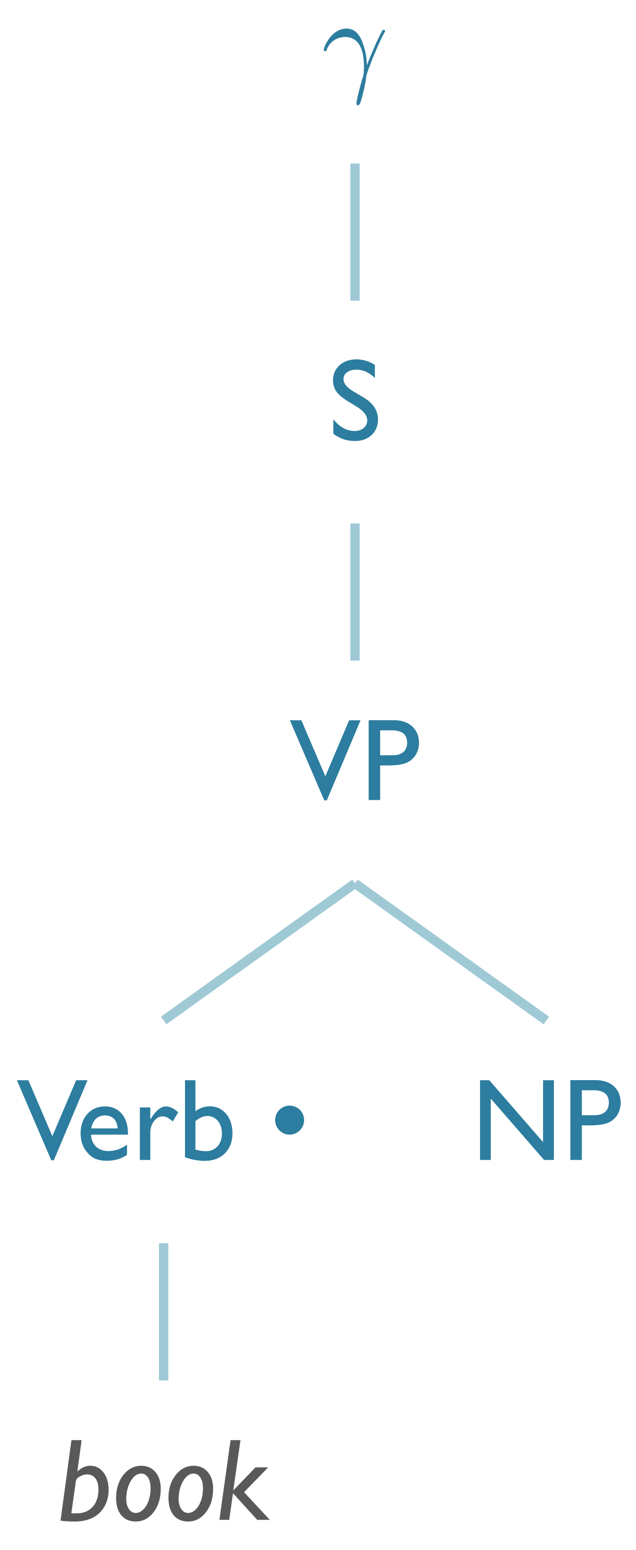
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]
S8: $VP \rightarrow \bullet\ Verb\ NP$ [0,0]
S12: $Verb \rightarrow book\ \bullet$ [0,1]



Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow \bullet VP$ [0,0]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]



Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP\ \bullet$ [0,1]
S8: $VP \rightarrow Verb\ \bullet\ NP$ [0,1]

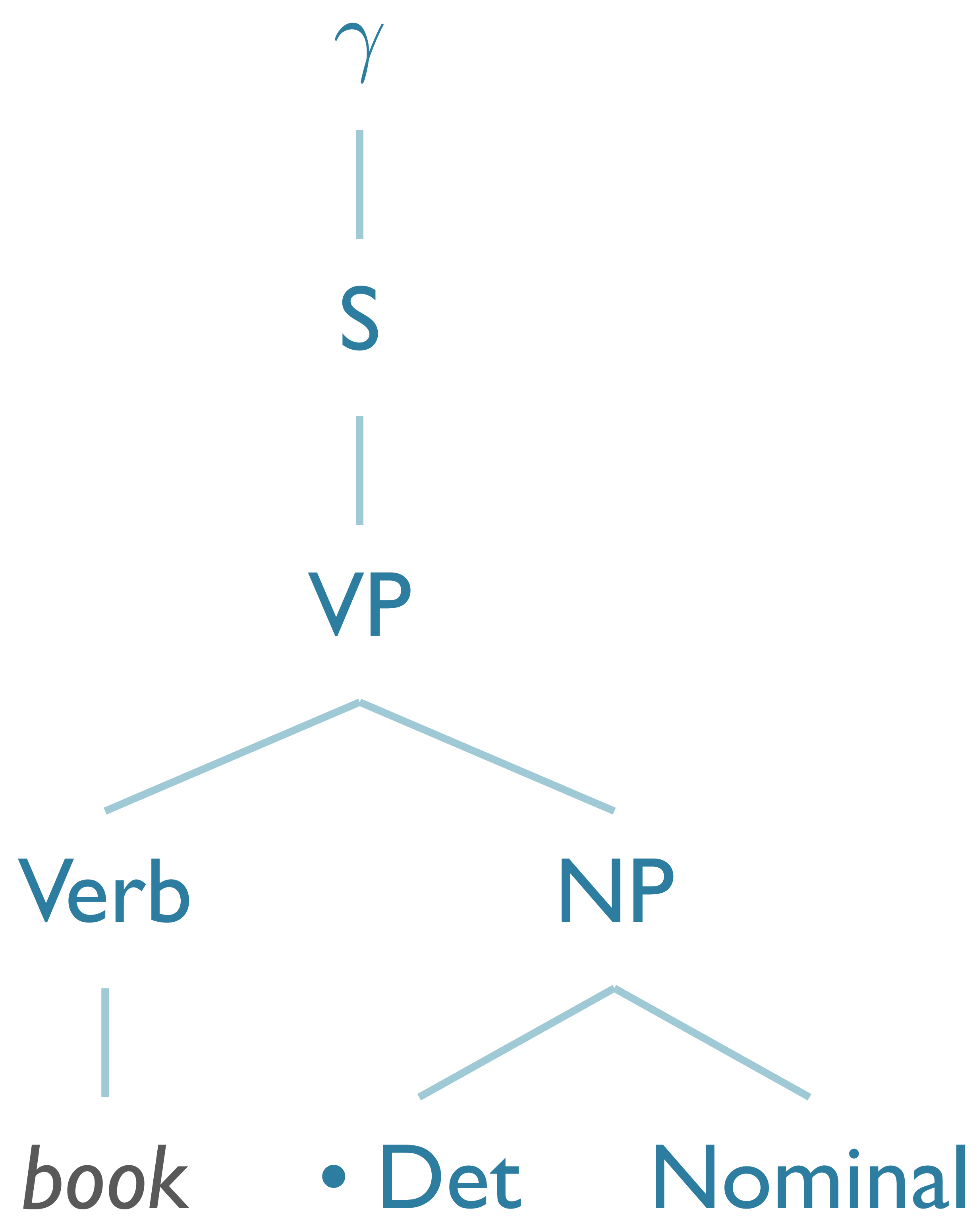
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]

