PCFGs:
Parsing & Evaluation

LING 571 — Deep Processing Techniques for NLP

Shane Steinert-Threlkeld

Roadmap

- CKY + back-pointers
- PCFGs
- PCFG Parsing (PCKY)
- Inducing a PCFG
- Evaluation
- [Earley parsing]
- HW3 + collaboration

CKY Parsing: Backpointers

Current CKY Algorithm

Limitations:

Only stores non-terminals in cell

- Not rules or cells corresponding to RHS

Stores SETS of non-terminals

- Multiple rules with same LHS collide

Currently only ***acceptance/recognition***

Backpointers

- Instead of list of possible nonterminals for that node, each cell should have:
  - Nonterminal for the node
  - Pointer to left and right children cells
    - Either direct pointer to cell, or indices

For example:

```
bp_2 = BackPointer()
bp_2.l_child = [X2, (1,4)]
bp_2.r_child = [PP, (4,6)]
```

CKY *Parser*

- Pair each nonterminal with back-pointer to cells from which it was derived
- Last step:
  - construct trees from back-pointers in $[\ 0,\ n\ ]$

NP, Pronoun [0,1]	**S** [0,2]	[0,3]	**S** [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	**VP, X2, S** [1,4]	[1,5]	**VP** [1,6]
		Det [2,3]	**NP** [2,4]	[2,5]	**NP** [2,6]
			Noun, Nom [3,4]	[3,5]	**Nom** [3,6]
				Prep [4,5]	**PP** [4,6]
					NNP, NP [5,6]

$VP \rightarrow VP\ PP$

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   bp_1 = BackPointer()
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
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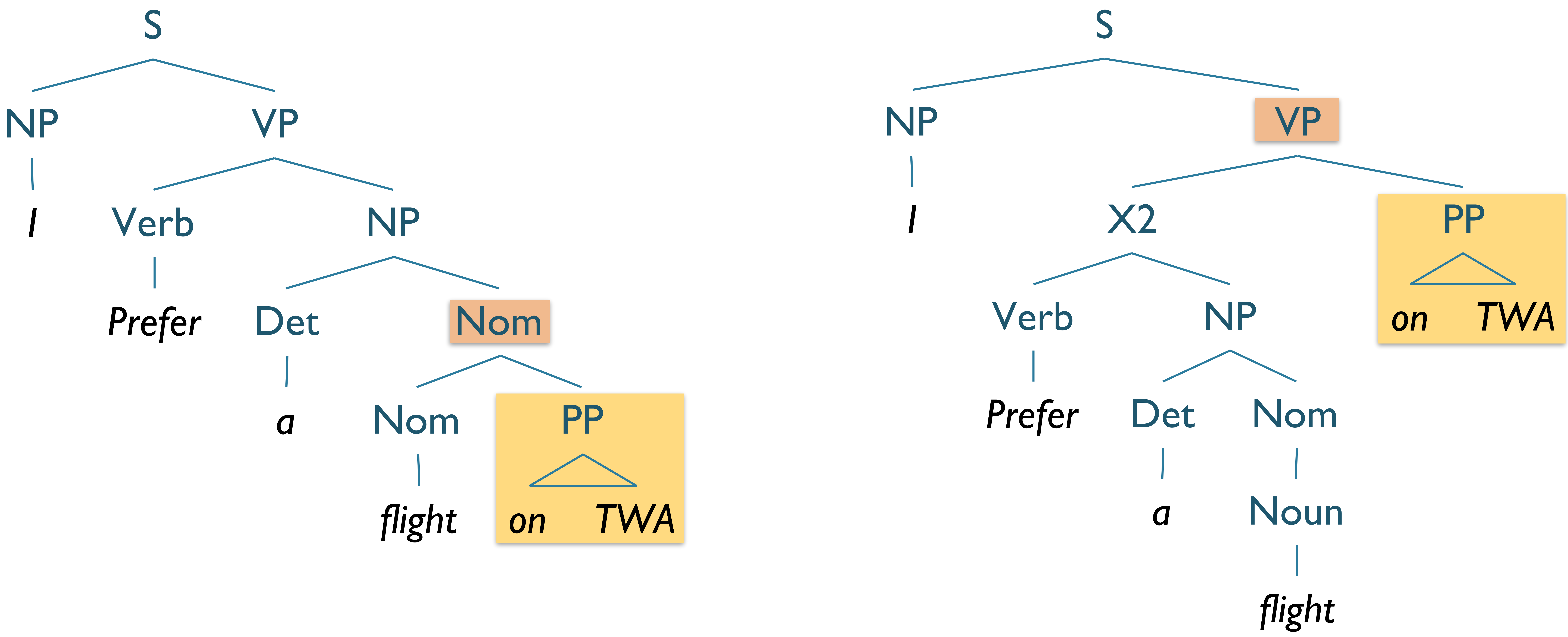
Resulting Parses



CKY Discussion

- Running time:
  - $O(n^3)$ where n is the length of the input string
  - Inner loop grows as square of # of non-terminals
- Expressiveness:
  - As implemented, requires CNF
    - Weak equivalence to original grammar
    - Doesn't capture full original structure
    - Back-conversion?
      - Can do binarization, terminal conversion
      - Unit productions requires change in CKY

CKY + Back-pointers Example

```
cky_table[0,6][S] = {(NP, (0,1),
                      VP, (1,6))}
```

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
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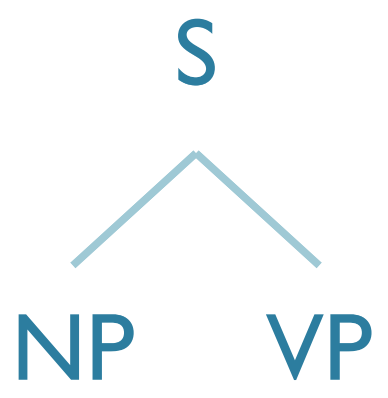

I *prefer* *a* *flight* *on* *TWA*

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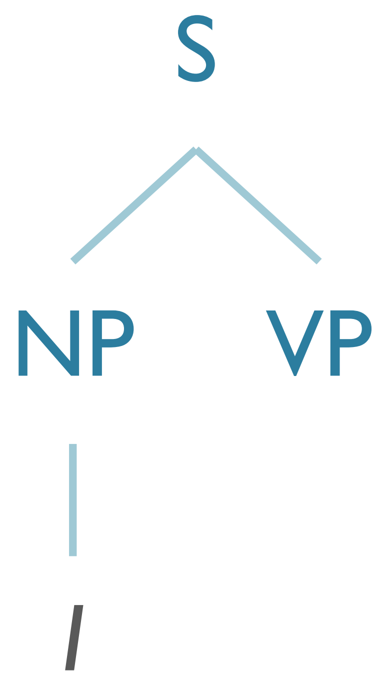

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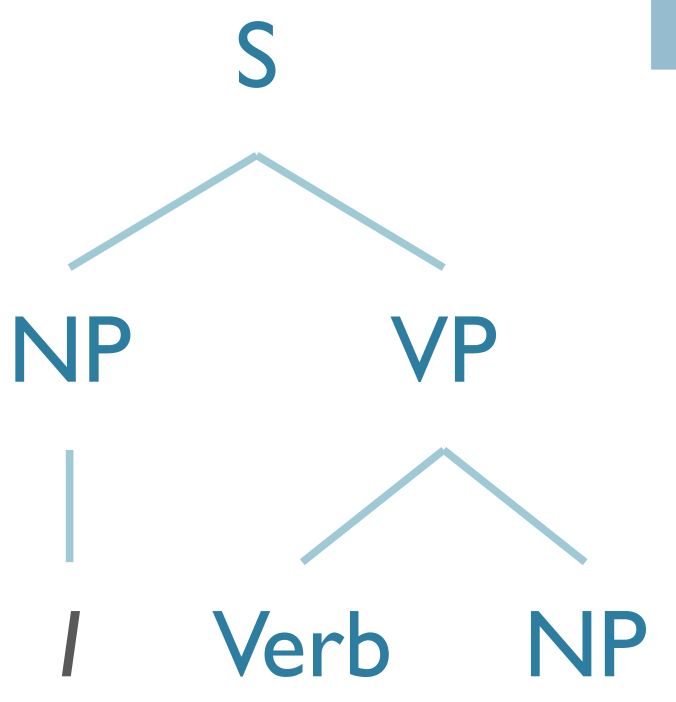

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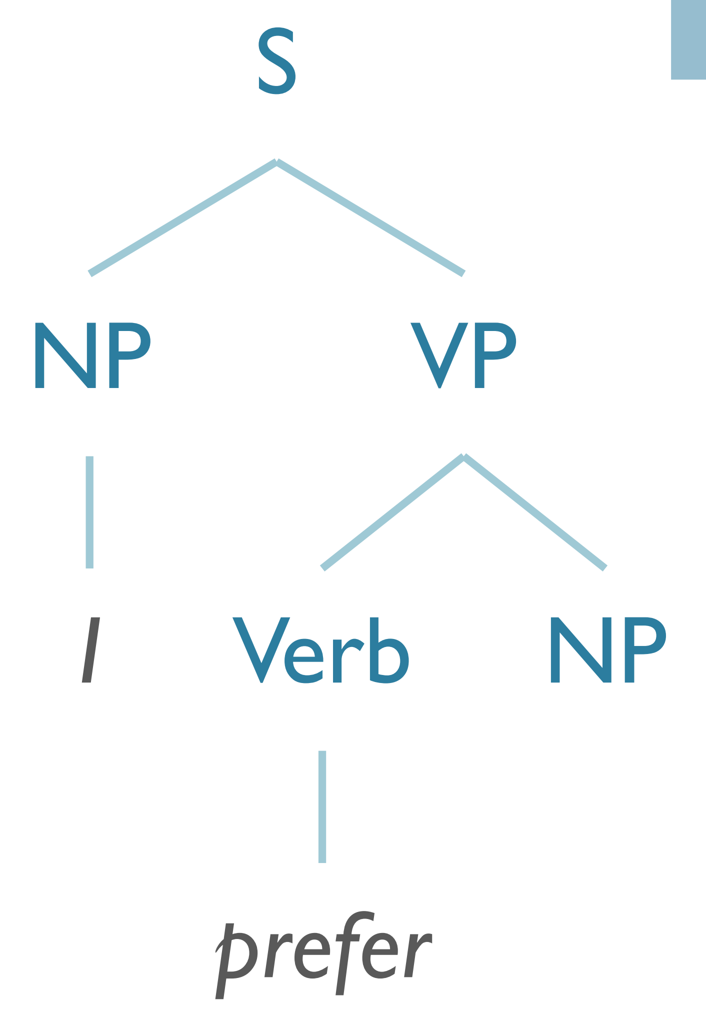

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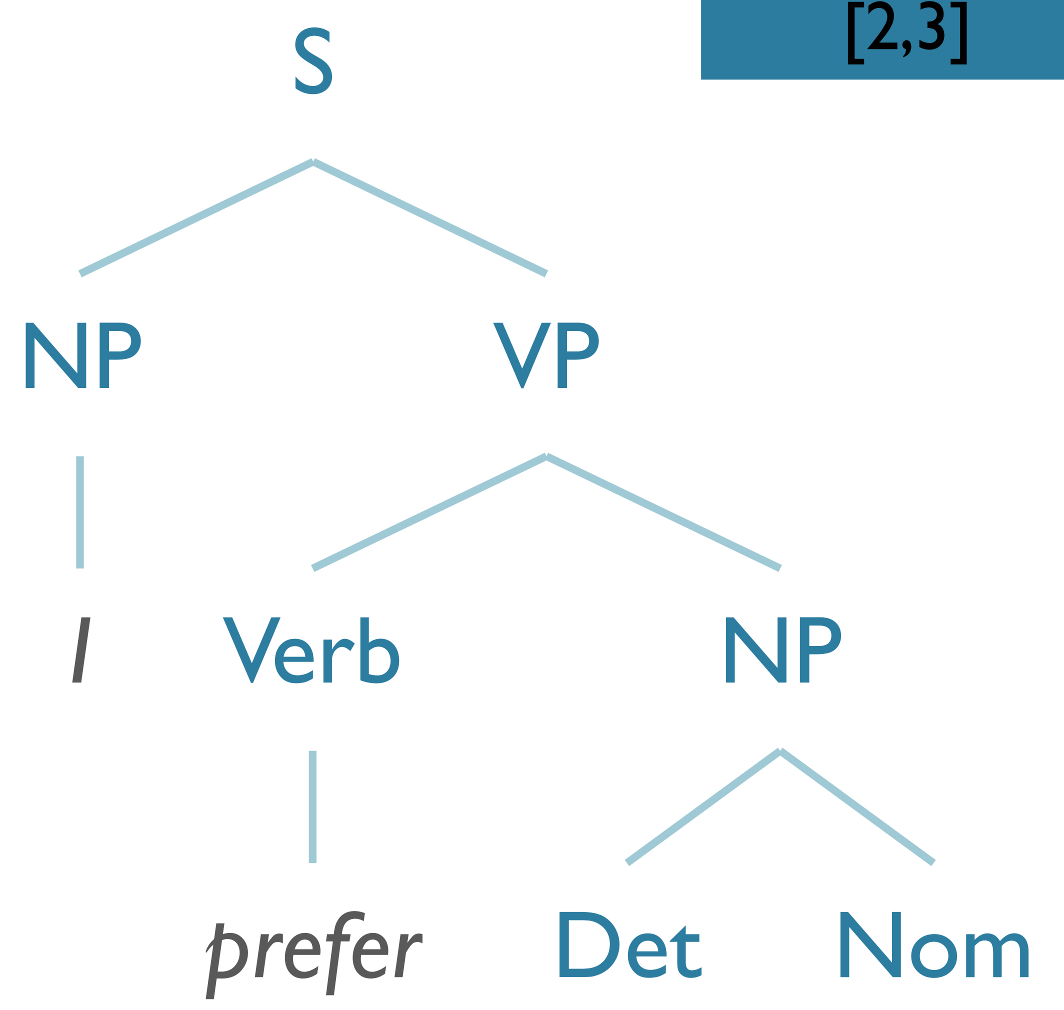

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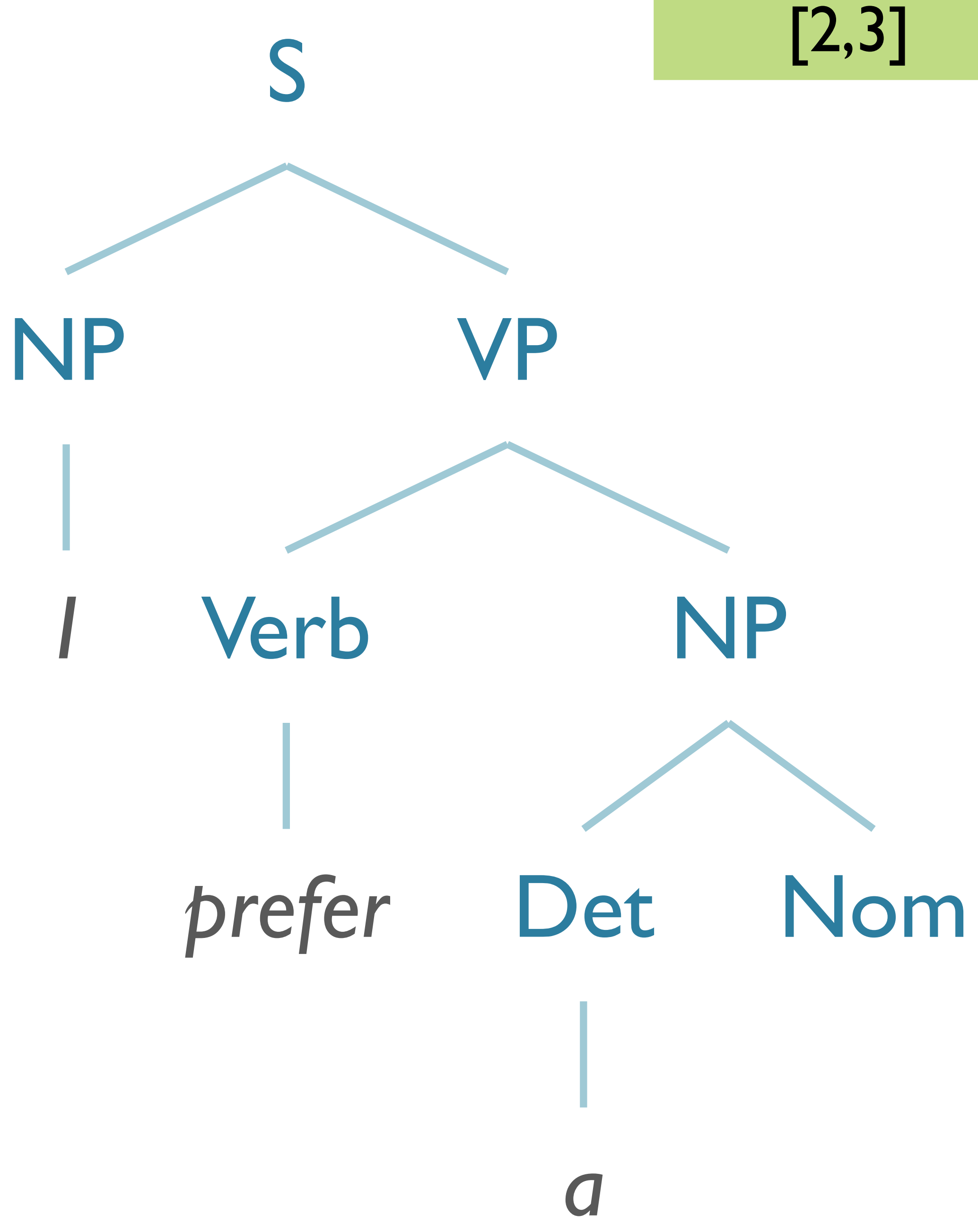

