

# Platonism, Structuralism and Mathematical Applicability

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## The Allegory of the Cave

- Interrelatedness of Plato's Metaphysics, Epistemology, Ethics and Philosophy of Math

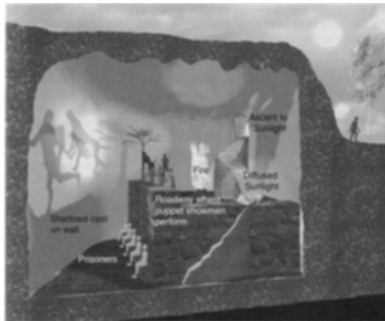


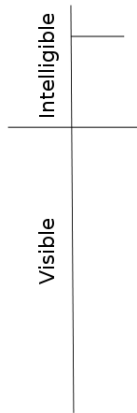
Figure: [faculty.salisbury.edu/~jdhatley](http://faculty.salisbury.edu/~jdhatley)

# The Role of Mathematics

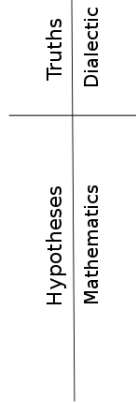
- Ethical significance extends beyond utility of calculations
- Plato implores his guardians to “go to calculation and to take it up, not after the fashion of private men, but to stay with it until they [the future guardians] come to the contemplation of the nature of numbers with intellection itself, not practicing it for the sake of buying and selling like merchants or tradesmen, but for war and for ease of turning the soul itself around from becoming to truth and being” (525c)

# The Divided Line and Ontology

## Divided Line



## Intelligible



# Mathematical Objects Are Forms

- Traditional (Gödel, i.e.): ontological realm separate from forms
- Although students of geometry, arithmetic, etc. “make use of the visible forms and reason about them, they are thinking not of these, but of *the ideals* which they resemble; not of the figures which they draw, but of the *absolute* square and the absolute diameter, and so on...” (510d, emphasis added)

## What is Structuralism?

- Truths not about specific objects, but relationships/structures
- “The compelling and immediate reason for rejecting the idea that mathematics is about particular objects is that for any mathematical theory the domain of objects which that theory is taken to be about can always be replaced by a domain consisting of different objects, so long as the second domain has a structure isomorphic to that of the first” (Isaacson 123)
- What then can we say about the ontological status of mathematical objects?

## The Formalist Solution

- Objects are empty signs with no meaning outside their structural relationships
- Makes no positive metaphysical claims and eliminates any hope of doing so

## The Relative Aspect of Structuralism

- For any given area of mathematical study, the choice of model is irrelevant as long as we are consistent with said choice
- For Peano Arithmetic, i.e.:
  - $M$ :  $S$  given by set  $\mathbb{N}$ , let  $c$  refer to '1' and  $s : \mathbb{N} \rightarrow \mathbb{N}$ :  
 $s(x) = x + 1$
  - $M'$ :  $S' = \{2n | n \in \mathbb{N}\}$ , let  $c$  refer to '2' and  $s : \mathbb{N} \rightarrow S'$ :  
 $s(x) = 2(x + 1)$
- Does this make truth relative?

## Ontological Ambiguity

- Implicit assignment of a particular object to the symbols of a model
- “a case of *systematic referential ambiguity*—in a sense we are always talking about ‘my number 1’, ‘my successor function’, etc.” (Reck and Price 350, emphasis in original)
- Underneath these structures, ontological commitment to the existence of (infinite) sets

## How Relativist Structuralism Resembles Platonism

- Sets are considered actually existent abstract objects
- Retains sense of math as discovery
- Relativist structuralism + ZFC allows mathematical practice to continue virtually unchanged

## The One Over Many Principle

- “There is one Form for each set of many things to which we give the same name” (596a)
- Relativist structuralism: we can choose what particular object to which to assign a name
  - Place-holders: under every model of  $PA$ ,  $3 = s(s(c))$ ; but choice of  $c$  changes

## Idealized Abstractions

- “The form of the bed”
  - Single, abstract object
- “The set of all beds”
  - Collection of particular objects; no other unique properties

## Minimal Platonism

- Relativist structuralism + ZFC represents a minimal version of platonism
- Mathematical objects are still truly existent abstract objects but they do not have the same sort of perfection and correspondence to sensible objects as Plato's forms

# Intuitions About Math's Applicability

- The good:
  - Language of physics
  - Phenomena appear to conform to mathematical models
- The bad:
  - Non-uniqueness of models
  - Theories constantly proven wrong, improved, etc.

## What Plato Has to Say

- “If a man, gaping up or squinting down, attempts to learn something of sensible things, I would deny that he ever learns—for there is no knowledge of such things” (529b)
- One over many vs. idealized abstraction interpretations on applicability
- Metaphysical contingency of physical objects

## Non-Uniqueness of Physical Models

- Electrons, wave-particle duality and mathematical indifference
- “In physical theorizing we also need the *ontological* distinction between theoretical objects and their physical realization. We need to maintain a level of description in which a physical theory can *talk about* electrons, as theoretical objects, without its having to *be about* electrons, as objects that are physically realized in the world. To talk about electrons (or unicorns) is not thereby to bring them into existence as physical objects” (Brading and Landry 573)

# The Problem of Representation

- Structural models represent multiple kinds of objects
- A model of physical phenomena must also represent particular physical objects
- Intermediate step: theories of data
  - Still a mathematical model; how do we represent its correspondence?

## The 'No Miracles' Solution

- If physical phenomena did not conform to some theory of data, the correspondence that we find would have to be called a miracle
- No positive description of representation
- “The *unreasonable* effectiveness of mathematics in the natural sciences”

## Logical Necessity of Physical Theories

- Another point of agreement between platonism and structuralism
  - Plato's metaphysics render mathematical truths more necessary
  - Structuralism's indifference towards specific model used (i.e. model of data)
- Possible answers: direct correspondence or idealized abstractions





## The Role of Philosophy and Remaining Humble

- The question of why physical phenomena conform to a theory of data falls under philosophy and not mathematics, given that mathematics makes no positive metaphysical claims about its objects.
- One must remain humble when using mathematics for its physical explanatory power
  - Never can a physical model have the same necessity of mathematics as long as the final objects represented are particular physical objects






## Unanswered Questions

- Why does mathematics appear to describe physical phenomena so accurately?
- Can we ever achieve physical necessity of the same order as logical necessity?
- Metaphysical and epistemological commitments required





## Further Reading

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## Further Reading (cont.)

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## Further Reading (cont.)

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# The End

- Questions?