S3: $S \rightarrow VP \bullet$ [0,1]

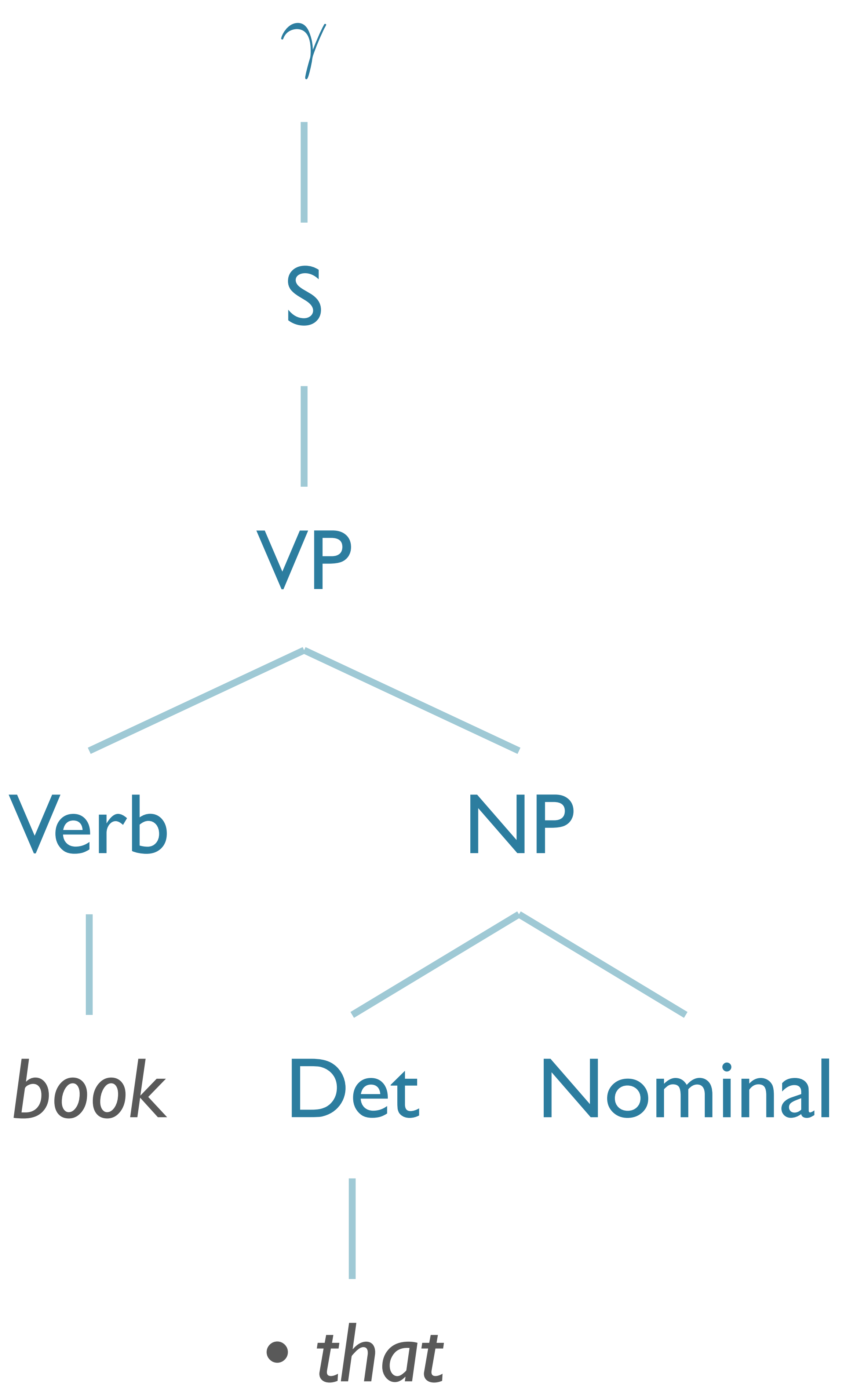
S8: $VP \rightarrow Verb \bullet NP$ [0,1]

S21: $NP \rightarrow \bullet\ Det\ Nominal$ [1,1]