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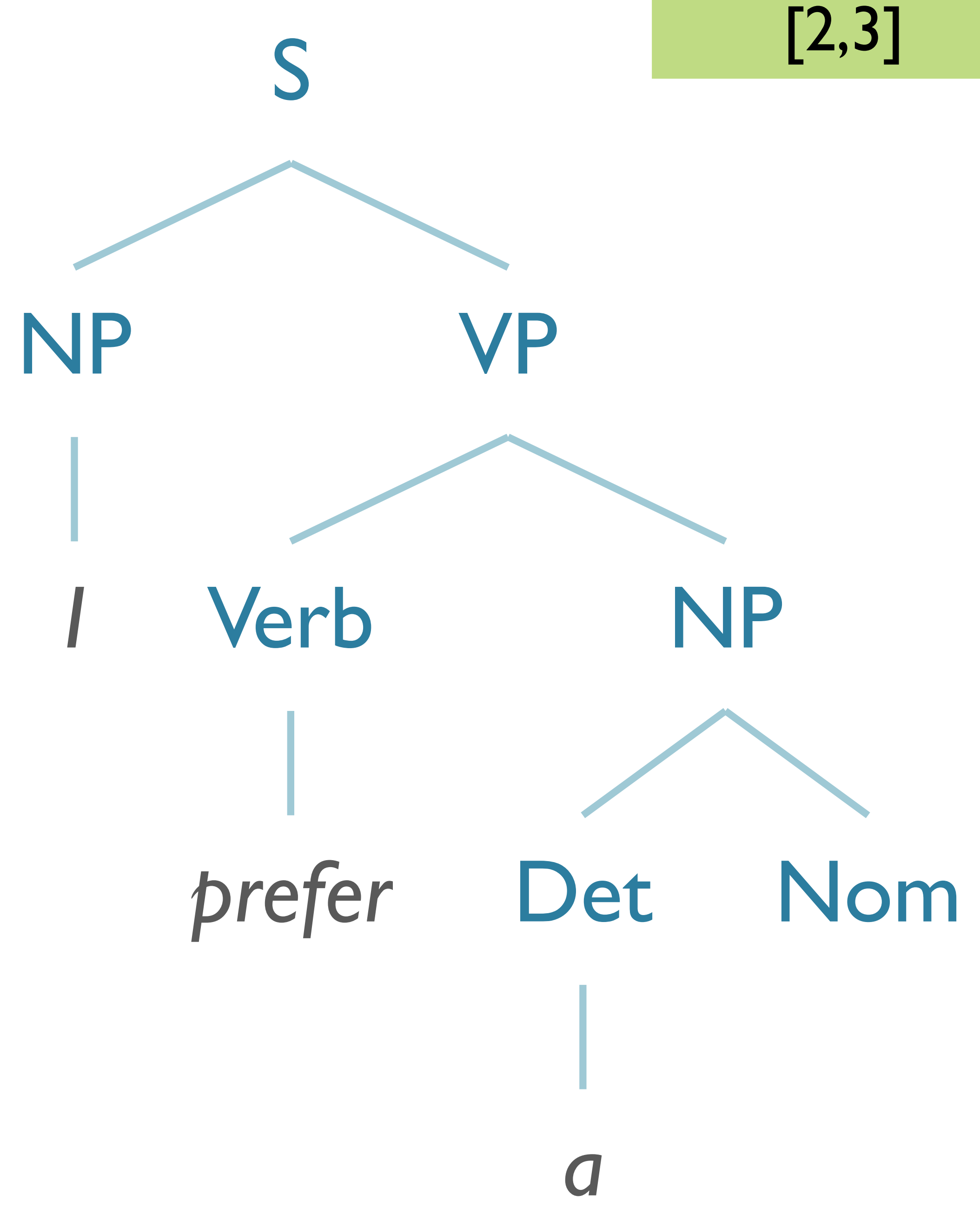

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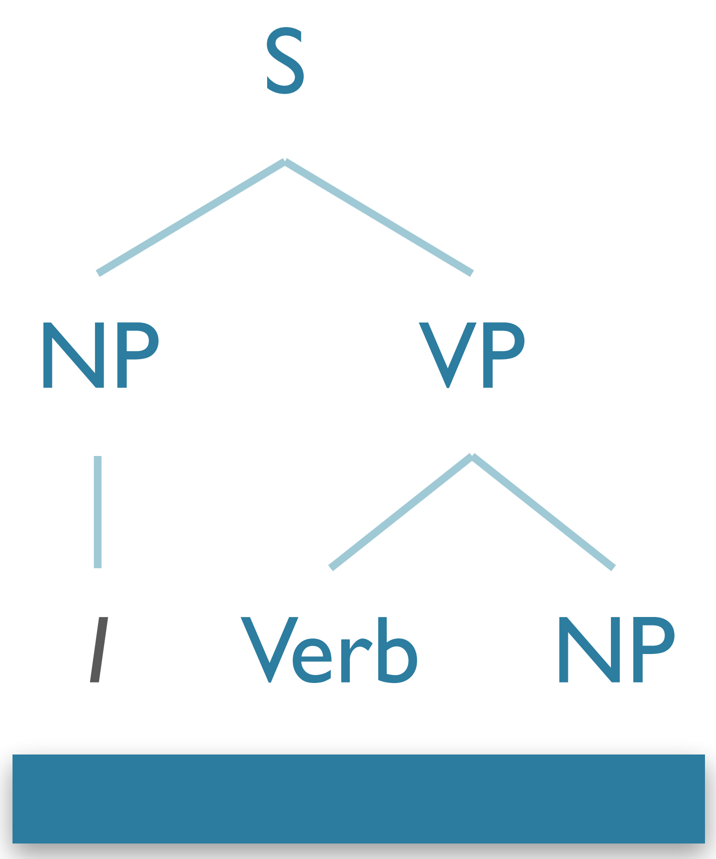

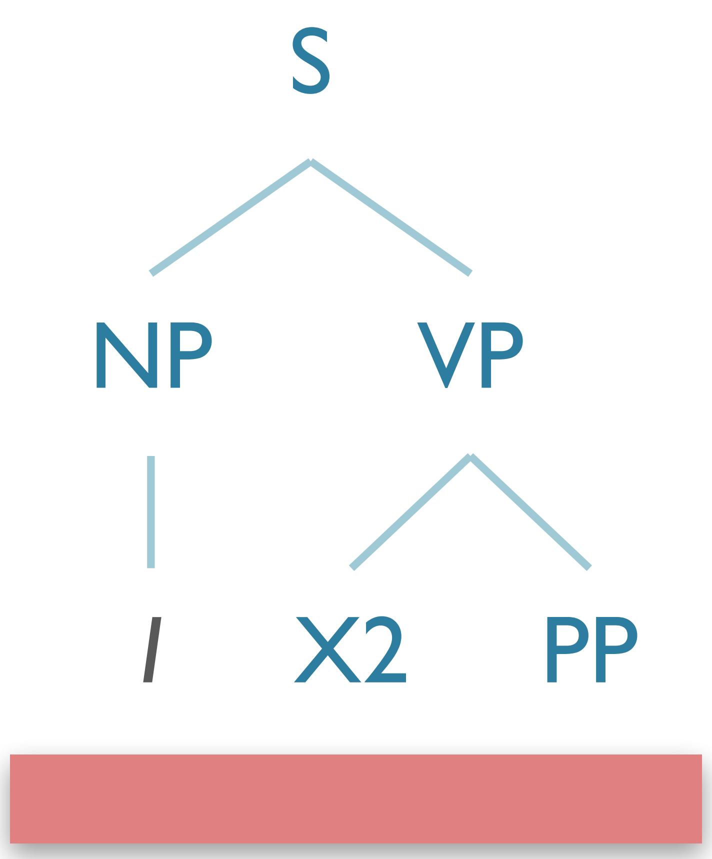

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Probabilistic Context-Free Grammars

Probabilistic Context-free Grammars: Roadmap

Motivation: Ambiguity

Approach:

- Definition
- Disambiguation
- Parsing
- Evaluation
- Enhancements

Motivation

What about ambiguity?

Current algorithm can ***represent*** it

…can't resolve it.

Probabilistic Parsing

- Provides strategy for solving disambiguation problem
  - Compute the probability of all analyses
  - Select the most probable
- Employed in language modeling for speech recognition
  - N-gram grammars predict words, constrain search
  - Also, constrain generation, translation

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S	a designated **start symbol**

PCFGs

- Augment each production with probability that LHS will be expanded as RHS
  - $P(A \rightarrow \beta)$
  - $P(A \rightarrow \beta | A)$
  - $P(\beta | A)$
  - $P(RHS \mid LHS)$
- NB: the first is often used; but the latter are what's really meant.

PCFGs

- Sum over all possible expansions is 1

$$\sum_{\beta} P(A \rightarrow \beta) = 1$$

- A PCFG is ***consistent*** if sum of probabilities of all sentences in language is 1
  - Recursive rules often yield inconsistent grammars *(Booth & Thompson, 1973)*

Example PCFG: Augmented $\mathcal{L}_1$

Grammar		Lexicon
$S \rightarrow NP\ VP$	[.80]	$Det \rightarrow that\ [.10] \mid a\ [.30] \mid the\ [.60]$
$S \rightarrow Aux\ NP\ VP$	[.15]	$Noun \rightarrow book\ [.10] \mid flight\ [.30] \mid meal\ [.15] \mid money\ [0.5]$
$S \rightarrow VP$	[.05]	$\mid flights\ [0.40] \mid dinner\ [.10]$
$NP \rightarrow Pronoun$	[.35]	$Verb \rightarrow book\ [.30] \mid include\ [.30] \mid prefer\ [.40]$
$NP \rightarrow Proper\text{-}Noun$	[.30]	$Pronoun \rightarrow I\ [.40] \mid she\ [.05] \mid me\ [.15] \mid you\ [.40]$
$NP \rightarrow Det\ Nominal$	[.20]	$Proper\text{-}Noun \rightarrow Houston\ [.60] \mid NWA\ [.40]$
$NP \rightarrow Nominal$	[.15]	$Aux \rightarrow does\ [.60] \mid can\ [.40]$
$Nominal \rightarrow Noun$	[.75]	$Preposition \rightarrow from\ [.30] \mid to\ [.30] \mid on\ [.20] \mid near\ [.15]$
$Nominal \rightarrow Nominal\ Noun$	[.20]	$\mid through\ [.05]$
$Nominal \rightarrow Nominal\ PP$	[.05]	
$VP \rightarrow Verb$	[.35]	
$VP \rightarrow Verb\ NP$	[.20]	
$VP \rightarrow Verb\ NP\ PP$	[.10]	
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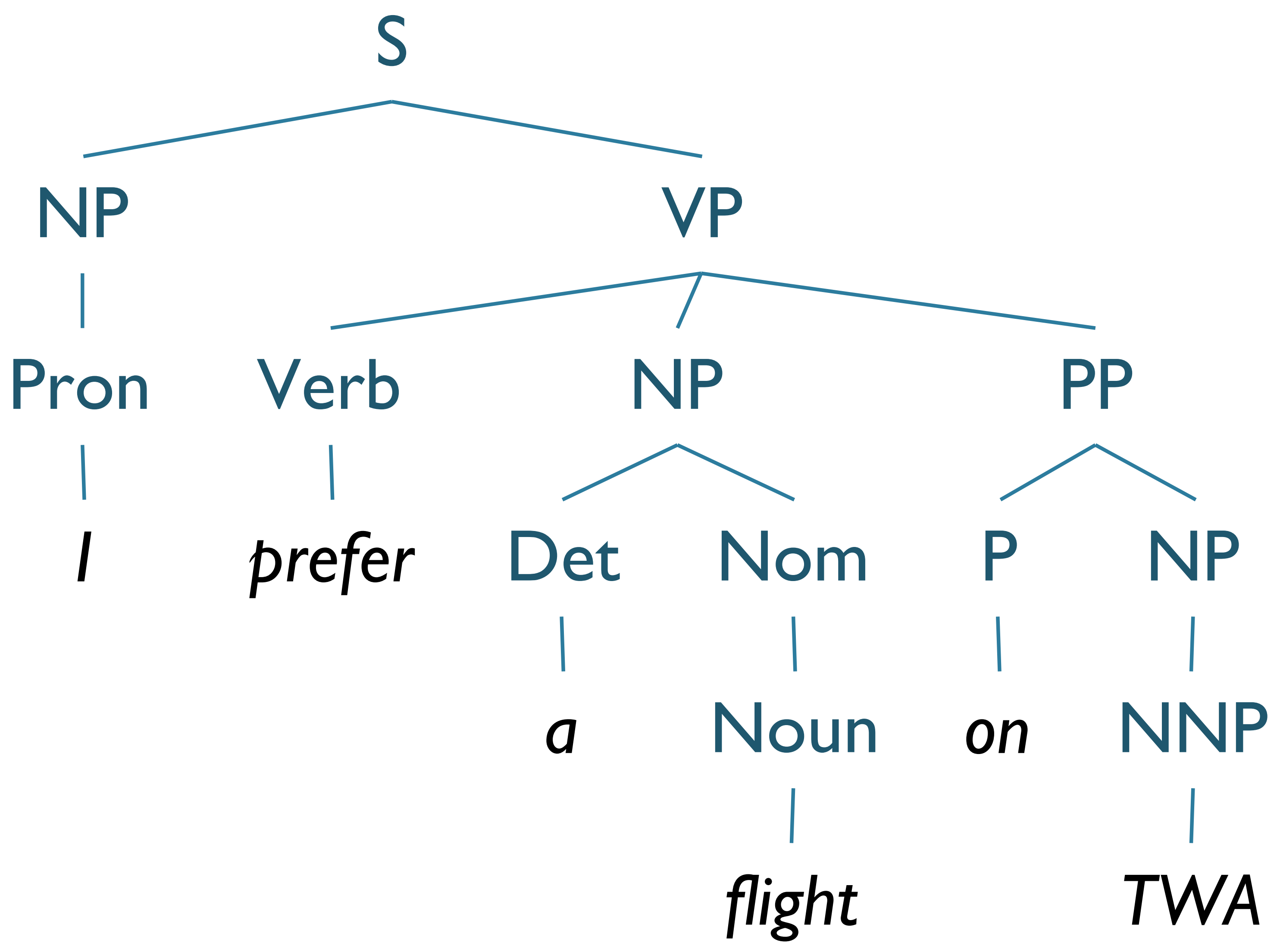
Disambiguation

- A PCFG assigns probability to each parse tree T for input S
- Probability of T: product of all rules used to derive T

$$P(T,S) = \prod_{i=1}^{n} P(RHS_i \mid LHS_i)$$

$$P(T,S) = P(T)P(S \mid T) = P(T)$$



S → NP VP	[0.8]
NP → Pron	[0.35]
Pron → I	[0.4]
VP → V NP PP	[0.1]
V → prefer	[0.4]
NP → Det Nom	[0.2]
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PP → P NP	[1.0]
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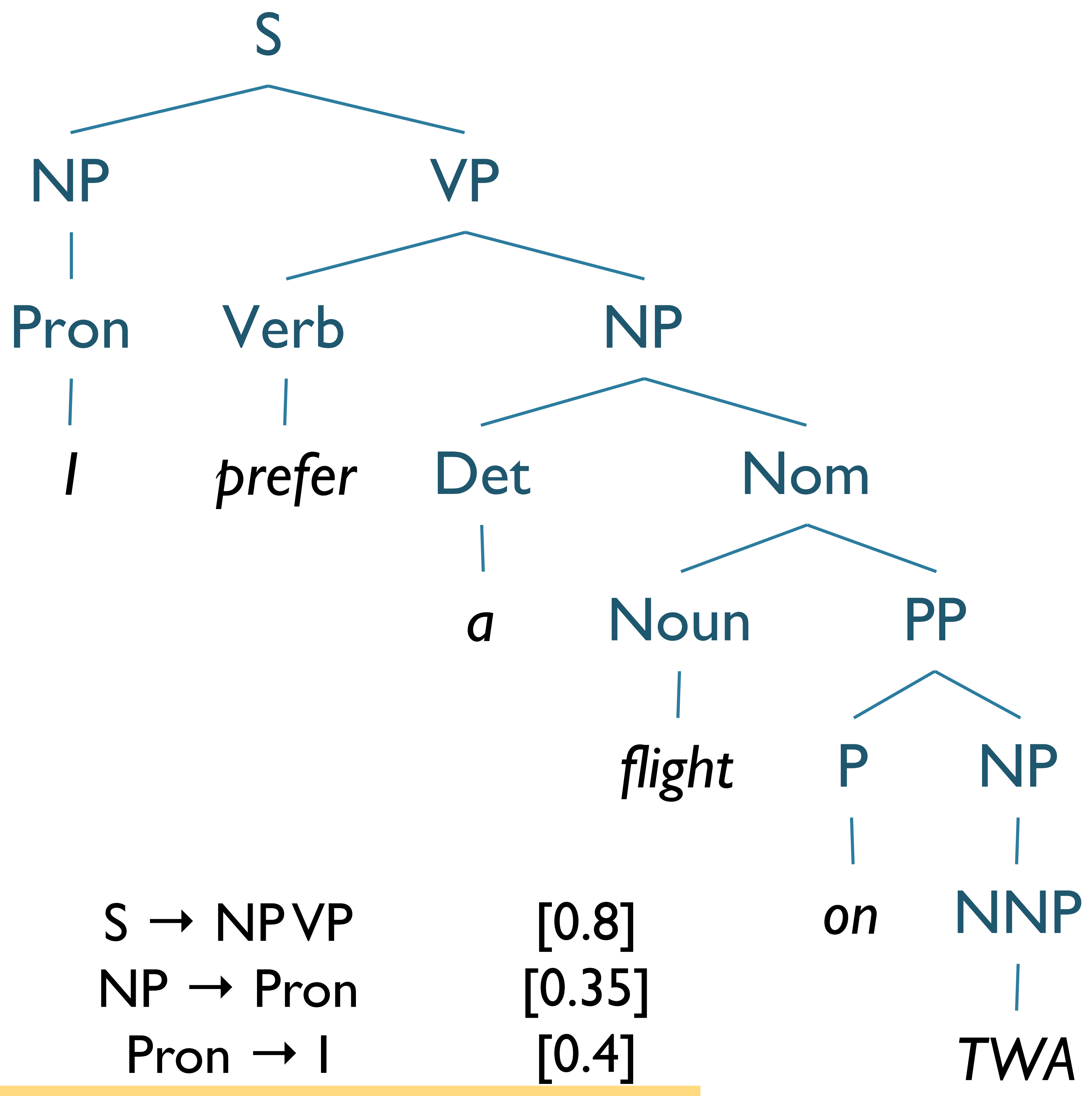