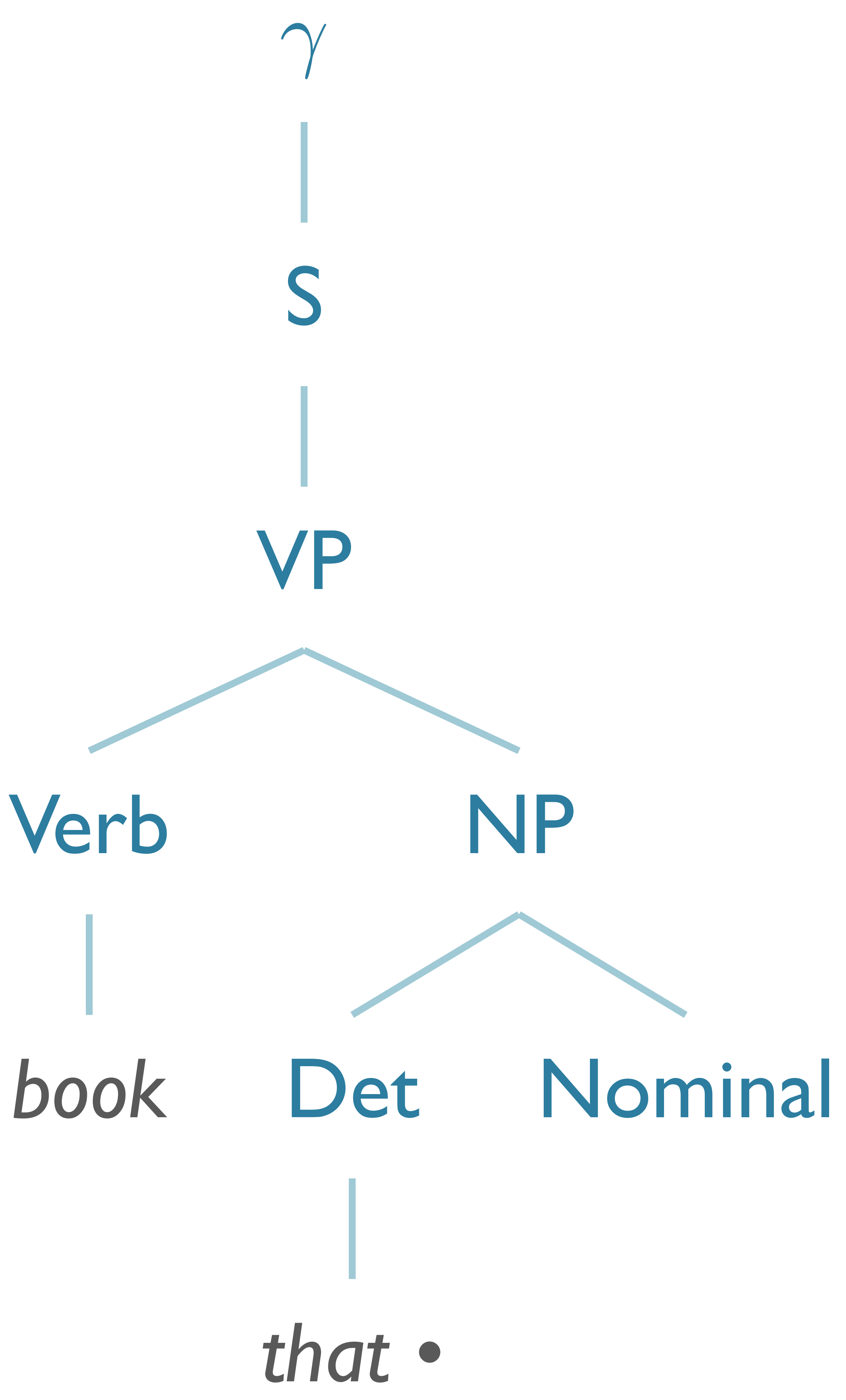
Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow \bullet Det\ Nominal$ [1,1]
S23: $Det \rightarrow \bullet$ *"that"* [1,1]



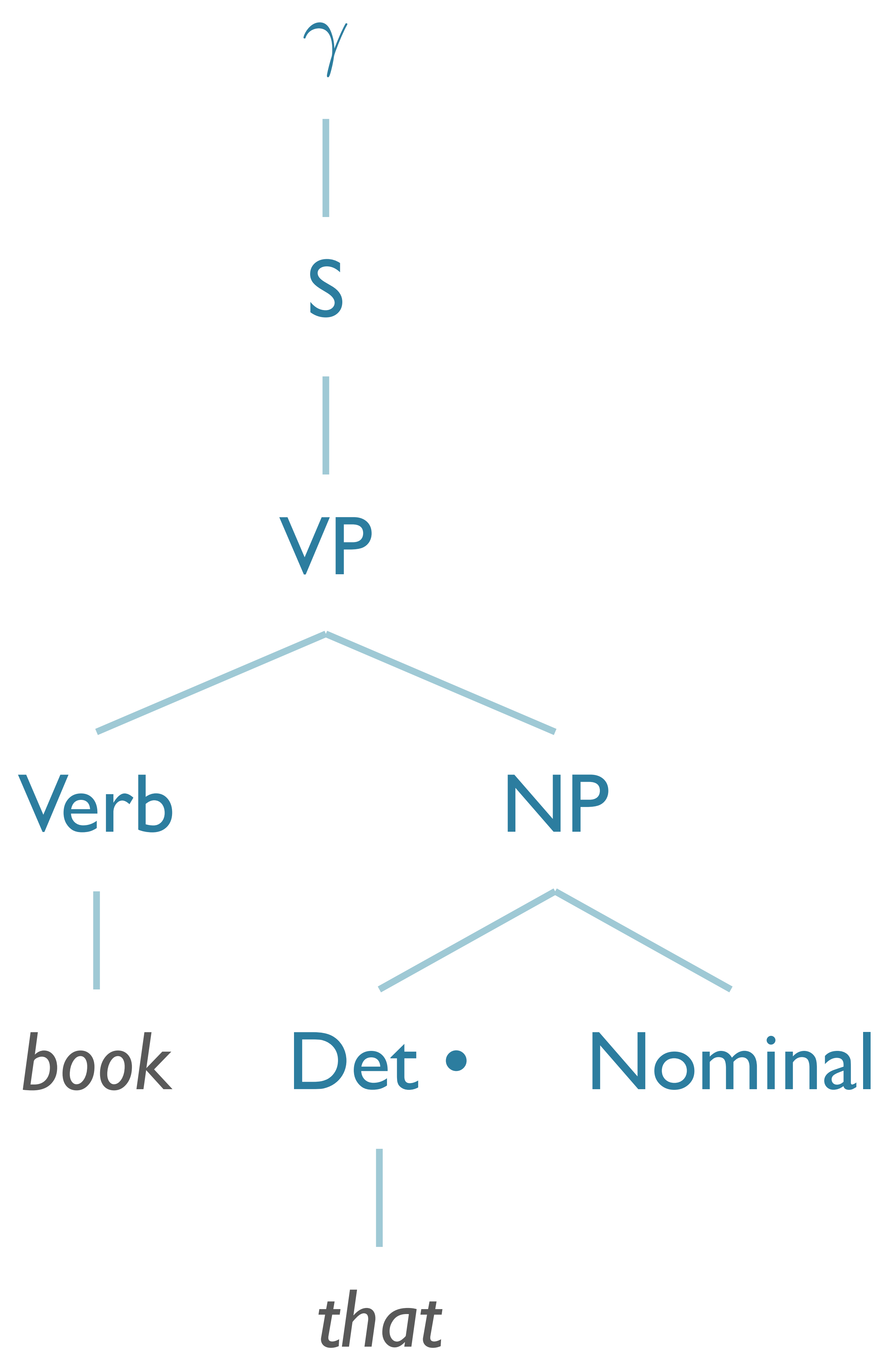
Book that flight

S0: *γ* → • *S* [0,0]
S3: *S* → *VP* • [0,1]
S8: *VP* → *Verb* • *NP* [0,1]
S21: *NP* → • *Det Nominal* [1,1]
S23: *Det* → *"that"* • [1,2]



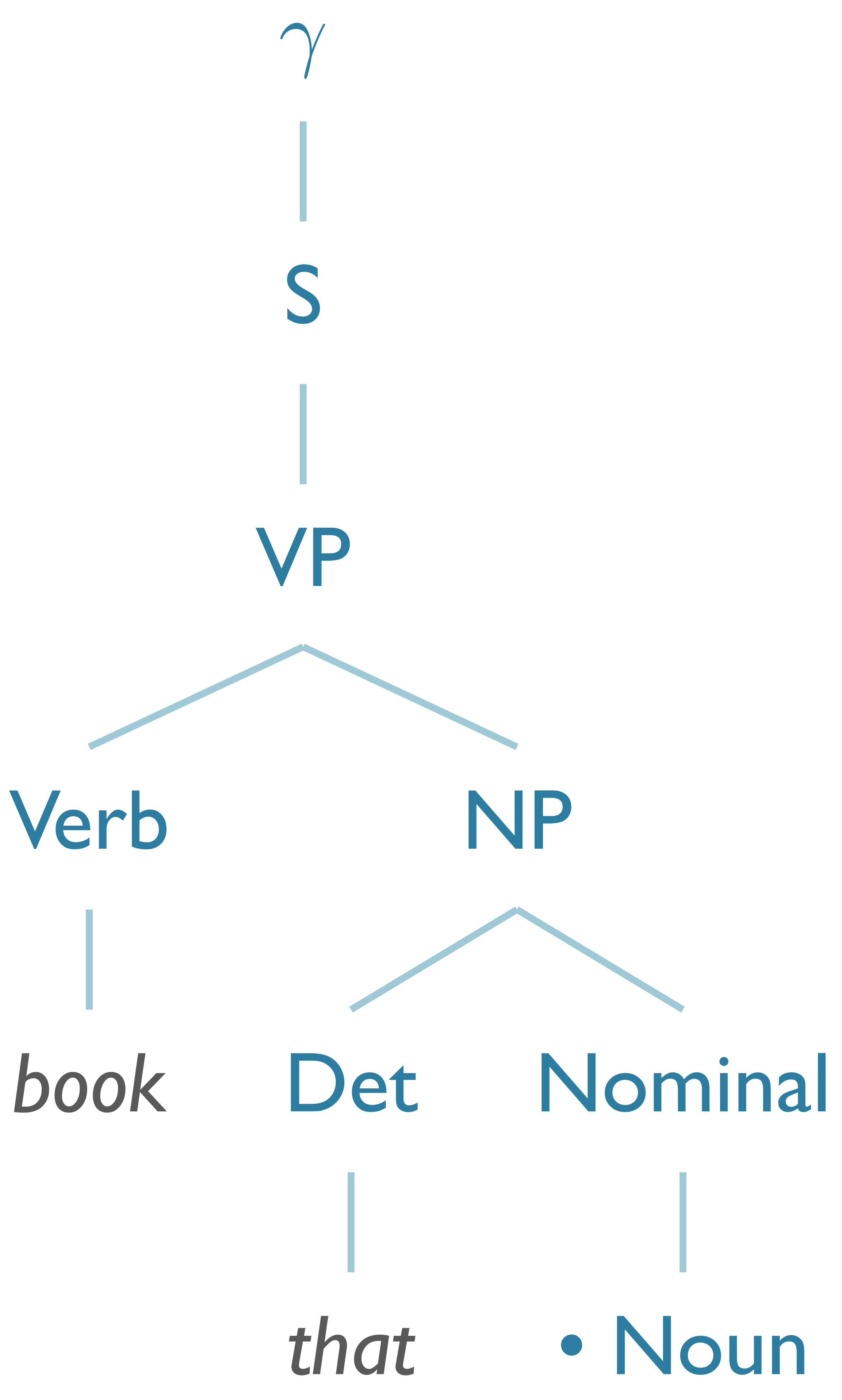
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]