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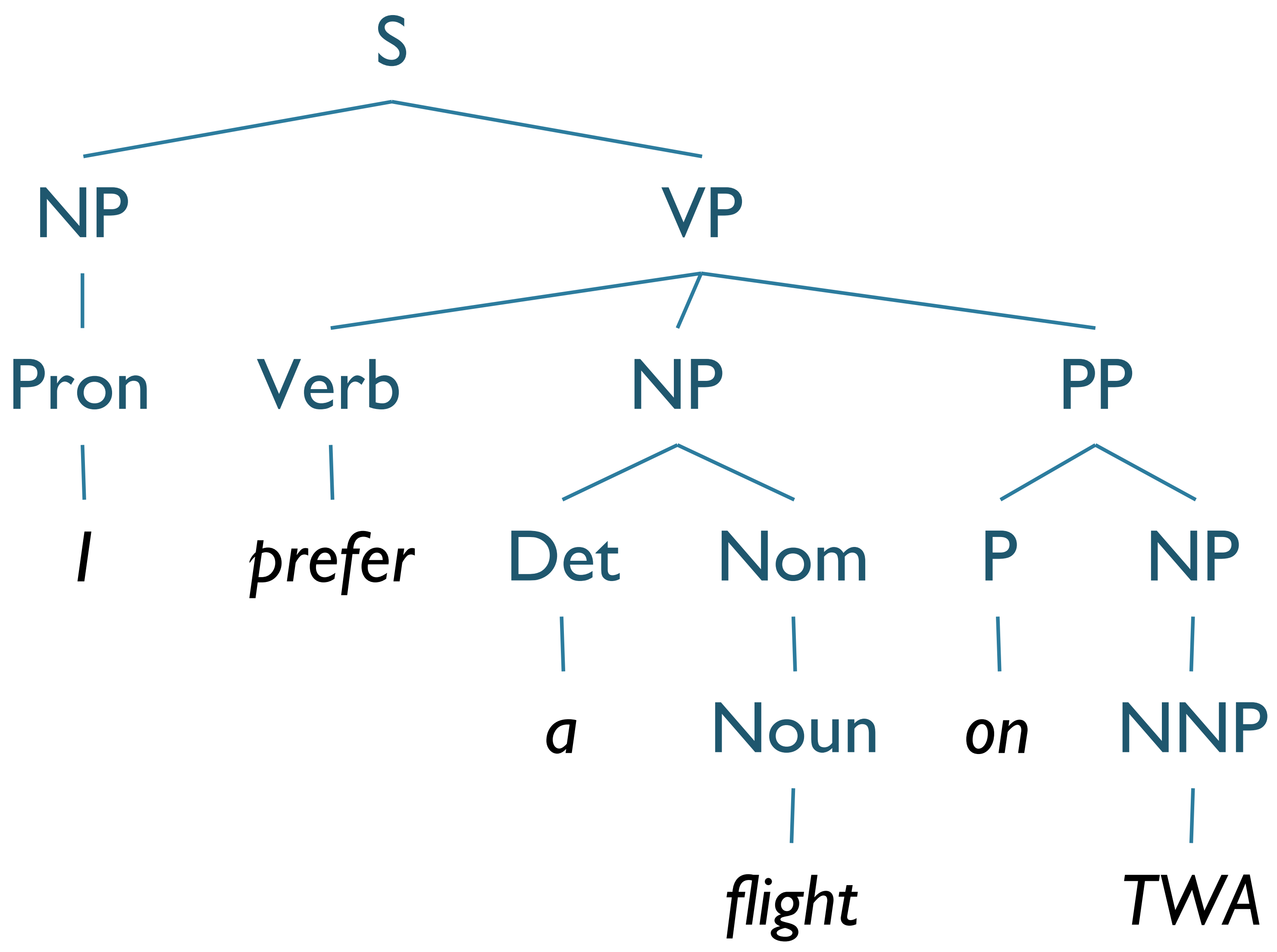

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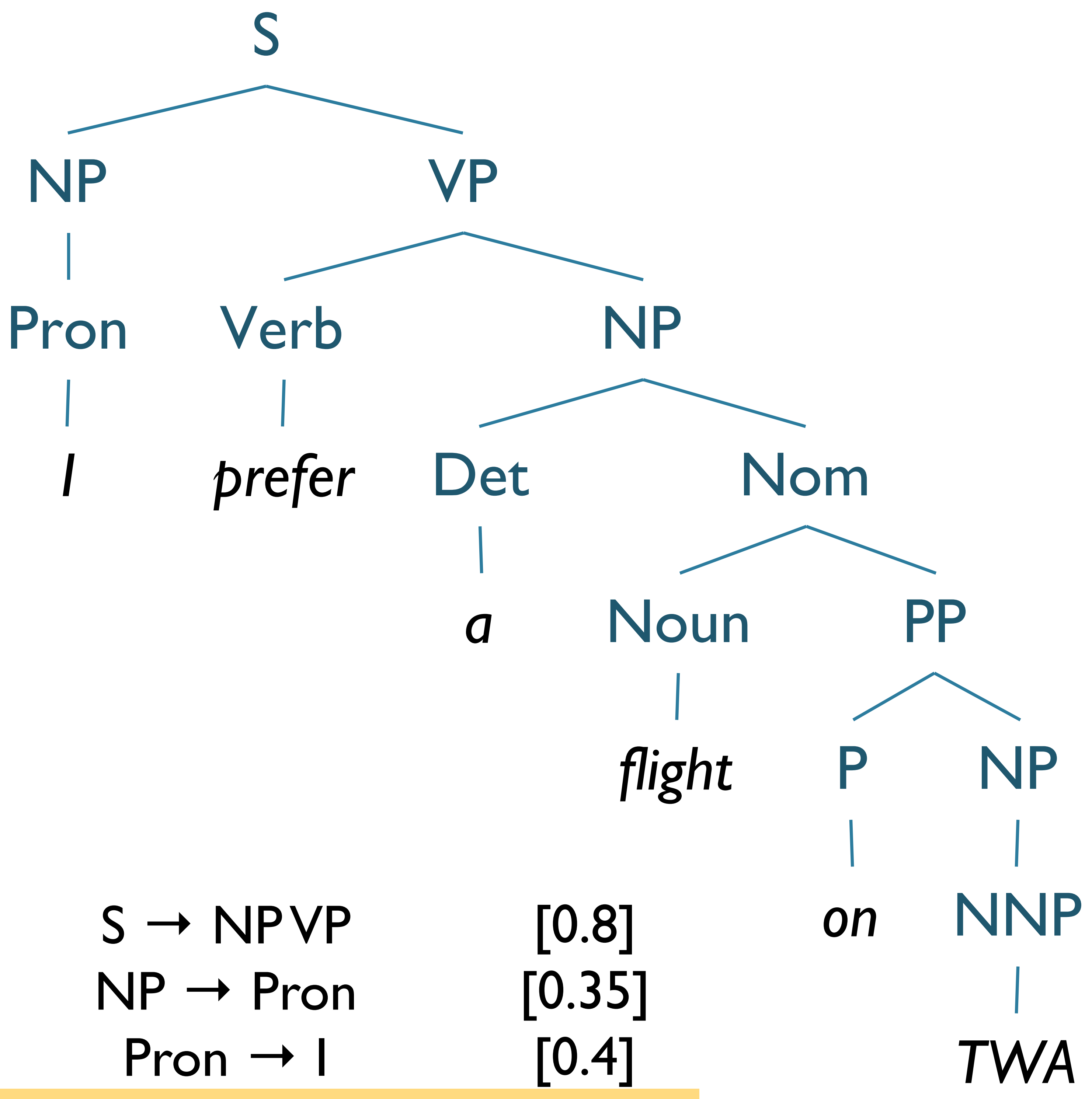
~1.452×10^{-6}



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~1.452×10^{-7}

Parsing Problem for PCFGs

- Select T such that $(s.t.)$

$$\hat{T}(S) = \underset{T\ s.t.\ S=yield(T)}{\operatorname{argmax}} P(T)$$

- String of words S is $yield$ of parse tree
- Select the tree $\hat{T}$ that maximizes the probability of the parse

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  - Approximate using conditioning on limited context: $P(w_i \mid w_{i-1}) = \dfrac{P(w_{i-1}, w_i)}{P(w_{i-1})}$
- PCFGs are able to give probability of entire string without as bad sparsity

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- To model a whole sentence with *n-grams* either:
    - Must use 10+-grams… too sparse
    - Approximate using conditioning on limited context: $P(w_i \mid w_{i-1}) = \dfrac{P(w_{i-1}, w_i)}{P(w_{i-1})}$
- PCFGs are able to give probability of entire string without as bad sparsity
- Model probability of *syntactically valid* sentences

Application: Language Modeling

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- PCFGs are able to give probability of entire string without as bad sparsity
- Model probability of *syntactically valid* sentences
  - Not just probability of sequence of words

PCFGs: Parsing

Probabilistic CKY (PCKY)

- Like regular CKY
  - Assumes grammar in Chomsky Normal Form (CNF)
    - $A \rightarrow B\ C$
    - $A \rightarrow \mathrm{w}$
  - Represent input with indices b/t words:
    - $_0$ Book $_1$ that $_2$ flight $_3$ through $_4$ Houston $_5$

Probabilistic CKY (PCKY)

- For input string length n and non-terminals V
  - Cell [i, j, A] in (n+1) × (n+1) × V matrix
  - Contains probability that A spans $[i, j]$

PCKY Algorithm

function PROBABILISTIC-CKY-PARSE(*words*, *grammar*) **returns** most probable parse and its probability

for j ← **from** 1 to LENGTH(*words*) **do**

for all { A | A → *words[j]* ∈ *grammar* }

$table[\ j{-}1,\ j,\ A\ ] \leftarrow P(A \rightarrow words[j])$

for i ← **from** j–2 **downto** 0 **do**

for $k \leftarrow i+1$ **to** j–1 **do**

for all { A | A → $B\ C$ ∈ *grammar*,

and $table[i, k, B] > 0$ **and** $table[\ k, j, C\ ] > 0$ }

if $(table[\ i, j, A\ ] < P(A \rightarrow BC) \times table[\ i, k, B\ ] \times table[\ k,j,C\ ])$ **then**

$table[\ i, j, A\ ] \leftarrow P(A \rightarrow BC) \times table[i,k,B] \times table[k,j,C]$

$back[\ i, j, A\ ] \leftarrow \{\ k, B, C\ \}$

return BUILD_TREE(*back*[1, LENGTH(*words*), S]), *table*[1,LENGTH(*words*), S]

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PCKY Grammar Segment

$S \rightarrow NP\ VP$	[0.80]	$Det \rightarrow$ the	[0.40]
$NP \rightarrow Det\ N$	[0.30]	$Det \rightarrow$ a	[0.40]
$VP \rightarrow V\ NP$	[0.20]	$V \rightarrow$ includes	[0.05]
		$N \rightarrow$ meal	[0.01]
		$N \rightarrow$ flight	[0.02]

PCKY Matrix

$S \rightarrow NP\ VP$ [0.80]
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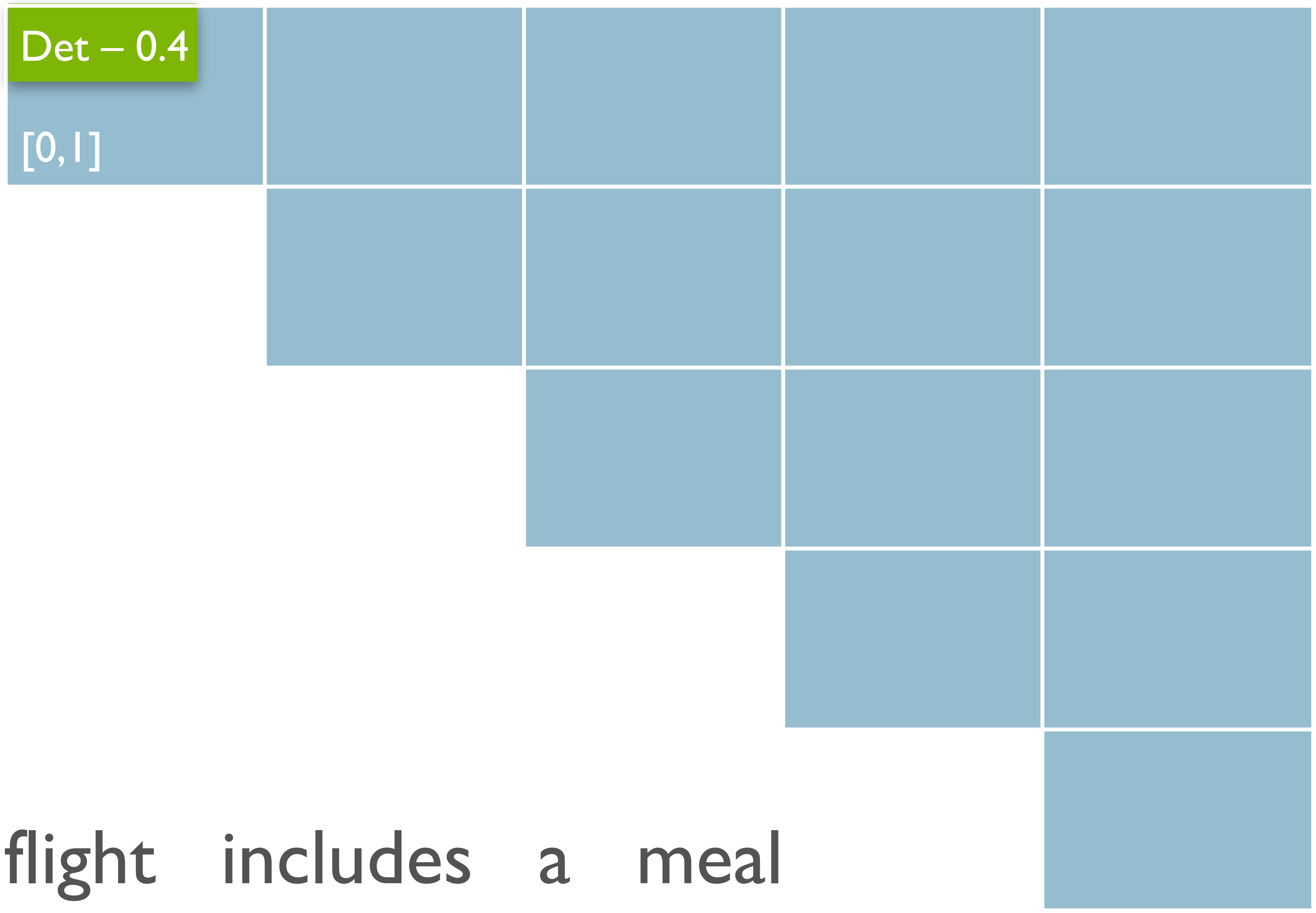

The flight includes a meal

0 1 2 3 4 5

PCKY Matrix

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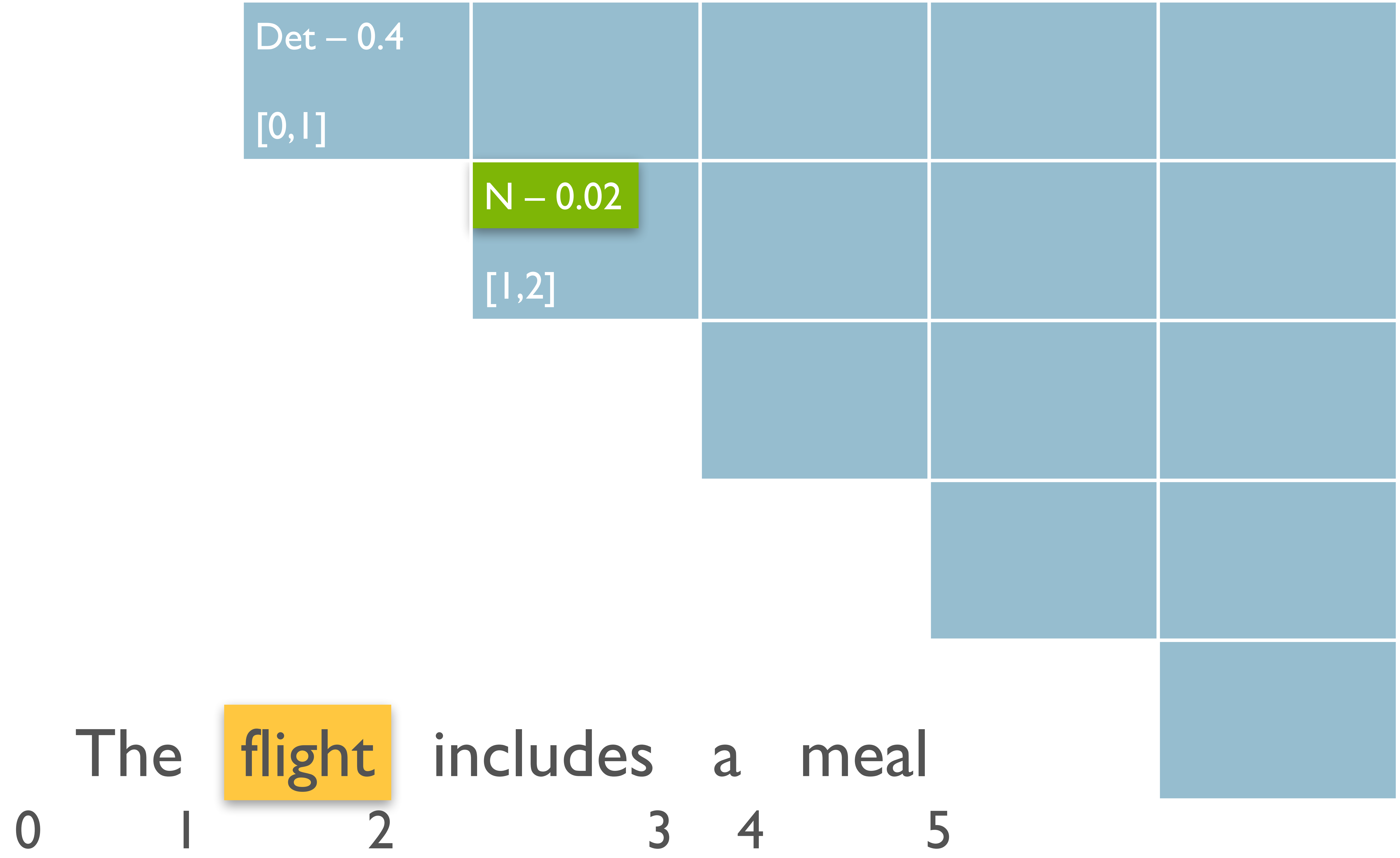