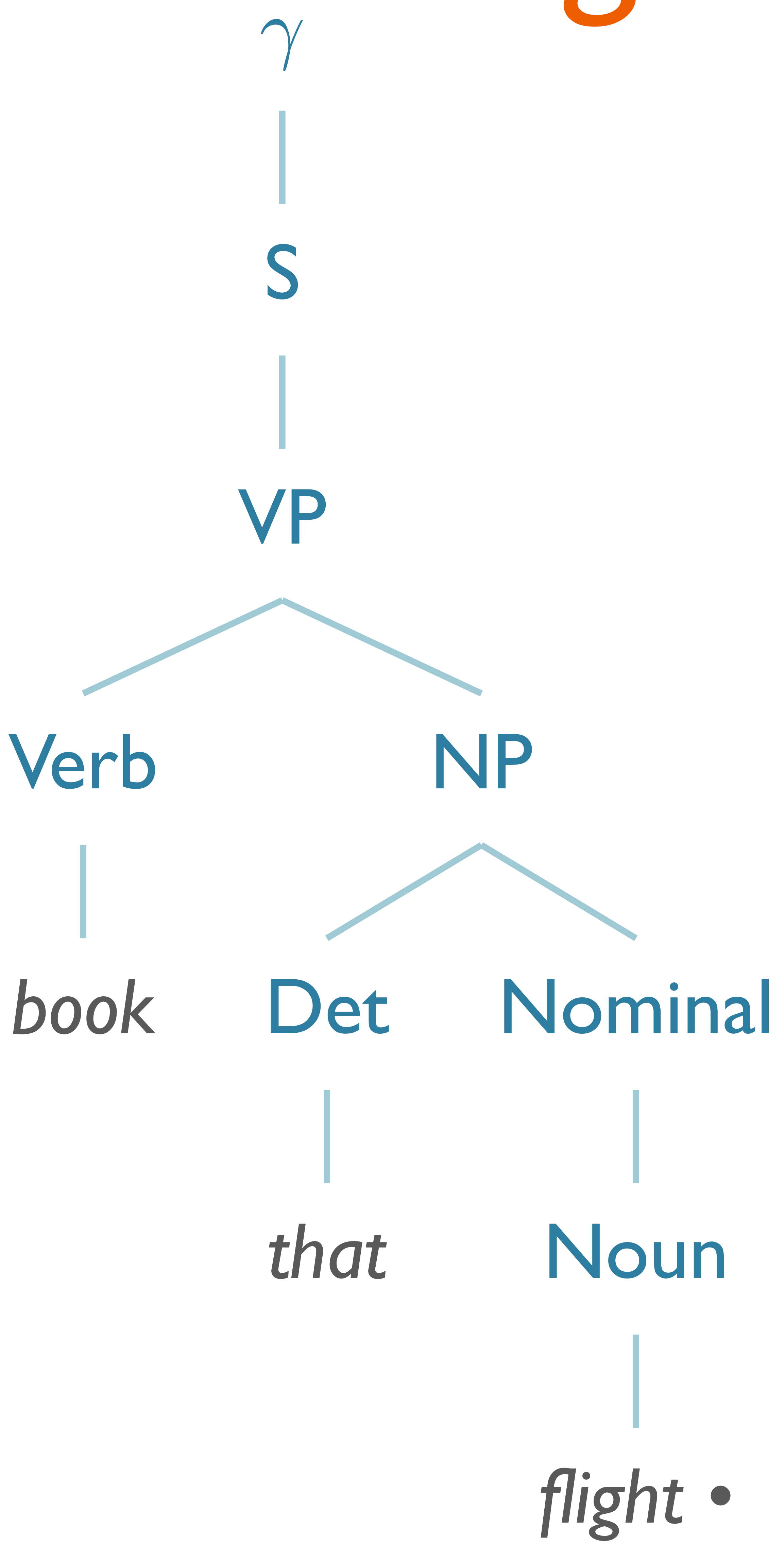
Book that flight

S0: *γ* → • *S* [0,0]
S3: *S* → *VP* • [0,1]
S8: *VP* → *Verb* • *NP* [0,1]
S21: *NP* → *Det* • *Nominal* [1,2]
S25: *Nominal* → • *Noun* [2,2]



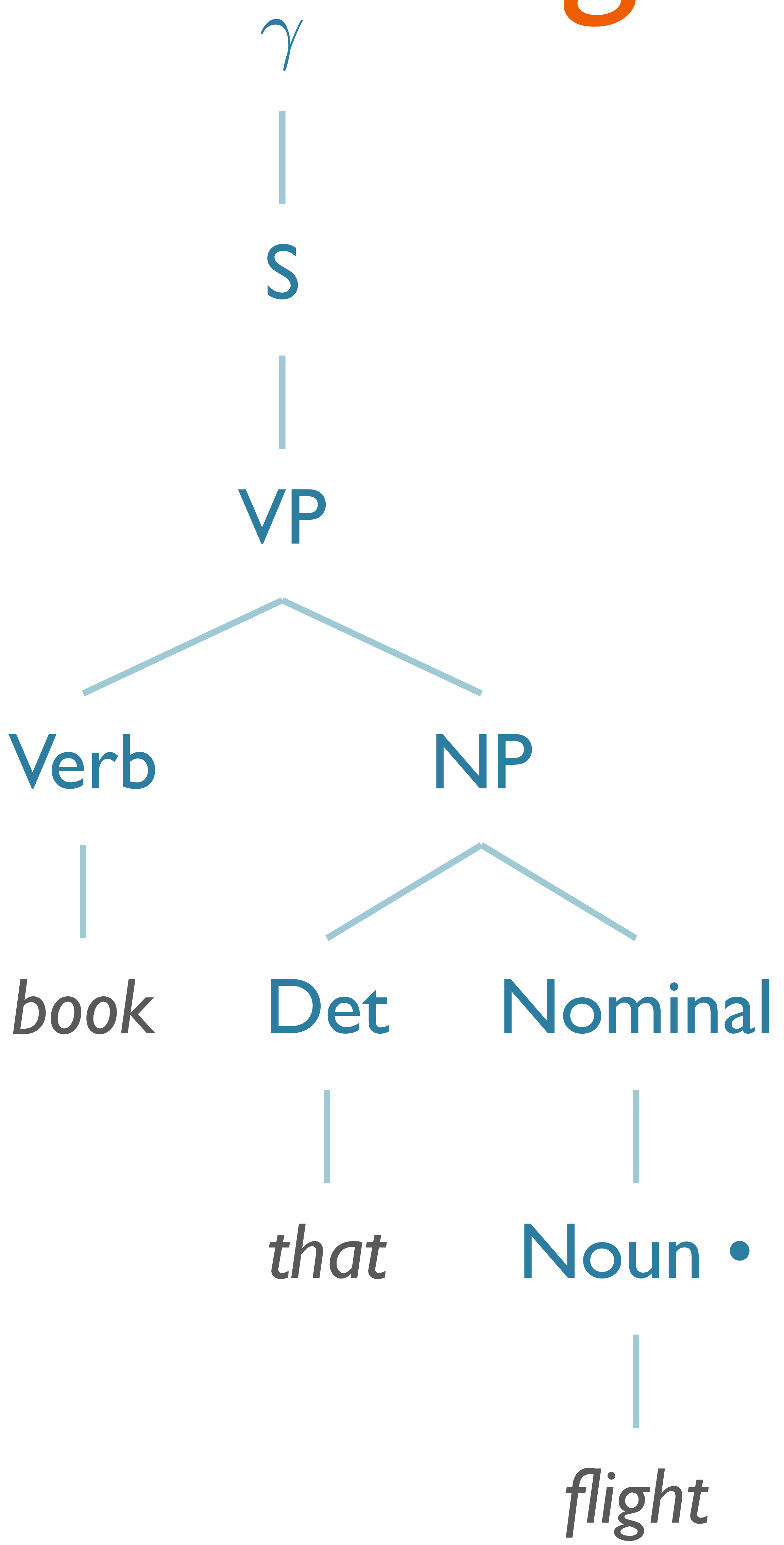
Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]
S25: $Nominal \rightarrow \bullet Noun$ [2,2]
S28: *Noun* → *"flight"* • [2,3]



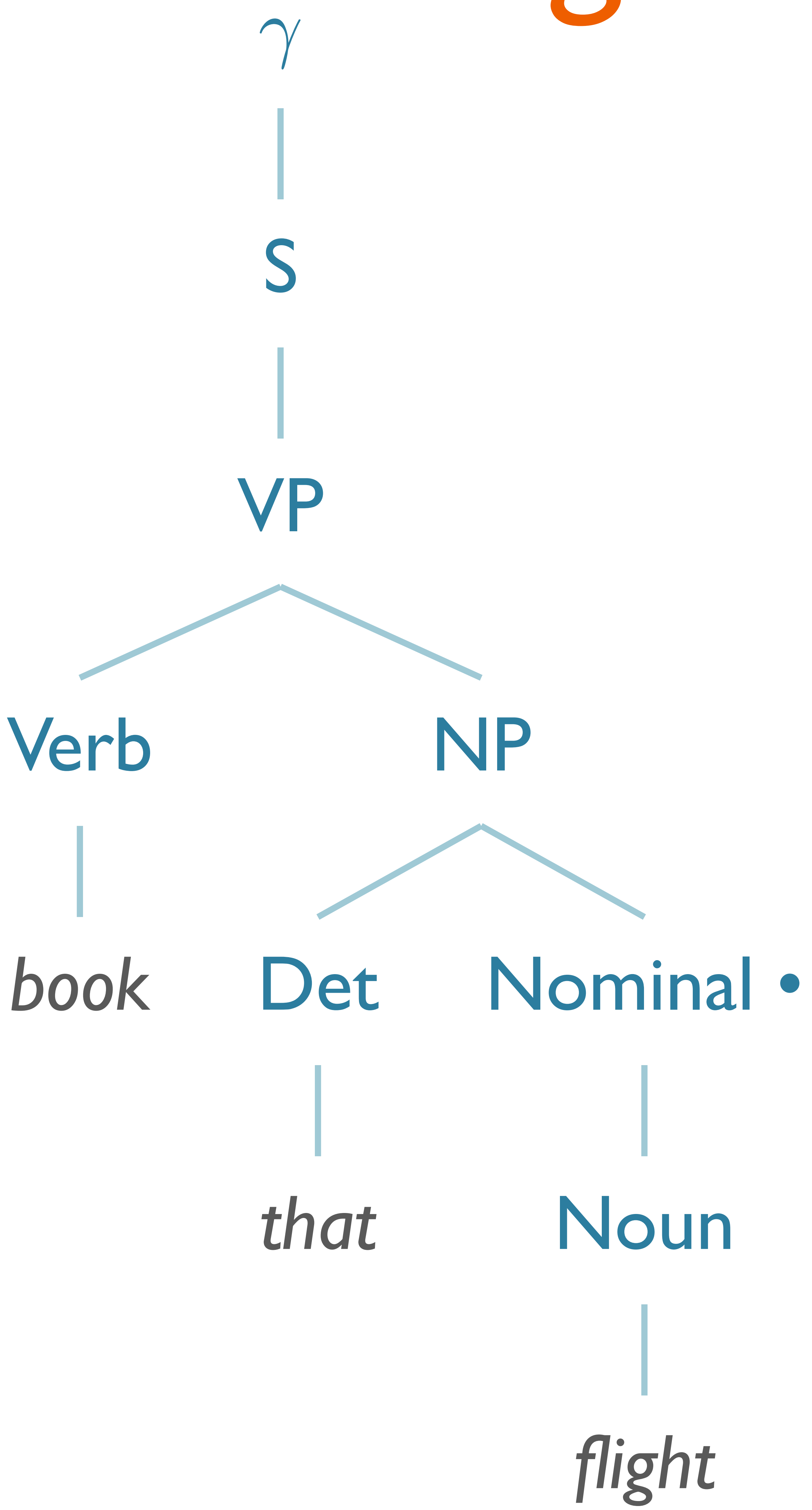
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP\ \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]
S25: $Nominal \rightarrow Noun\ \bullet$ [2,3]



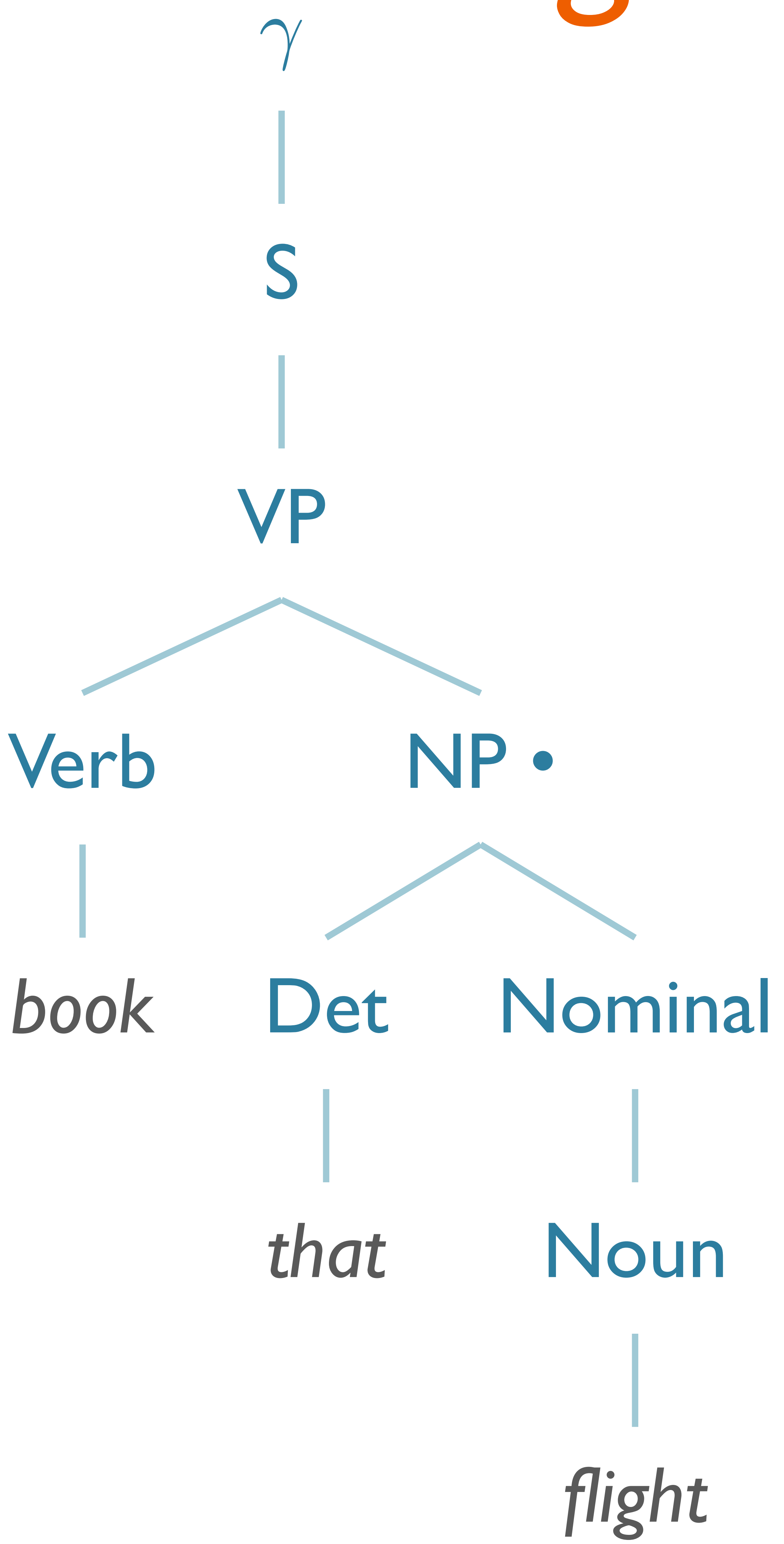
Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det\ Nominal \bullet$ [1,3]