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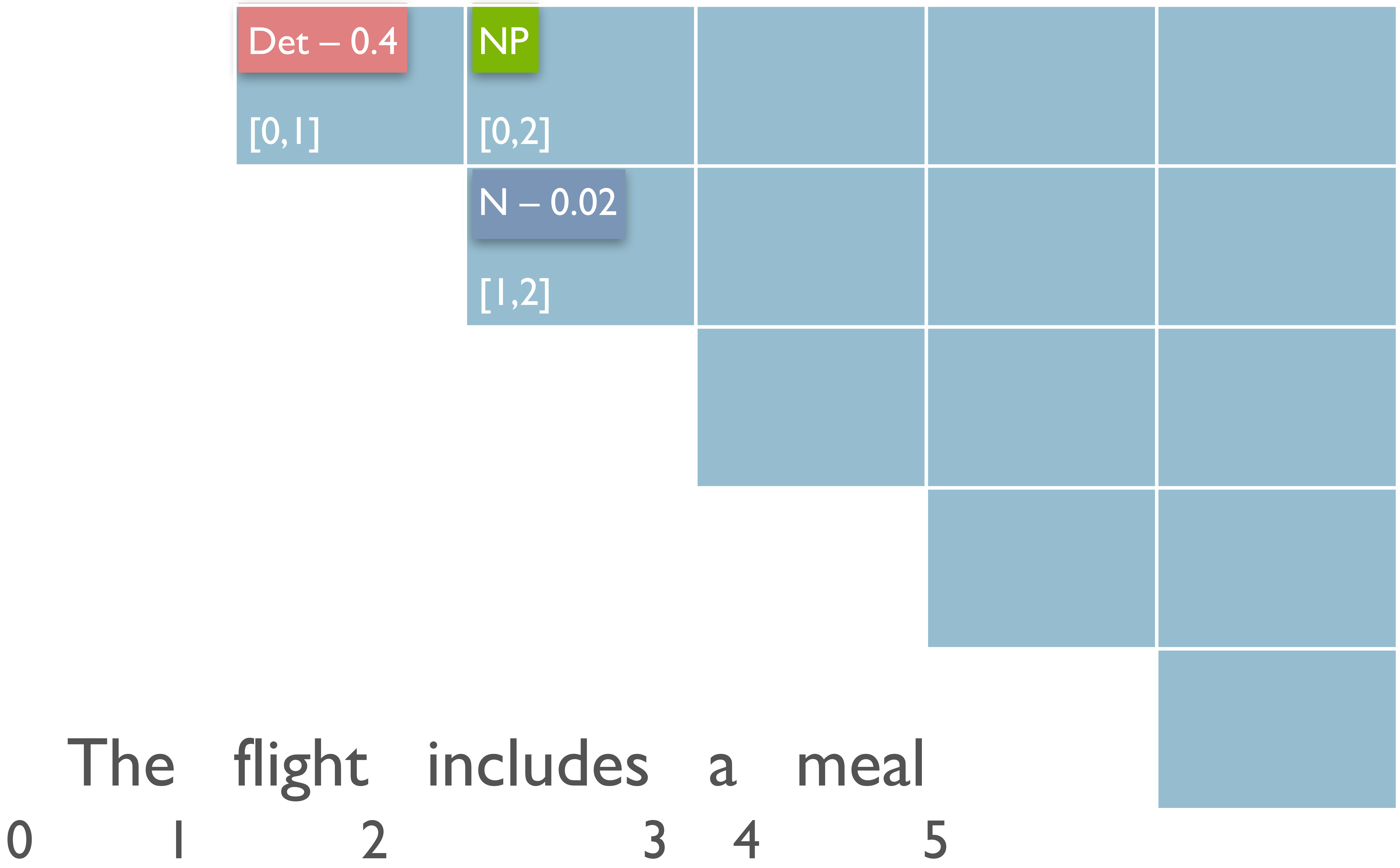

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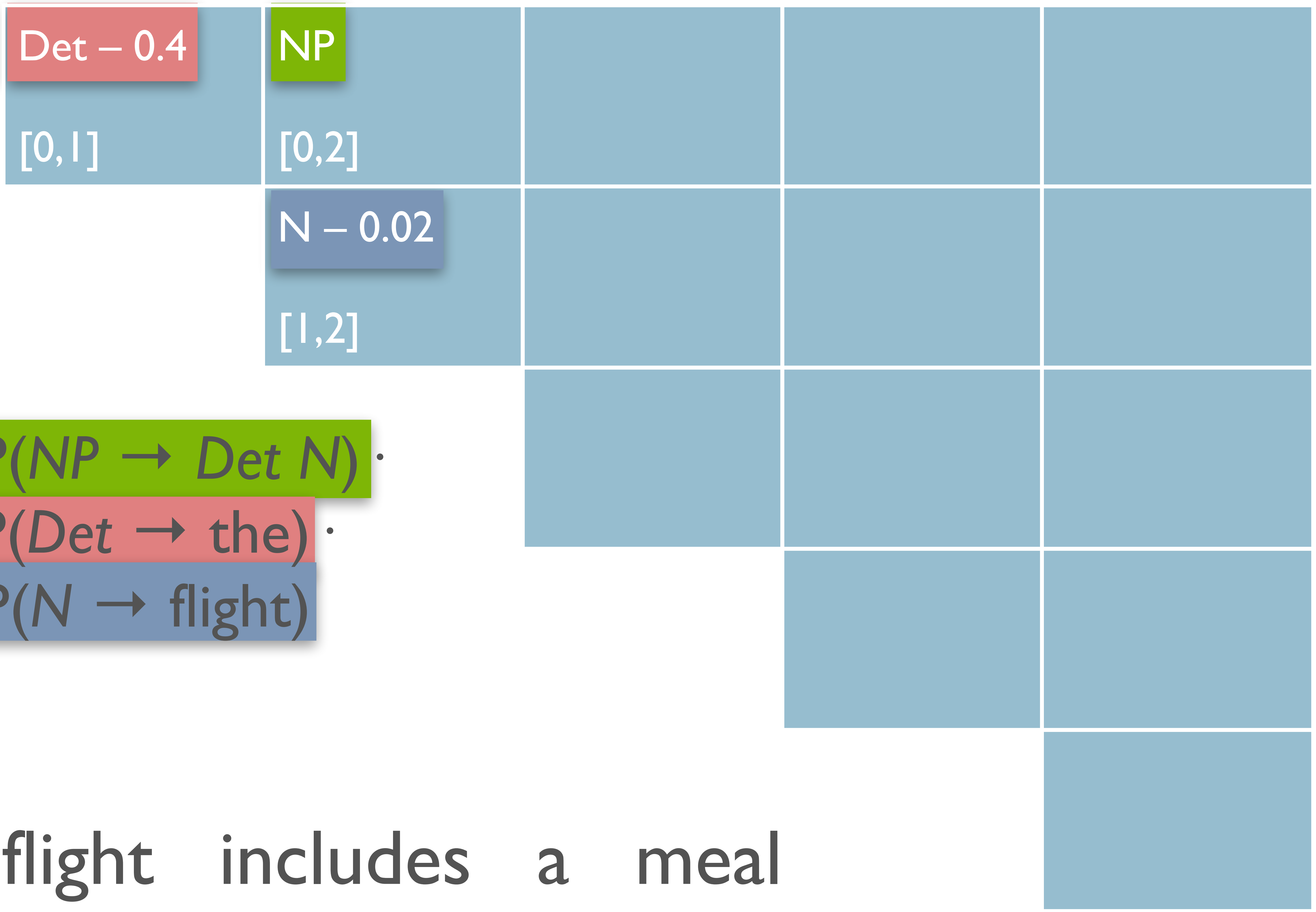

$$P = P(NP \rightarrow Det\ N) \cdot P(Det \rightarrow \text{the}) \cdot P(N \rightarrow \text{flight})$$

The flight includes a meal

0 1 2 3 4 5

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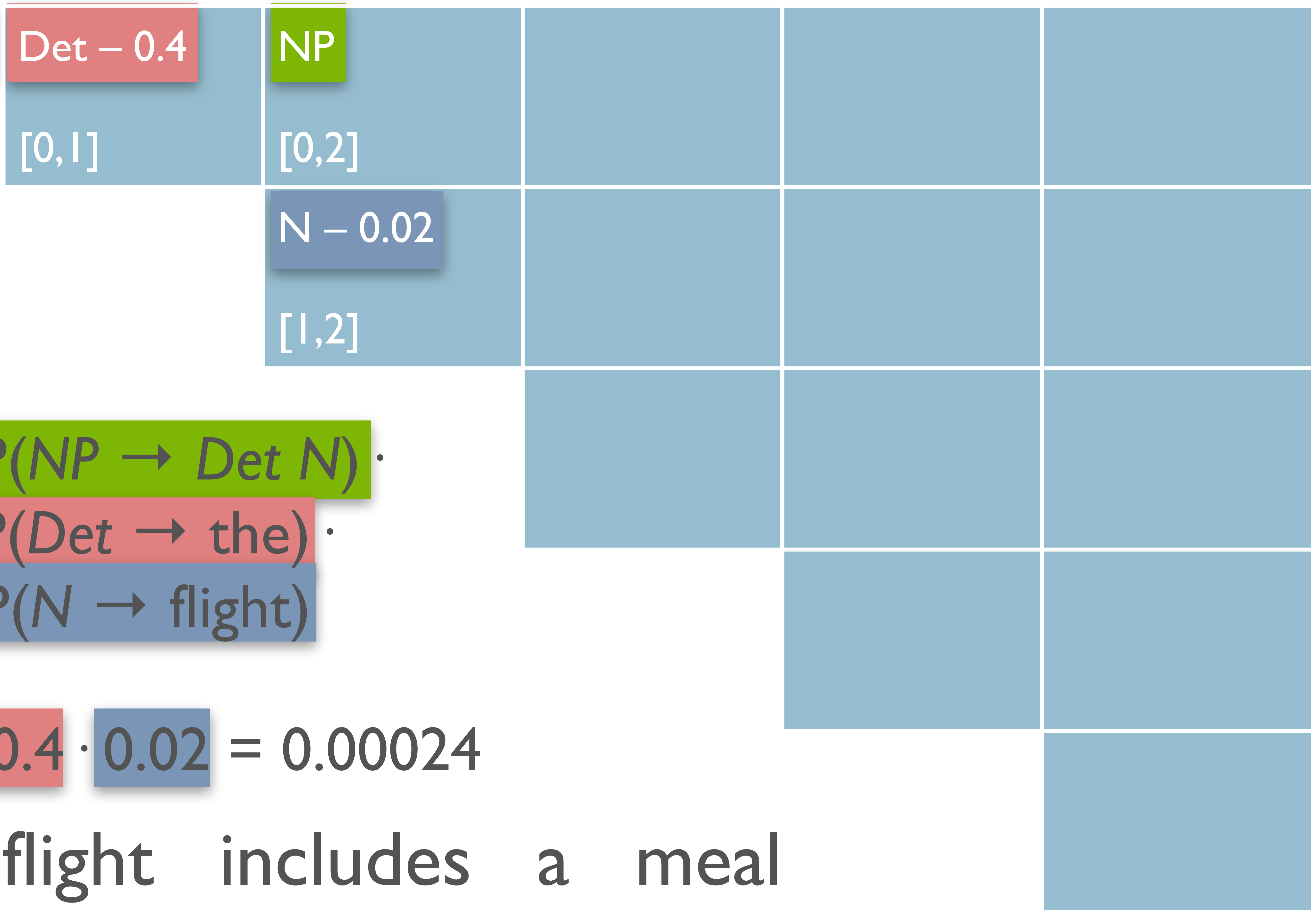

$$P = P(NP \rightarrow Det\ N) \cdot P(Det \rightarrow \text{the}) \cdot P(N \rightarrow \text{flight})$$

$$P = 0.3 \cdot 0.4 \cdot 0.02 = 0.00024$$

The flight includes a meal
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Det – 0.4
[0,1]

NP – 0.0024
[0,2]

N – 0.02
[1,2]

$$P = P(NP \rightarrow Det\ N) \cdot P(Det \rightarrow a) \cdot P(N \rightarrow flight)$$

$$P = 0.3 \cdot 0.4 \cdot 0.02 = 0.00024$$

The flight includes a meal
0 1 2 3 4 5

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$N \rightarrow$ flight [0.02]

Det – 0.4 [0,1]	NP – 0.0024 [0,2]	[0,3]	[0,4]	S – 2.304×10^{-8} [0,5]
	N – 0.02 [1,2]	[1,3]	[1,4]	[1,5]
		V – 0.05 [2,3]	[2,4]	VP – 1.2×10^{-5} [2,5]
			Det – 0.4 [3,4]	NP – 0.0012 [3,5]
				N – 0.01 [4,5]

The flight includes a meal

0 1 2 3 4 5

Inducing a PCFG

Learning Probabilities

- Simplest way:
  - Use treebank of parsed sentences

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$$P(\alpha \rightarrow \beta \,|\, \alpha) = \frac{Count(\alpha \rightarrow \beta)}{\sum_\gamma Count(\alpha \rightarrow \gamma)} = \frac{Count(\alpha \rightarrow \beta)}{Count(\alpha)}$$

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$$P(\alpha \rightarrow \beta \,|\, \alpha) = \frac{Count(\alpha \rightarrow \beta)}{\sum_\gamma Count(\alpha \rightarrow \gamma)} = \frac{Count(\alpha \rightarrow \beta)}{Count(\alpha)}$$

- Alternative: Learn probabilities by re-estimating
  - (Later)

Probabilistic Parser Development Paradigm

	Train	Dev	Test
Size	Large (eg. WSJ 2–21, 39,830 sentences)	Small (e.g. WSJ 22)	Small/Med (e.g. WSJ, 23, 2,416 sentences)
Usage	Estimate rule probabilities	Tuning/Verification, Check for Overfit	Held Out, Final Evaluation

Parser Evaluation

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    - Constituents in output match those in reference

Parser Evaluation

- Assume a ‘gold standard’ set of parses for test set
- How can we tell how good the parser is?
- How can we tell how good a parse is?
  - Maximally strict: identical to ‘gold standard’
  - Partial credit:
    - Constituents in output match those in reference
      - Same start point, end point, non-terminal symbol

Parseval

- How can we compute parse score from constituents?
- Multiple Measures:

$$\text{Labeled Recall (LR)} = \frac{\text{\# of } \textbf{correct} \text{ constituents in } \textbf{hypothetical} \text{ parse}}{\text{\# of } \textbf{total} \text{ constituents in } \textbf{reference} \text{ parse}}$$

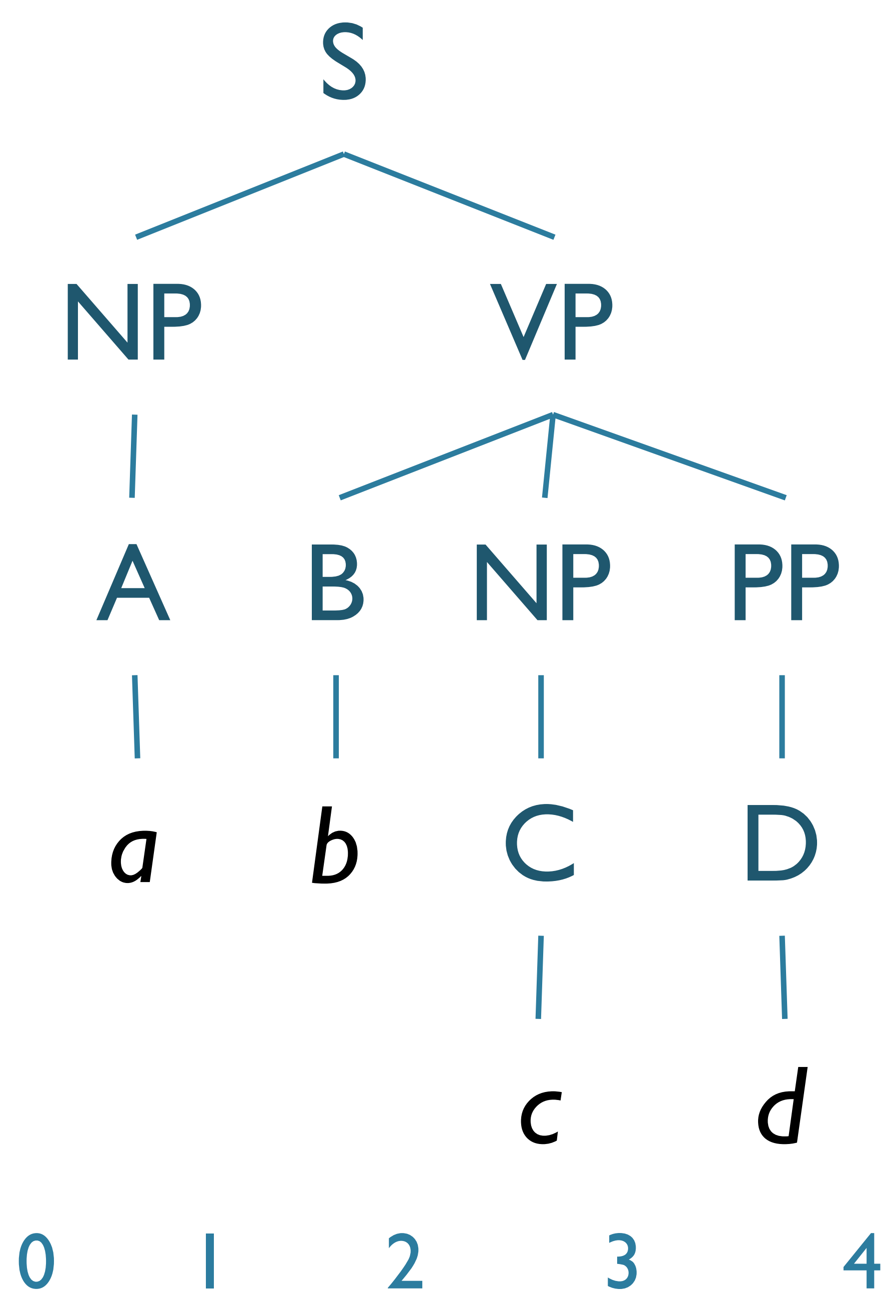
$$\text{Labeled Precision (LP)} = \frac{\text{\# of } \textbf{correct} \text{ constituents in } \textbf{hypothetical} \text{ parse}}{\text{\# of } \textbf{total} \text{ consituents in } \textbf{hypothetical} \text{ parse}}$$