Book that flight

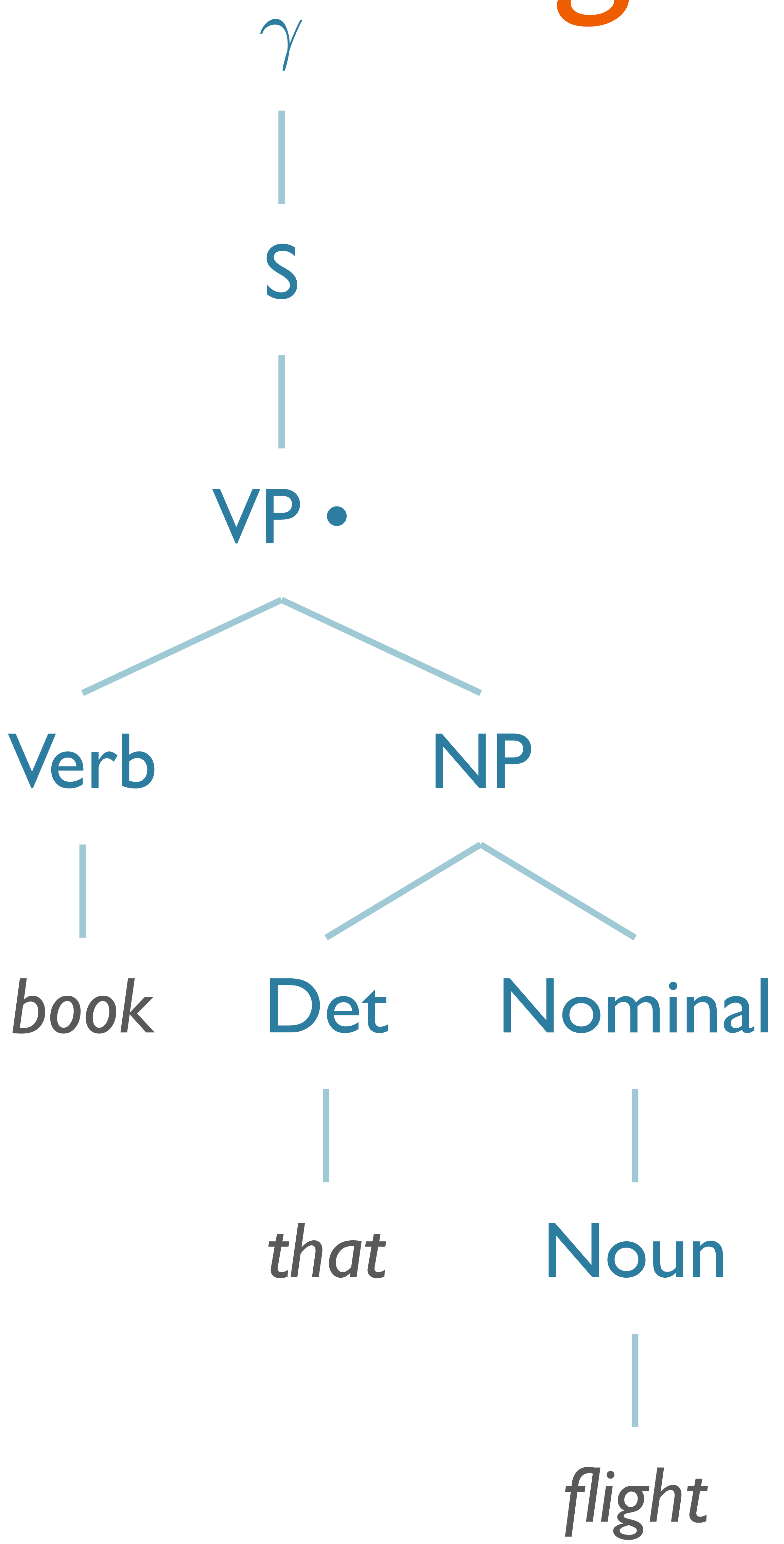
S0: $\gamma \rightarrow \bullet S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb\ NP \bullet$ [0,3]



Book that flight

S0: $\gamma \rightarrow \bullet S$ [0,0]

S3: $S \rightarrow VP \bullet$ [0,3]

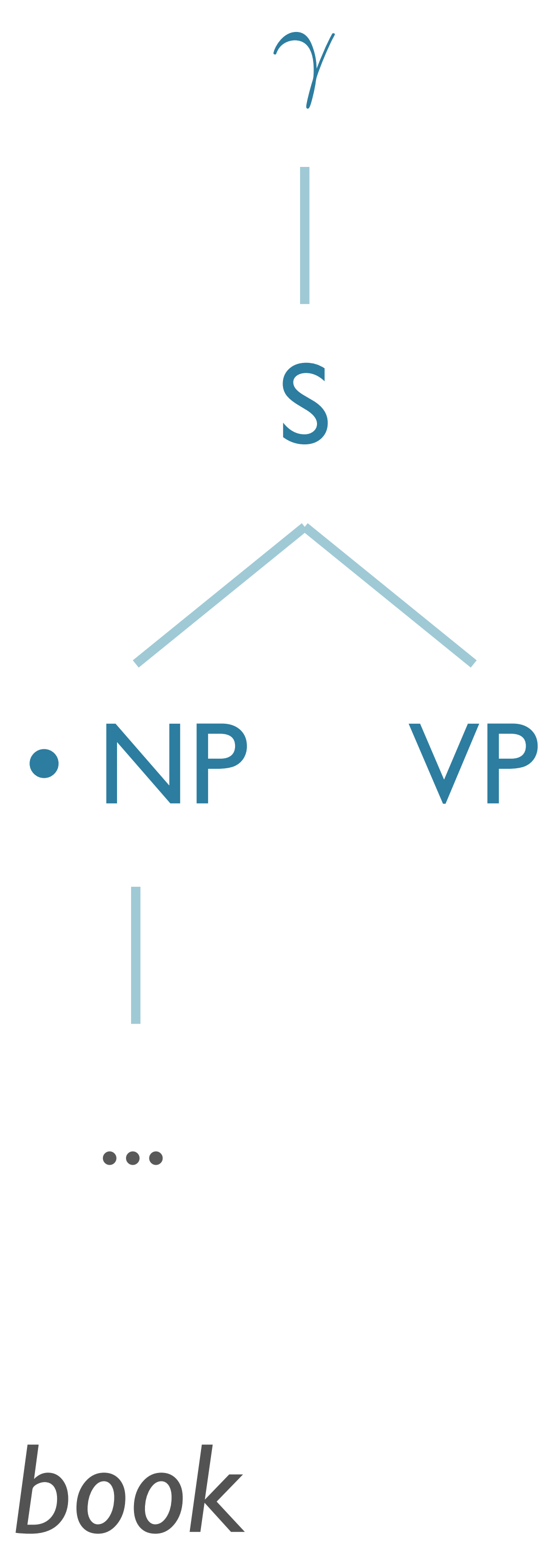


What About Dead Ends?

Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S1: $S \rightarrow \bullet\ NP\ VP$ [0,0]

NP → • *Pronoun*
NP → • *Proper-Noun*
NP → • *Det Nominal*



Book that flight

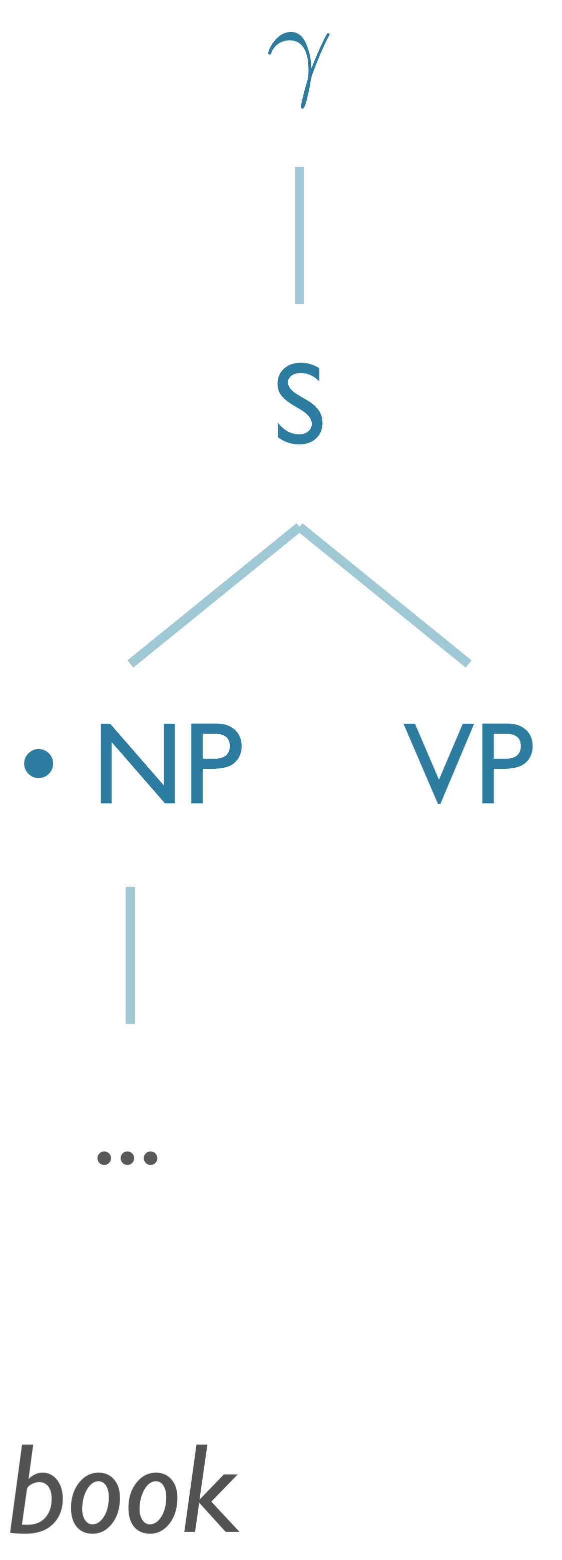
S0: $\gamma \rightarrow \bullet\ S$ [0,0]

S1: $S \rightarrow \bullet\ NP\ VP$ [0,0]

~~*NP* → • *Pronoun*~~

~~*NP* → • *Proper-Noun*~~

~~*NP* → • *Det Nominal*~~



What About Recursion?

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- We now have a top-down parser in hand. Does it enter infinite loops on rules like S -> S ‘and’ S?

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- We now have a top-down parser in hand. Does it enter infinite loops on rules like S -> S ‘and’ S?
- No!

```
procedure ENQUEUE(state, chart-entry)
  if state is not already in chart-entry then
     PUSH(state, chart-entry)
  end
```

What about recursion?

- We now have a top-down parser in hand. Does it enter infinite loops on rules like S -> S ‘and’ S?
- No!

procedure ENQUEUE(*state*, *chart-entry*)
  if *state* is not already in *chart-entry* **then**
    PUSH(*state*, *chart-entry*)
  end

Exercise: parse ‘table and chair’ using the very simple grammar

Nom -> Nom ‘and’ Nom | ‘table’ | ‘chair’