Parseval

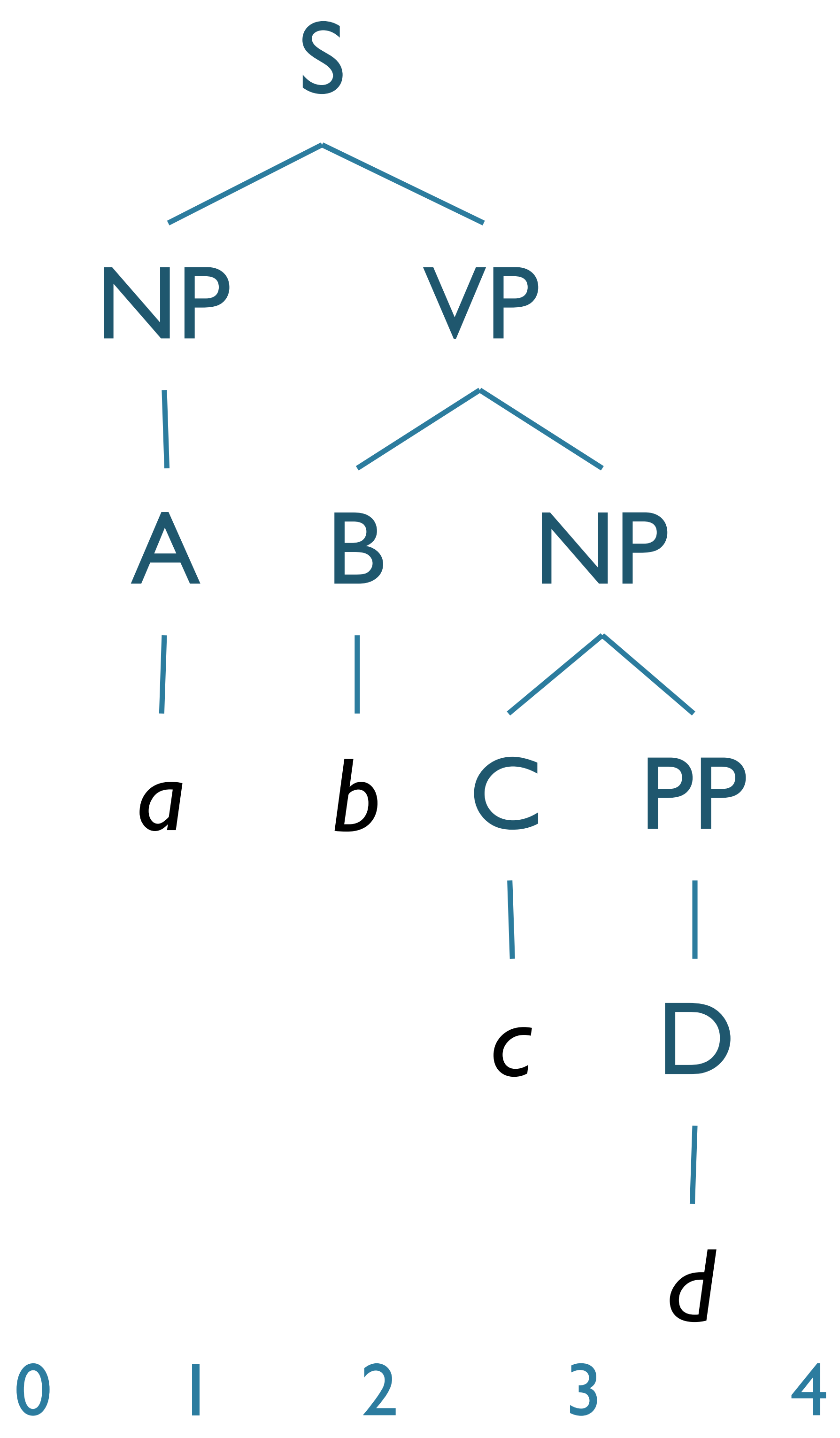
- **F-measure:**
  - Combines precision and recall
  - Let $\beta \in \mathbb{R}$, $\beta > 0$ that adjusts *P* vs. *R* s.t. $\beta \propto \dfrac{R}{P}$
  - F_β -measure is then: $F_\beta = (1 + \beta^2) \cdot \dfrac{P \cdot R}{\beta^2 \cdot P + R}$
  - With F1-measure as $F_1 = \dfrac{2PR}{P + R}$

Evaluation: Example

Reference



Hypothesis



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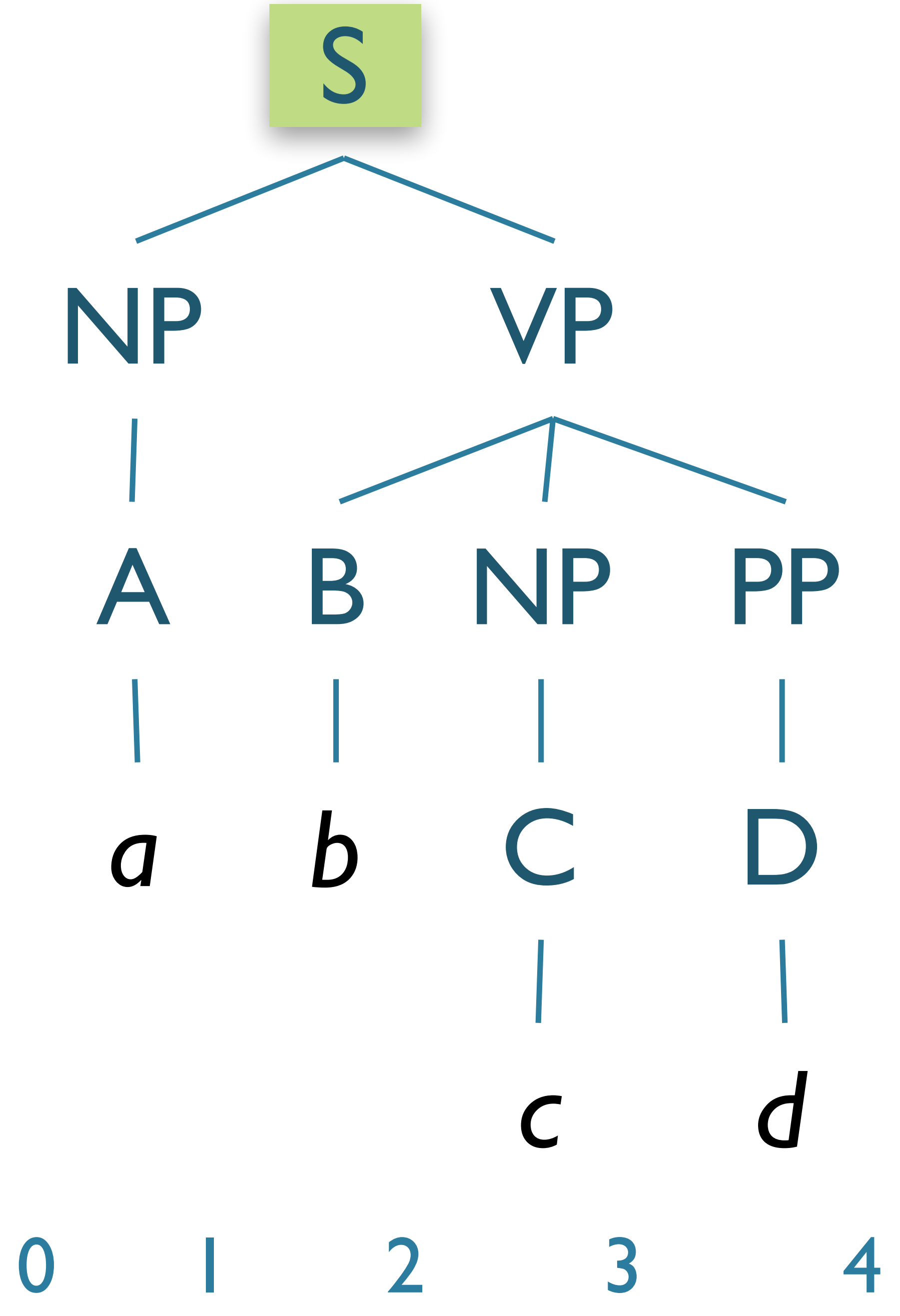
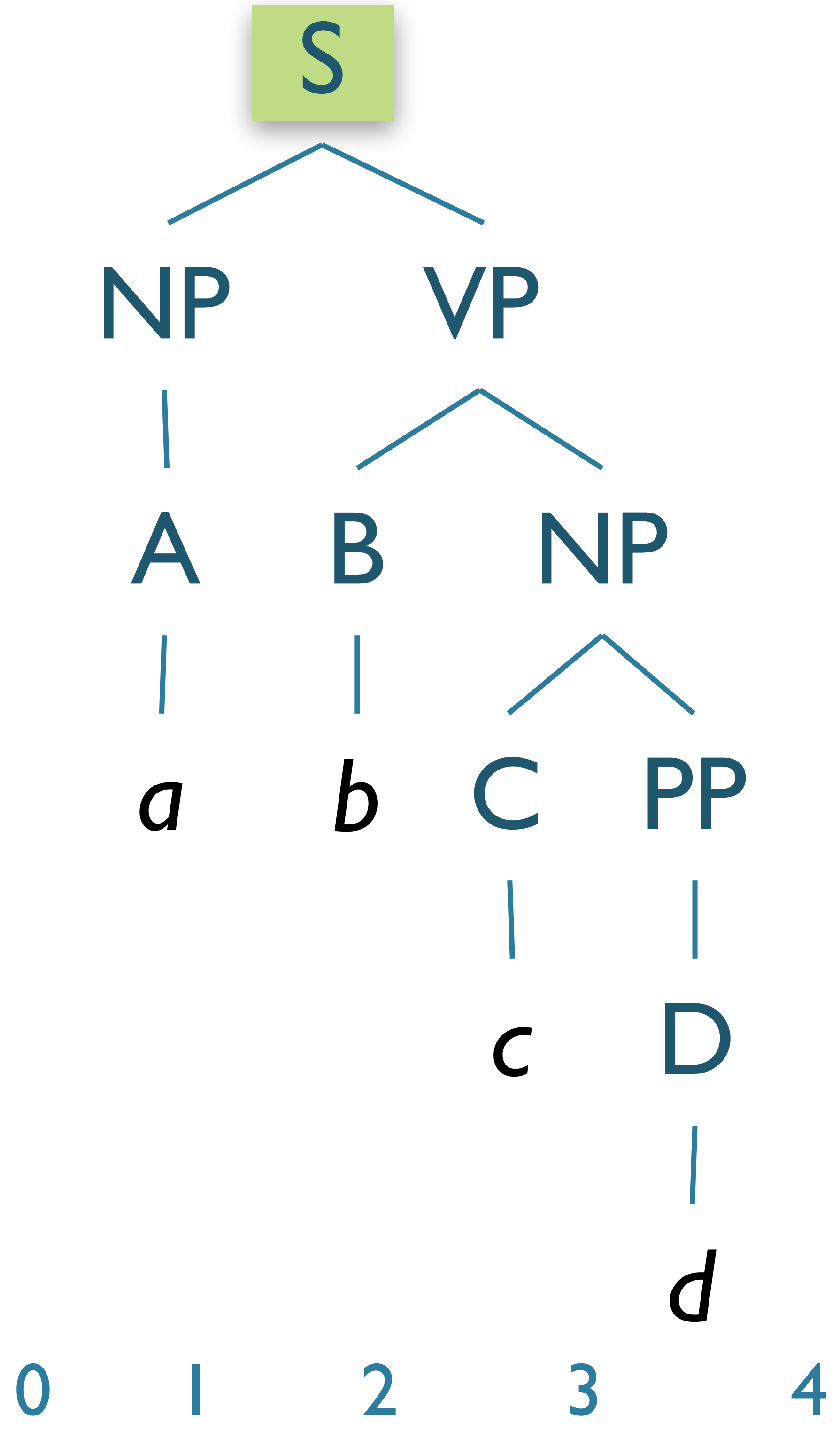


S(0,4)

Hypothesis



S(0,4)

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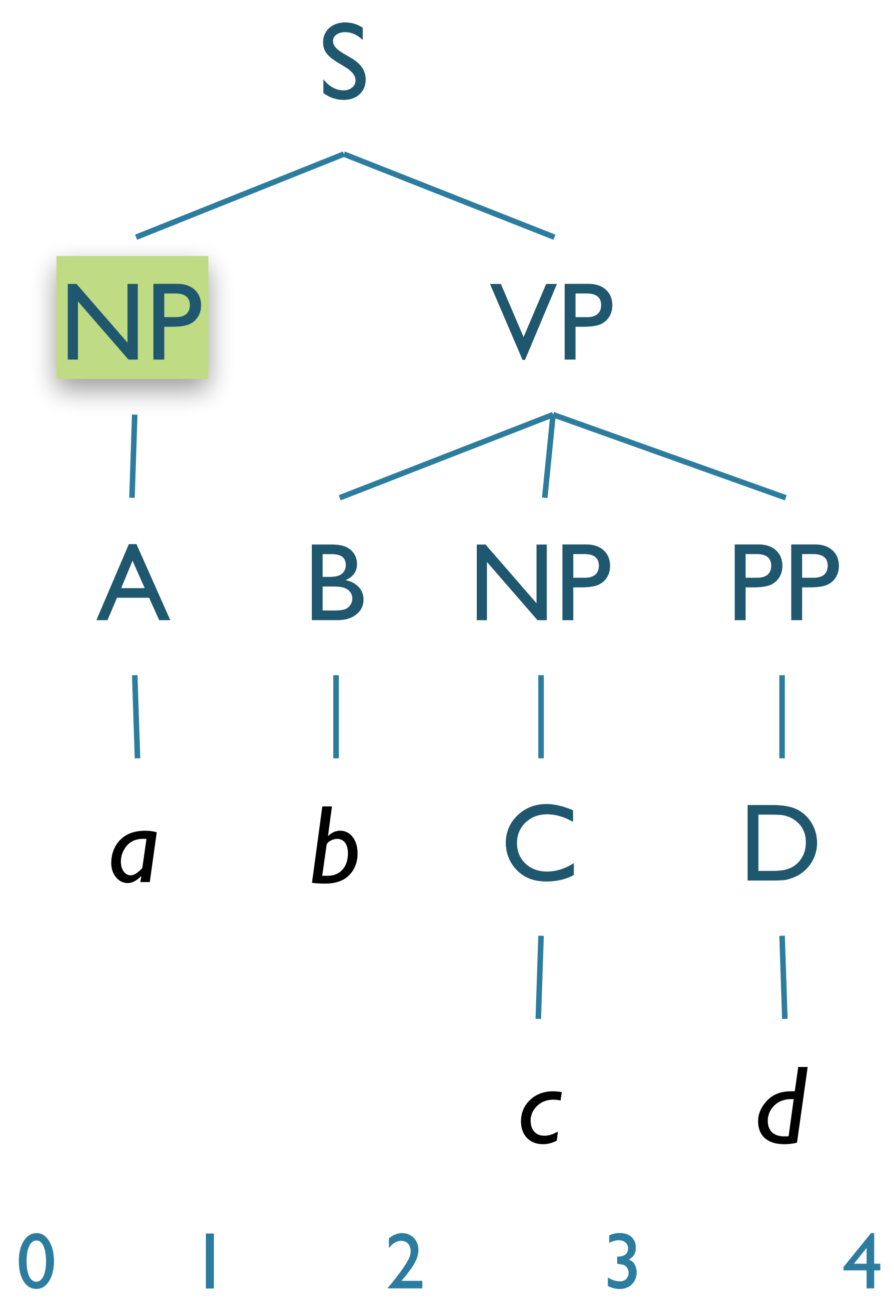
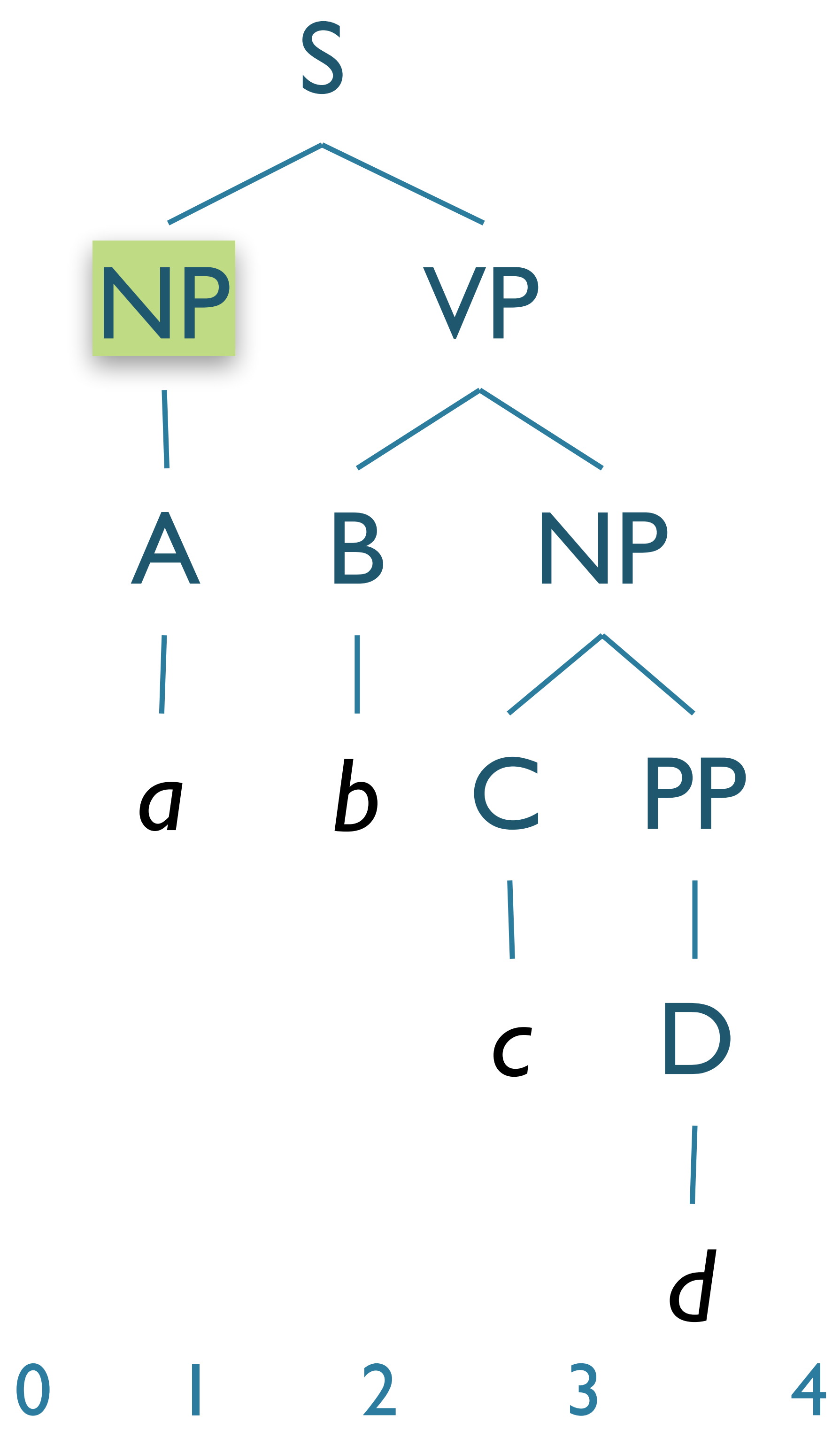


S(0,4)

NP(0,1)

Hypothesis

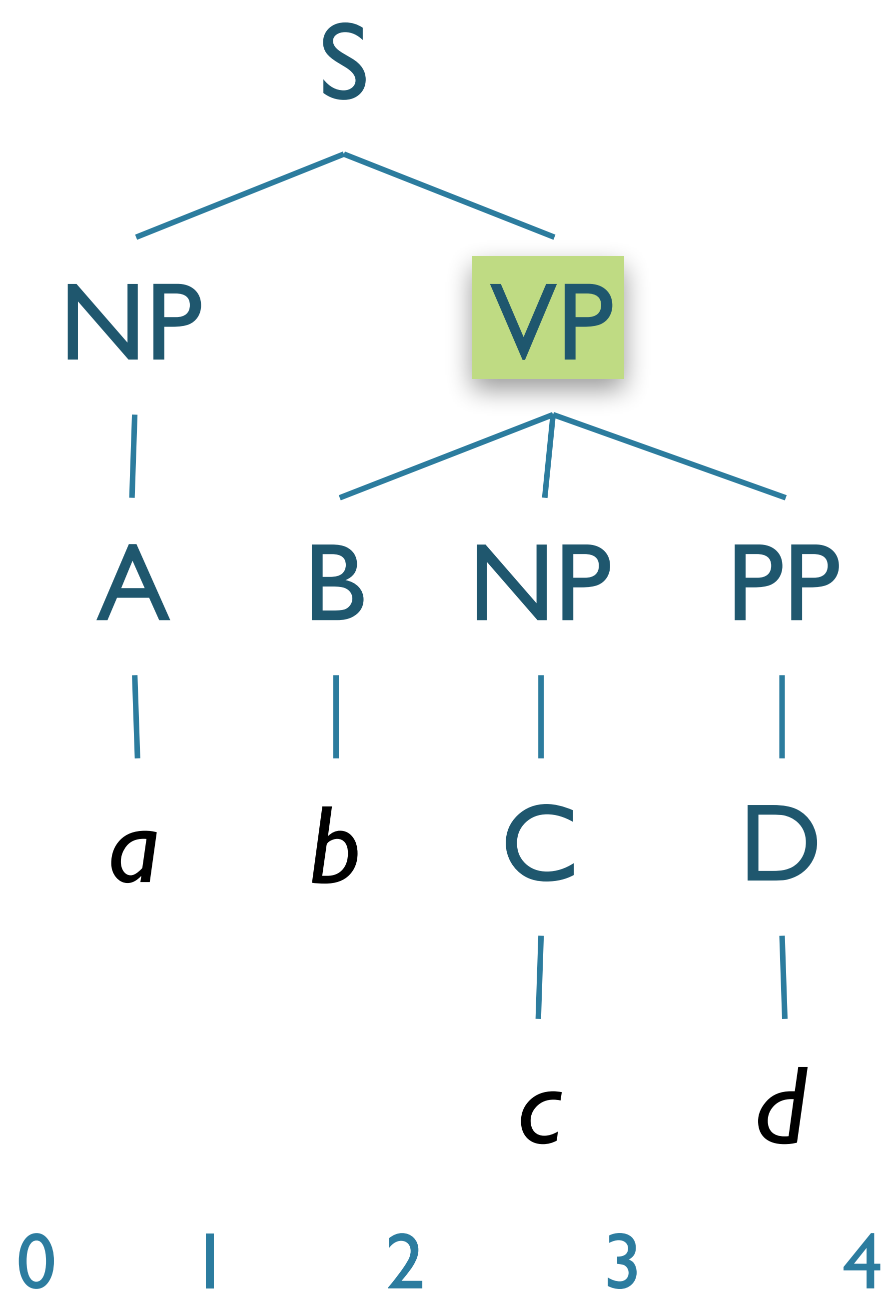


S(0,4)

NP(0,1)

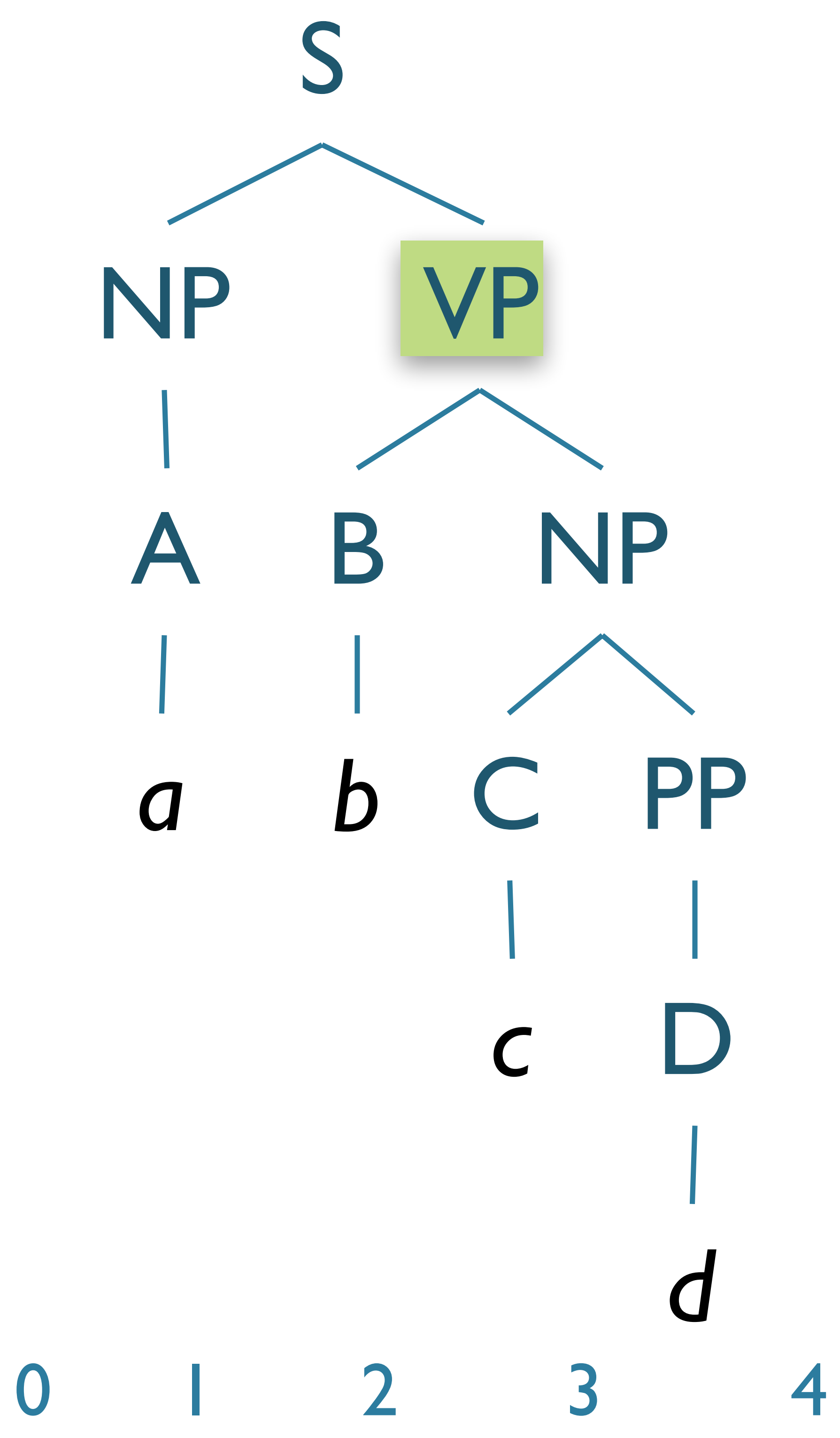
Evaluation: Example

Reference



S(0,4)
NP(0,1)
VP(1,4)

Hypothesis



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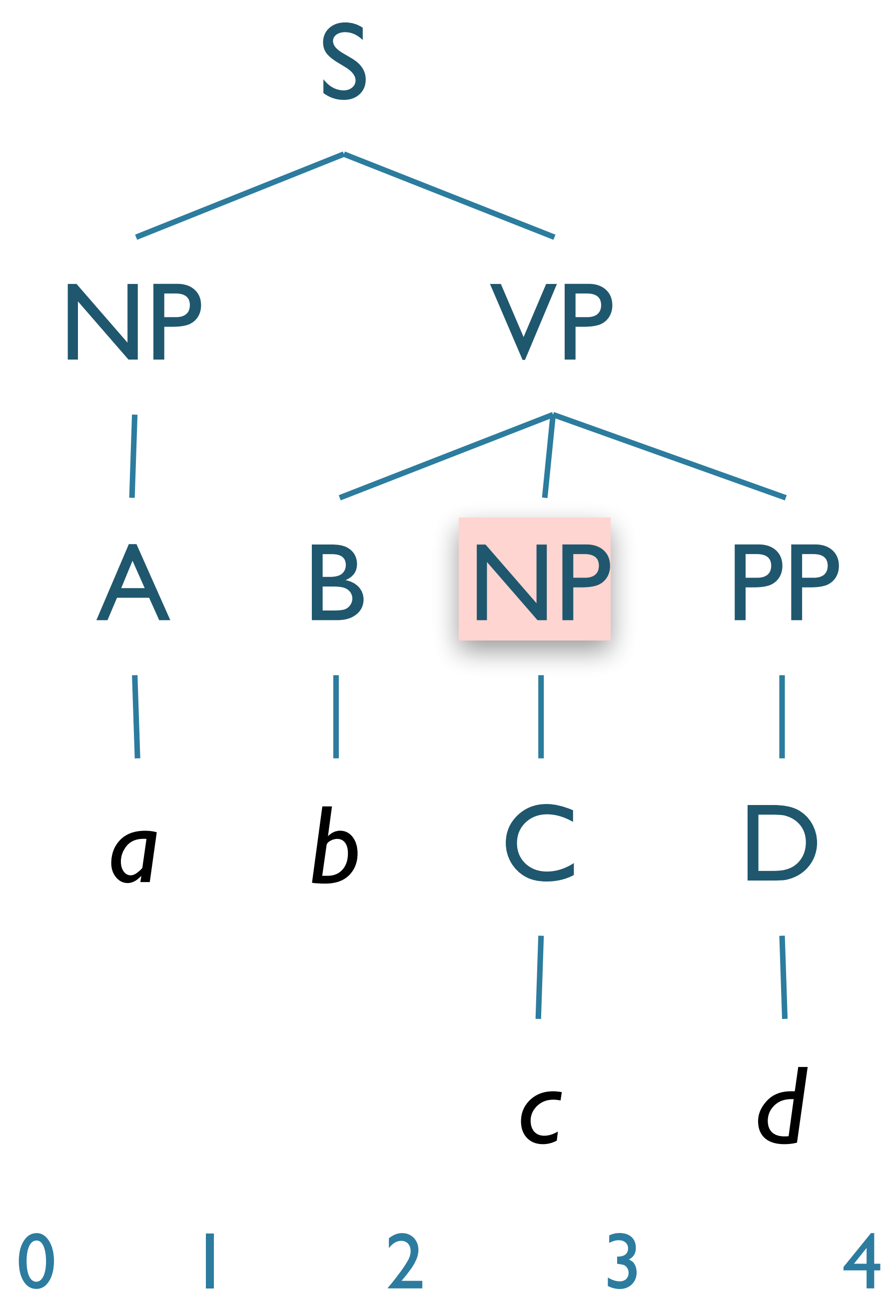
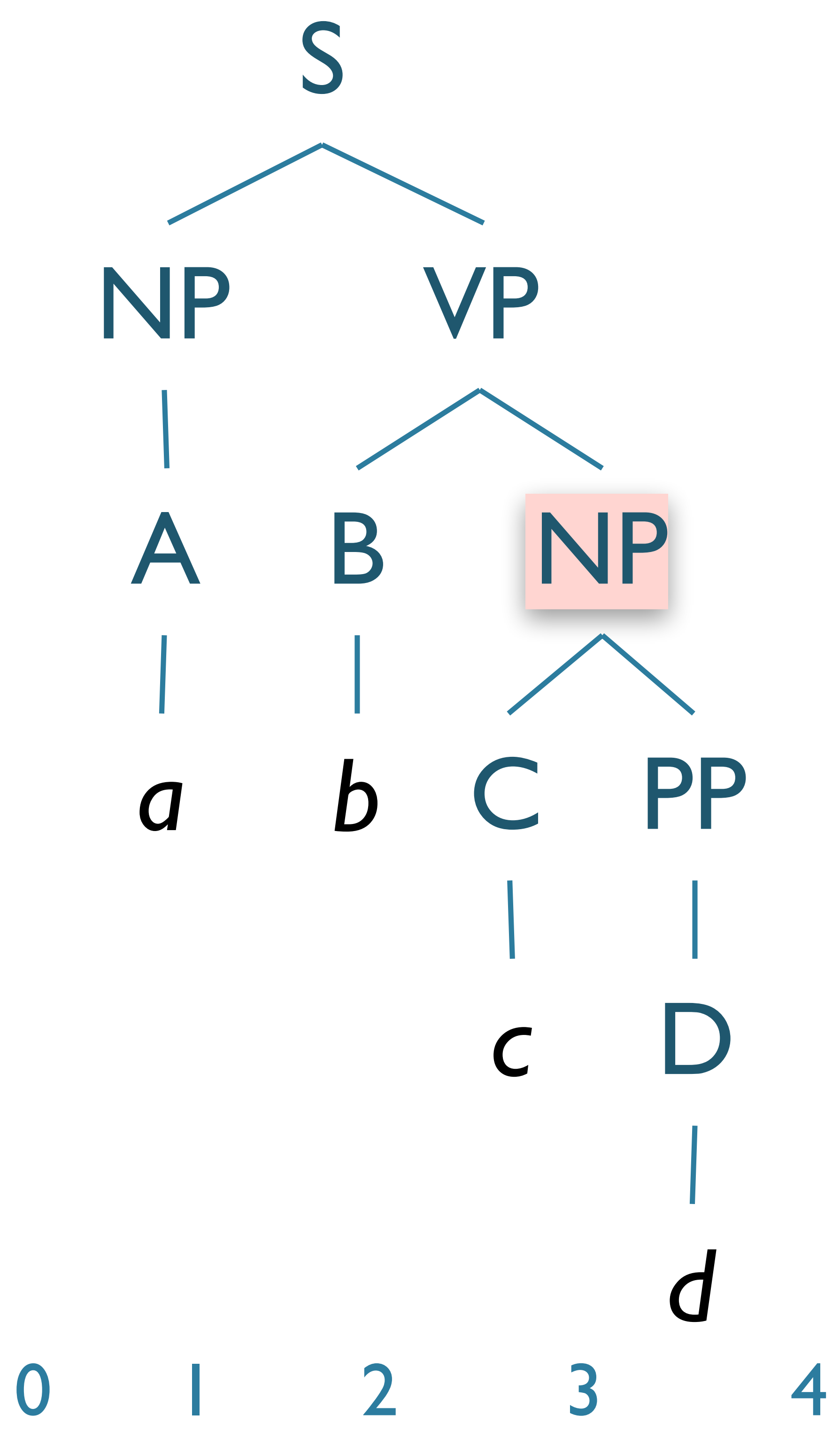


S(0,4)
NP(0,1)
VP(1,4)
NP(2,3)

Hypothesis



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Evaluation: Example

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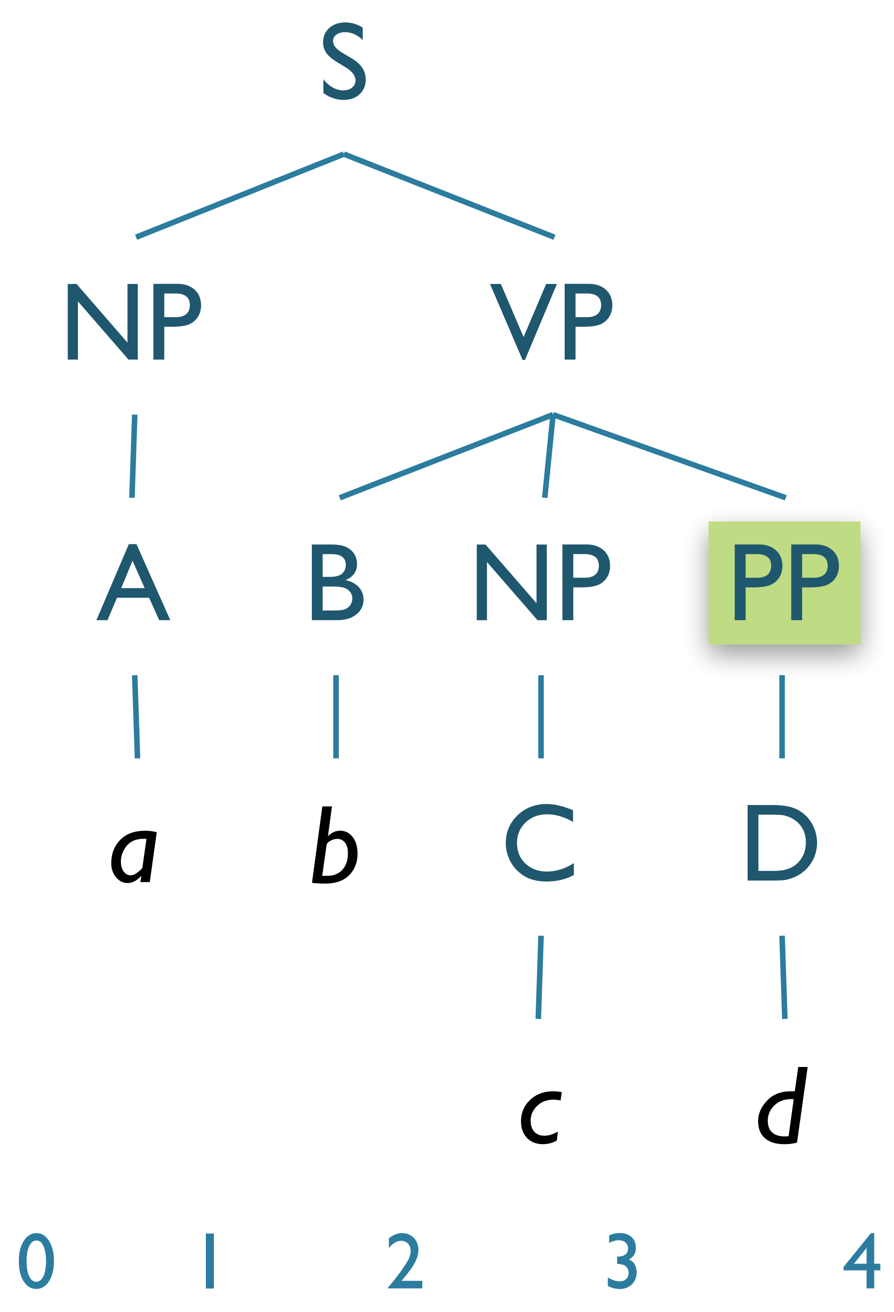
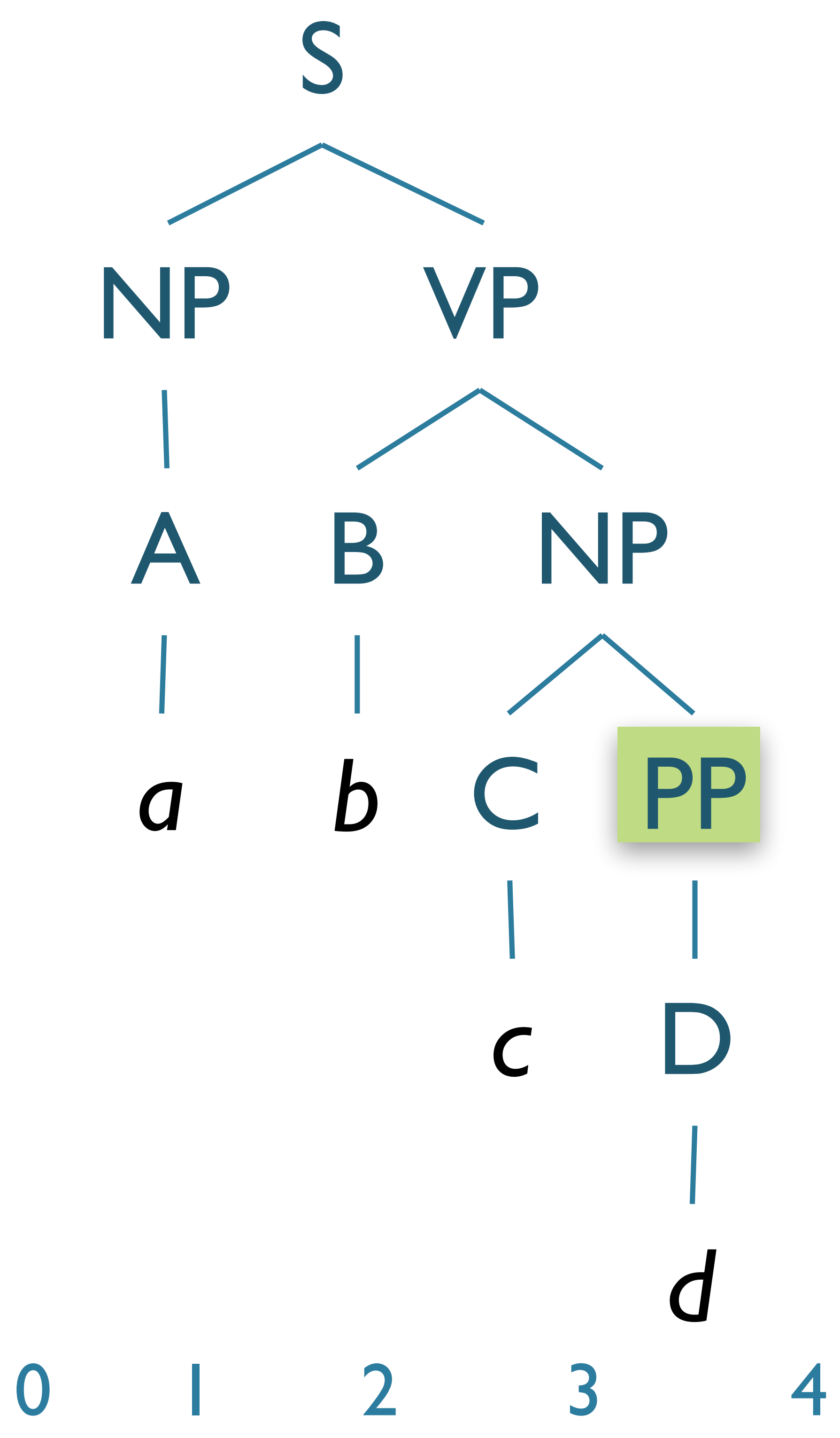


S(0,4)
NP(0,1)
VP(1,4)
NP(2,3)
PP(3,4)

Hypothesis



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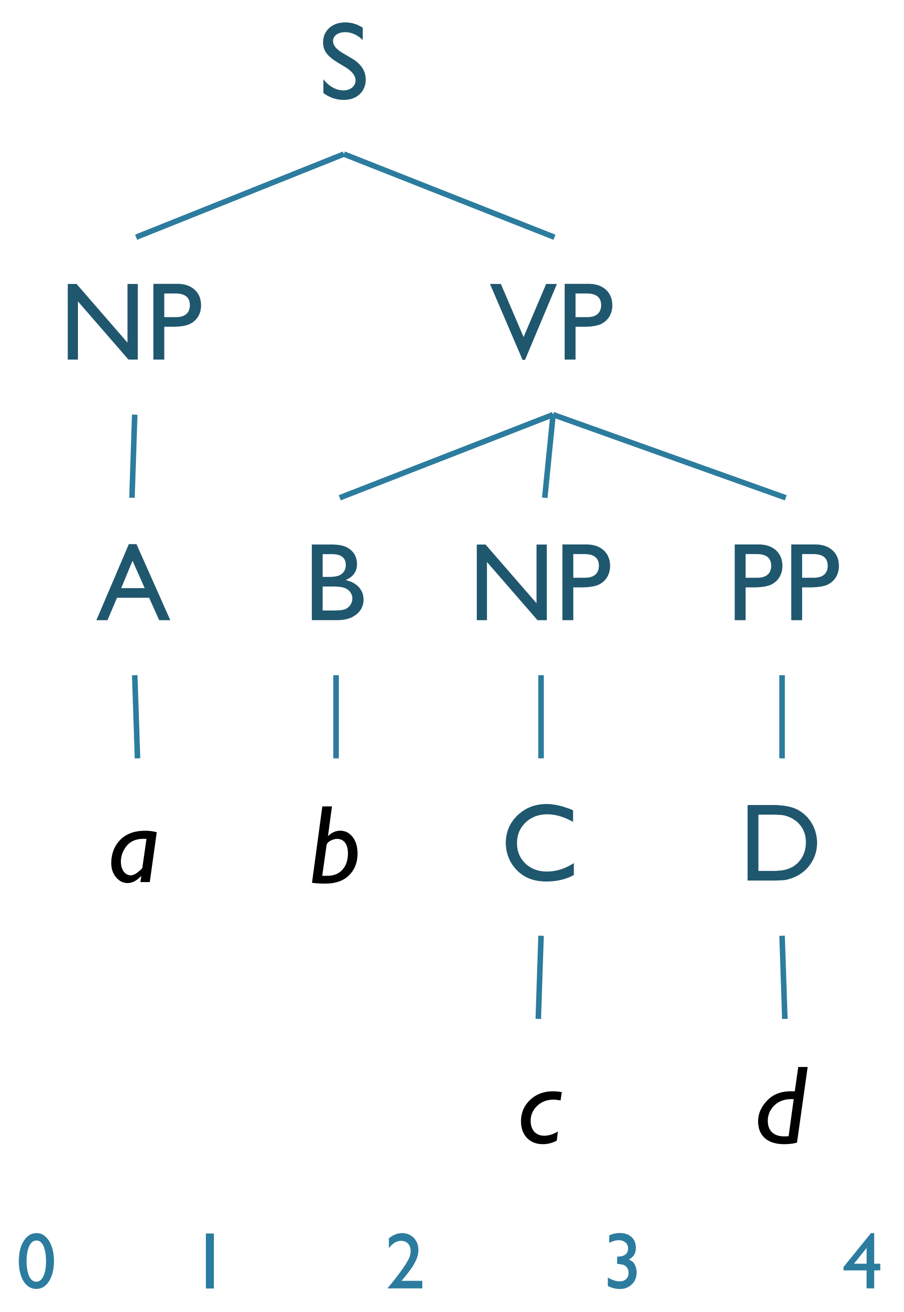

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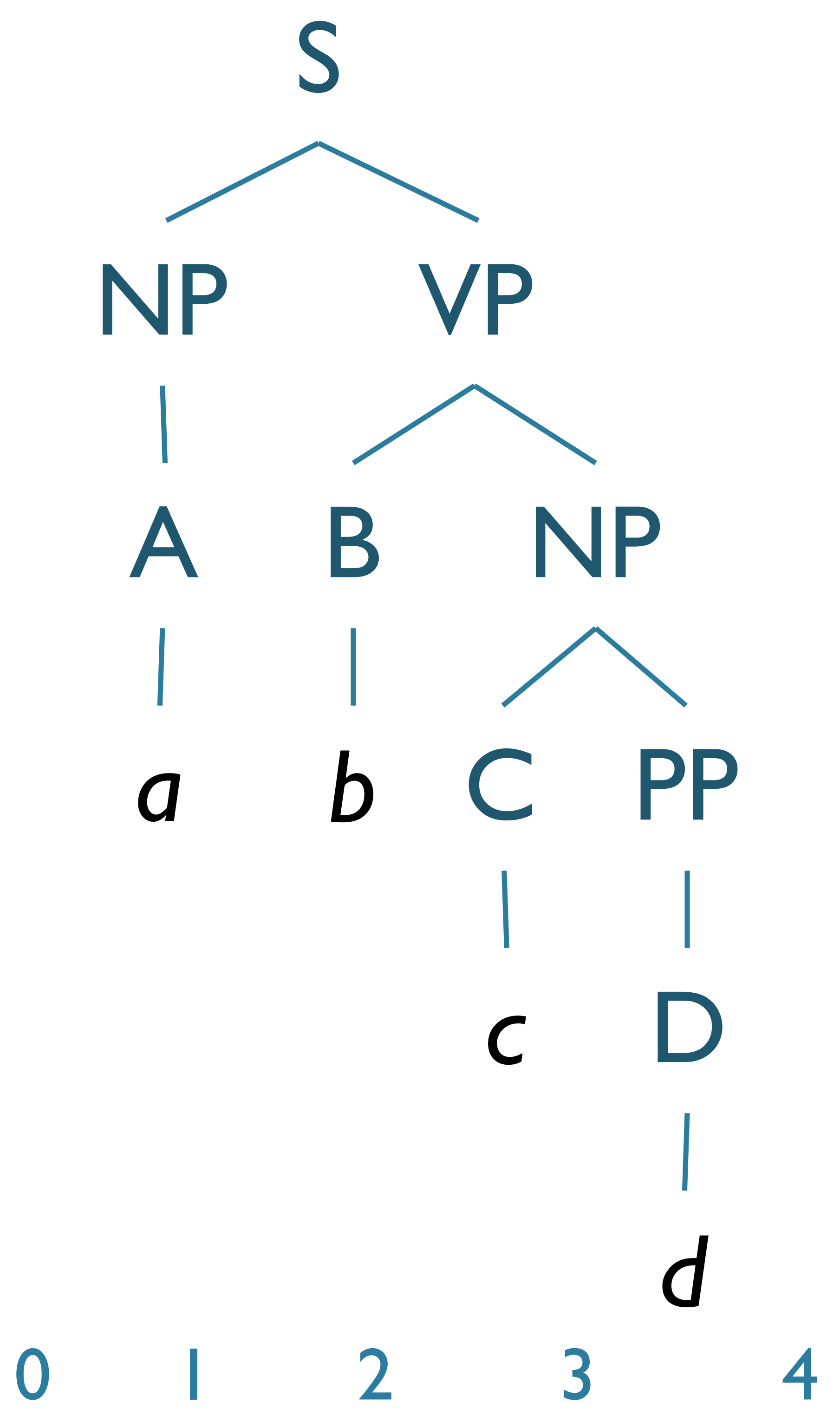
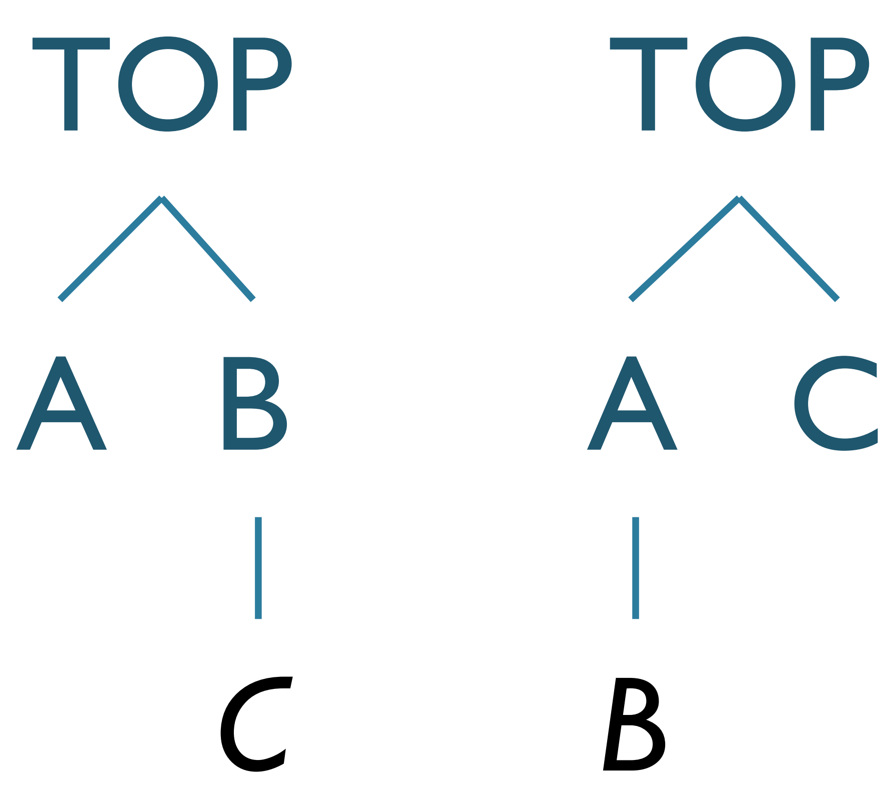


S(0,4)
NP(0,1)
VP(1,4)
NP(2,4)
PP(3,4)

LP: 4/5
LR: 4/5
F_1: 4/5

Parser Evaluation

- Crossing Brackets:
  - # of constituents where produced parse has bracketings that overlap for the siblings:
  - ((A B) C) — { (0,2), (2,3) }
    and hyp. has
    (A (B C)) — { (0,1), (1, 3) }




```
/* crossing is counted based on the brackets */
/* in test rather than gold file (by Mike)   */
for(j=0;j<bn2;j++){
  for(i=0;i<bn1;i++){
    if(bracket1[i].result != 5 &&
       bracket2[j].result != 5 &&
       ((bracket1[i].start < bracket2[j].start &&
         bracket1[i].end   > bracket2[j].start &&
         bracket1[i].end   < bracket2[j].end) ||
        (bracket1[i].start > bracket2[j].start &&
         bracket1[i].start < bracket2[j].end &&
         bracket1[i].end   > bracket2[j].end))){
```

from evalb.c

State-of-the-Art Parsing

- Parsers trained/tested on Wall Street Journal PTB
  - LR: 94%+;
  - LP: 94%+;
  - Crossing brackets: 1%

- Standard implementation of Parseval:
  - **`evalb`**

Evaluation Issues

- Only evaluating constituency
- There are other grammar formalisms:
  - LFG (Constraint-based)
  - Dependency Structure
- **Extrinsic** evaluation
  - How well does getting the correct parse match the semantics, etc?

Earley Parsing

Earley vs. CKY

- CKY doesn't capture full original structure
  - Can back-convert binarization, terminal conversion
  - Unit non-terminals require change in CKY

Earley vs. CKY

- CKY doesn't capture full original structure
  - Can back-convert binarization, terminal conversion
  - Unit non-terminals require change in CKY
- Earley algorithm
  - Supports parsing efficiently with arbitrary grammars
  - Top-down search
  - Dynamic programming
    - Tabulated partial solutions
  - Some bottom-up constraints

Earley Algorithm

- Another dynamic programming solution
  - Partial parses stored in “chart”
  - Compactly encodes ambiguity
  - $O(N^3)$
- Chart entries contain:
  - Subtree for a single grammar rule
  - Progress in completing subtree
  - Position of subtree w.r.t. input

Earley Algorithm

- First, left-to-right pass fills out a chart with *N+1* states
  - Chart entries — sit between words in the input string
  - Keep track of states of the parse at those positions
  - For each word position, chart contains set of states representing all partial parse trees generated so far
    - e.g. `chart[0]` contains all partial parse trees generated at the beginning of sentence

Chart Entries

- Three types of constituents:
  - Predicted constituents
  - In-progress constituents
  - Completed constituents

Parse Progress

- Represented by Dotted Rules
  - Position of • indicates type of constituent
- $_0$ Book $_1$ that $_2$ flight $_3$
  - *S* → • *VP* [0,0] (predicted)
  - *NP* → *Det* • *Nom* [1,2] (in progress)
  - *VP* → *V NP* • [0,3] (completed)
- [x,y] tells us what portion of the input is spanned so far by rule
- Each state s_i: *<dotted rule>*, [*<back pointer>*, *<current position>*]

$_0$ Book $_1$ that $_2$ flight $_3$

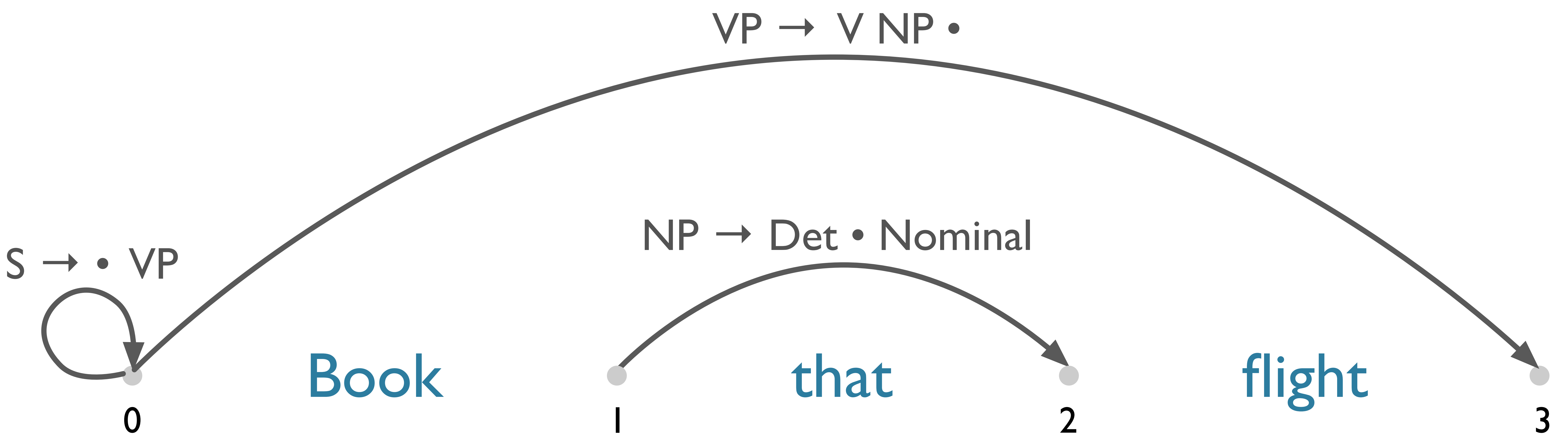
- $S \rightarrow \cdot\ VP$, [0,0]
  - First 0 means S constituent begins at the start of input
  - Second 0 means the dot is here too
  - So, this is a top-down prediction

$_0$ Book $_1$ that $_2$ flight $_3$

- $S \rightarrow \cdot\ VP$, [0,0]
  - First 0 means S constituent begins at the start of input
  - Second 0 means the dot is here too
  - So, this is a top-down prediction
- $NP \rightarrow Det \cdot Nom$, [1,2]
  - the NP begins at position 1
  - the dot is at position 2
  - so, Det has been successfully parsed
  - Nom predicted next

$_0$ Book $_1$ that $_2$ flight $_3$ (continued)

- *VP → V NP* • [0,3]
  - Successful VP parse of entire input



Successful Parse

- Final answer found by looking at last entry in chart
- If entry resembles $S \rightarrow \alpha \bullet [0,N]$ then input parsed successfully
- Chart will also contain record of all possible parses of input string, given the grammar

Parsing Procedure for the Earley Algorithm

- Move through each set of states in order, applying one of three operations:
  - **predictor**: add predictions to the chart
  - **scanner**: read input and add corresponding state to chart
  - **completer**: move dot to right when new constituent found
- Results (new states) added to current or next set of states in chart
- No backtracking and no states removed: keep complete history of parse

Earley Algorithm

```
function EARLEY-PARSE(words, grammar) returns chart
  ENQUEUE((γ→ • S, [0,0]), chart[0])
  for i ← from 0 to LENGTH(words) do
    for each state in chart[i] do
      if INCOMPLETE?(state) and
          NEXT-CAT(state) is not a part of speech then
        PREDICTOR(state)
      elseif INCOMPLETE?(state) and
          NEXT-CAT(state) is a part of speech then
        SCANNER(state)
      else
        COMPLETER(state)
      end
    end
  return(chart)
```

Earley Algorithm

procedure PREDICTOR(($A \rightarrow \alpha \bullet B\ \beta$, $[i,j]$))
  for each (B → γ) **in** GRAMMAR-RULES-FOR(B,$grammar$) **do**
    ENQUEUE(($B \rightarrow \bullet\ \gamma$, $[j,j]$), $chart[j]$)
  end

procedure SCANNER(($A \rightarrow \alpha \bullet B\ \beta$,$[i,j]$))
  if B ⊂ PARTS-OF-SPEECH($word[j]$) **then**
    ENQUEUE((B → $word[j] \bullet$, $[j,j+1]$), $chart[j+1]$)

procedure COMPLETER(($B \rightarrow \gamma \bullet$, $[j,k]$))
 for each ($A \rightarrow \alpha \bullet B\ \beta$, $[i,j]$) **in** $chart[j]$ **do**
    ENQUEUE(($A \rightarrow \alpha\ B \bullet\ \beta$, $[i,k]$), $chart[k]$)
  end

3 Main Subroutines of Earley

- Predictor
  - Adds predictions into the chart
- Scanner
  - Reads the input words and enters states representing those words into the chart
- Completer
  - Moves the dot to the right when new constituents are found

Predictor

- Intuition:
  - Create new state for top-down prediction of new phrase
- Applied when non part-of-speech non-terminals are to the right of a dot:
  - $S \rightarrow \bullet\ VP$ [0,0]
- Adds new states to *current* chart
  - One new state for each expansion of the non-terminal in the grammar
    $VP \rightarrow \bullet\ V$ [0,0]
    $VP \rightarrow \bullet\ V\ NP$ [0,0]

Chart[0]

S0	$\gamma \rightarrow \cdot S$	[0,0]	Dummy start state
S1	$S \rightarrow \cdot NP\ VP$	[0.0]	Predictor
S2	$S \rightarrow \cdot Aux\ NP\ VP$	[0,0]	Predictor
S3	$S \rightarrow \cdot VP$	[0,0]	Predictor
S4	$NP \rightarrow \cdot Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \cdot Proper\text{-}Noun$	[0,0]	Predictor
S6	$NP \rightarrow \cdot Det\ Nominal$	[0,0]	Predictor
S7	$VP \rightarrow \cdot Verb$	[0,0]	Predictor
S8	$VP \rightarrow \cdot Verb\ NP$	[0,0]	Predictor
S9	$VP \rightarrow \cdot Verb\ NP\ PP$	[0,0]	Predictor
S10	$VP \rightarrow \cdot Verb\ PP$	[0,0]	Predictor
S11	$VP \rightarrow \cdot VP\ PP$	[0,0]	Predictor

Chart[1]

S12	*Verb → book •*	[0,1]	Scanner
S13	*VP → Verb •*	[0,1]	Completer
S14	*VP → Verb • NP*	[0,1]	Completer
S15	*VP → Verb • NP PP*	[0,1]	Completer
S16	*VP → Verb • PP*	[0,1]	Completer
S17	*S → VP •*	[0,1]	Completer
S18	*VP → VP • PP*	[0,1]	Completer
S19	*NP → • Pronoun*	[1,1]	Predictor
S20	*NP → • Proper-Noun*	[1,1]	Predictor
S21	*NP → • Det Nominal*	[1,1]	Predictor
S22	*PP → • Prep NP*	[1,1]	Predictor

Book that flight

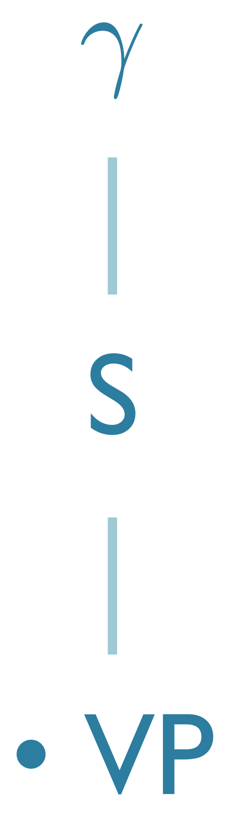
S0: $\gamma \rightarrow \bullet\ S$ [0,0]

γ

$\bullet$ S

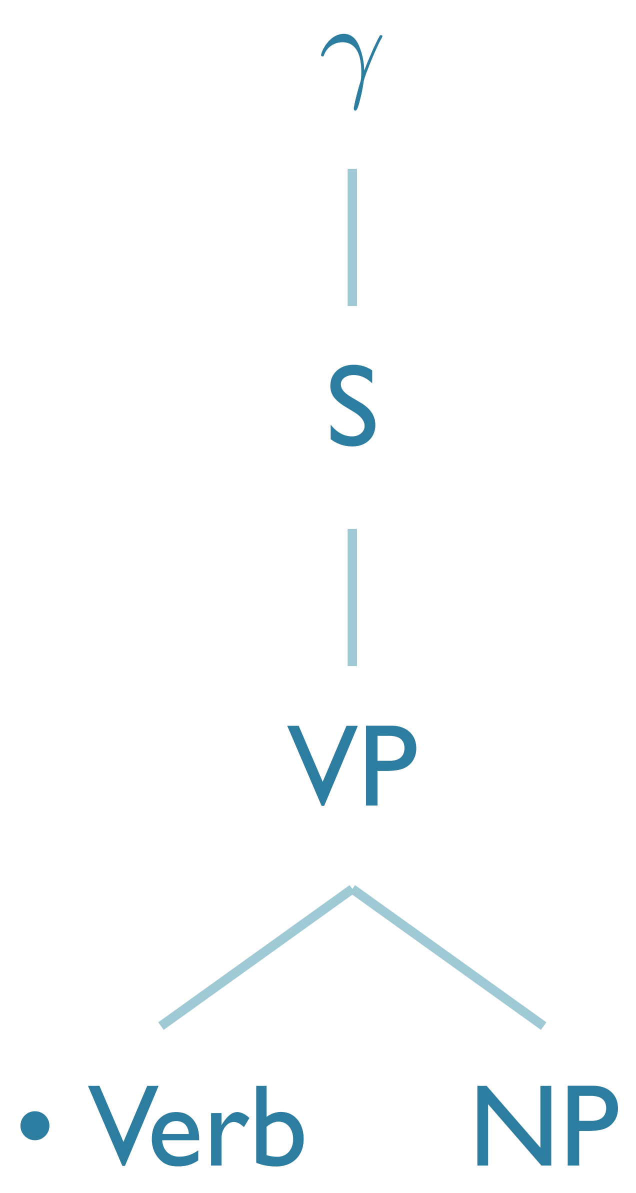
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]



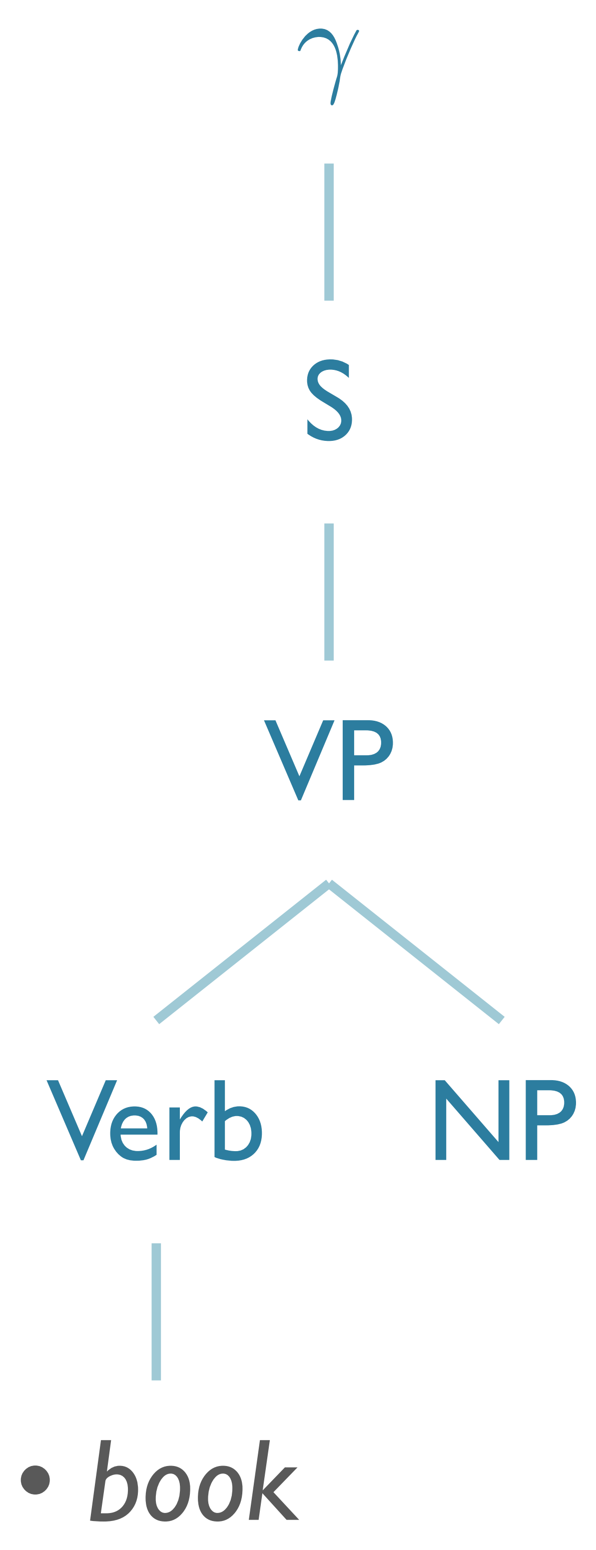
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]
S8: $VP \rightarrow \bullet\ Verb\ NP$ [0,0]



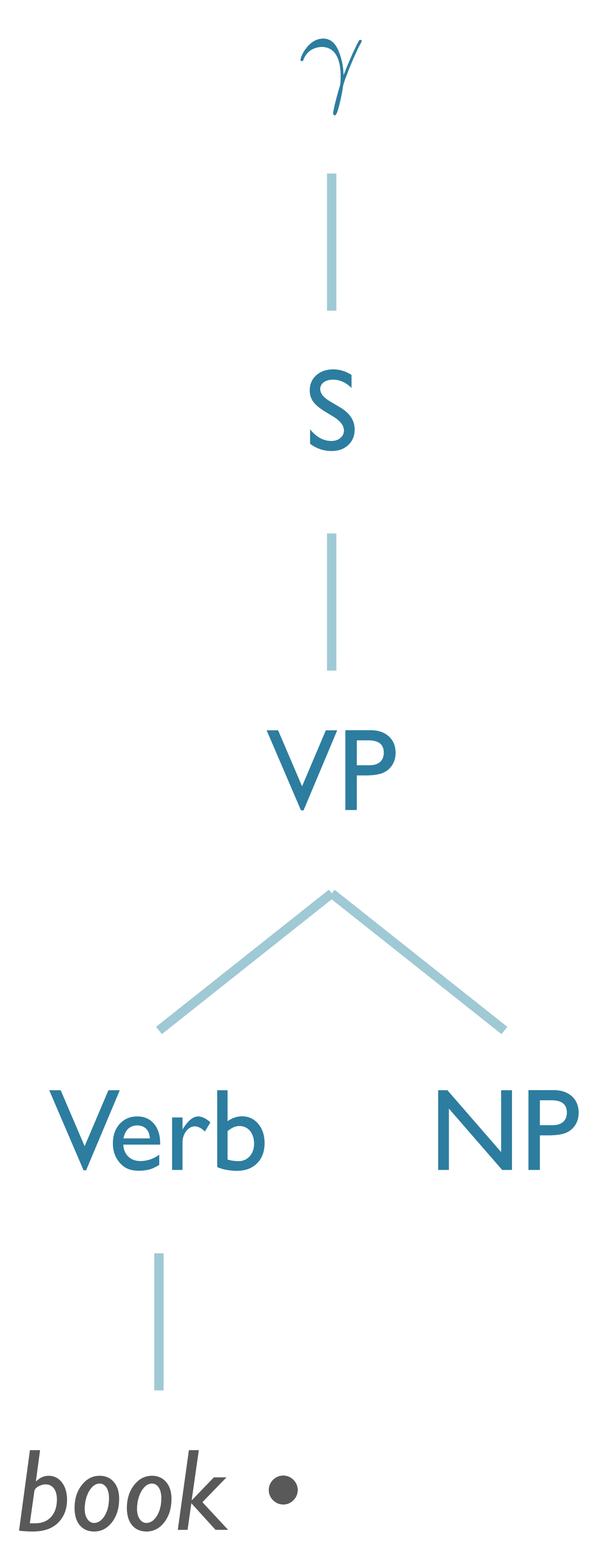
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]
S8: $VP \rightarrow \bullet\ Verb\ NP$ [0,0]
S12: $Verb \rightarrow \bullet\ book$ [0,0]



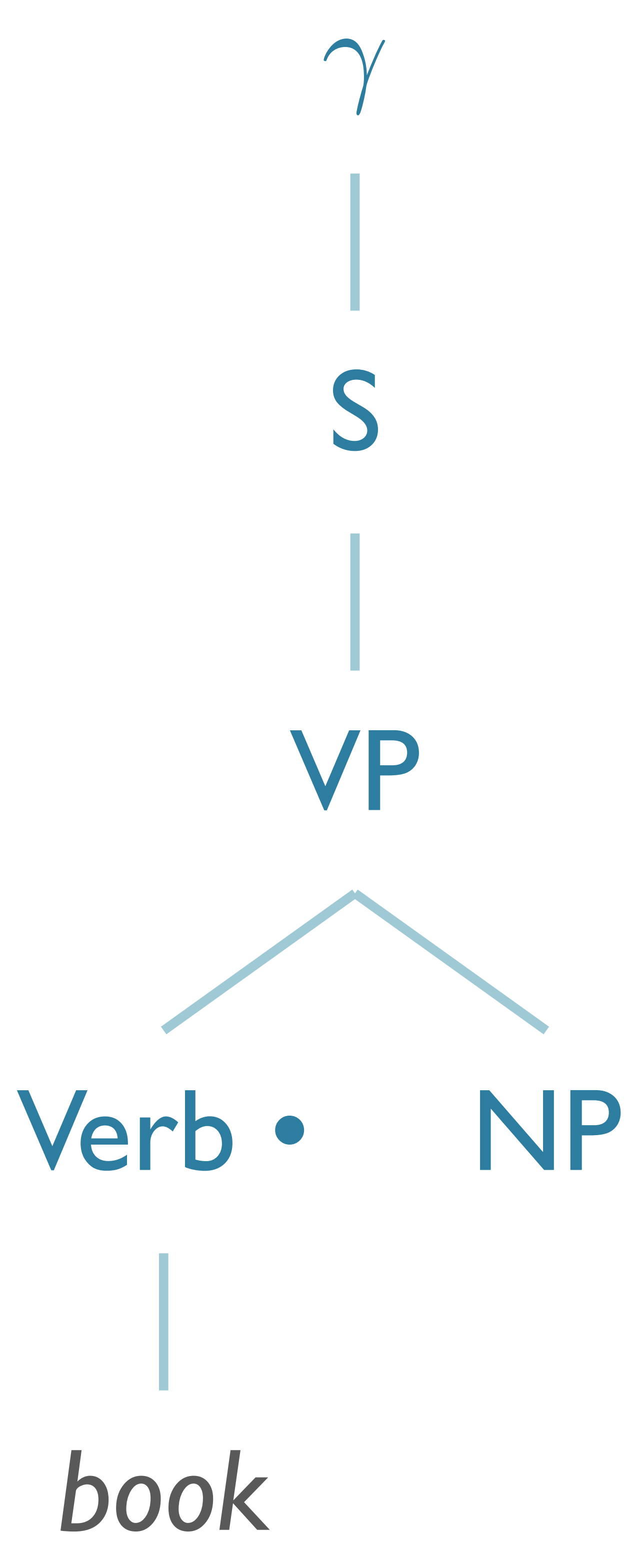
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]
S8: $VP \rightarrow \bullet\ Verb\ NP$ [0,0]
S12: $Verb \rightarrow book\ \bullet$ [0,1]



Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow \bullet\ VP$ [0,0]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]

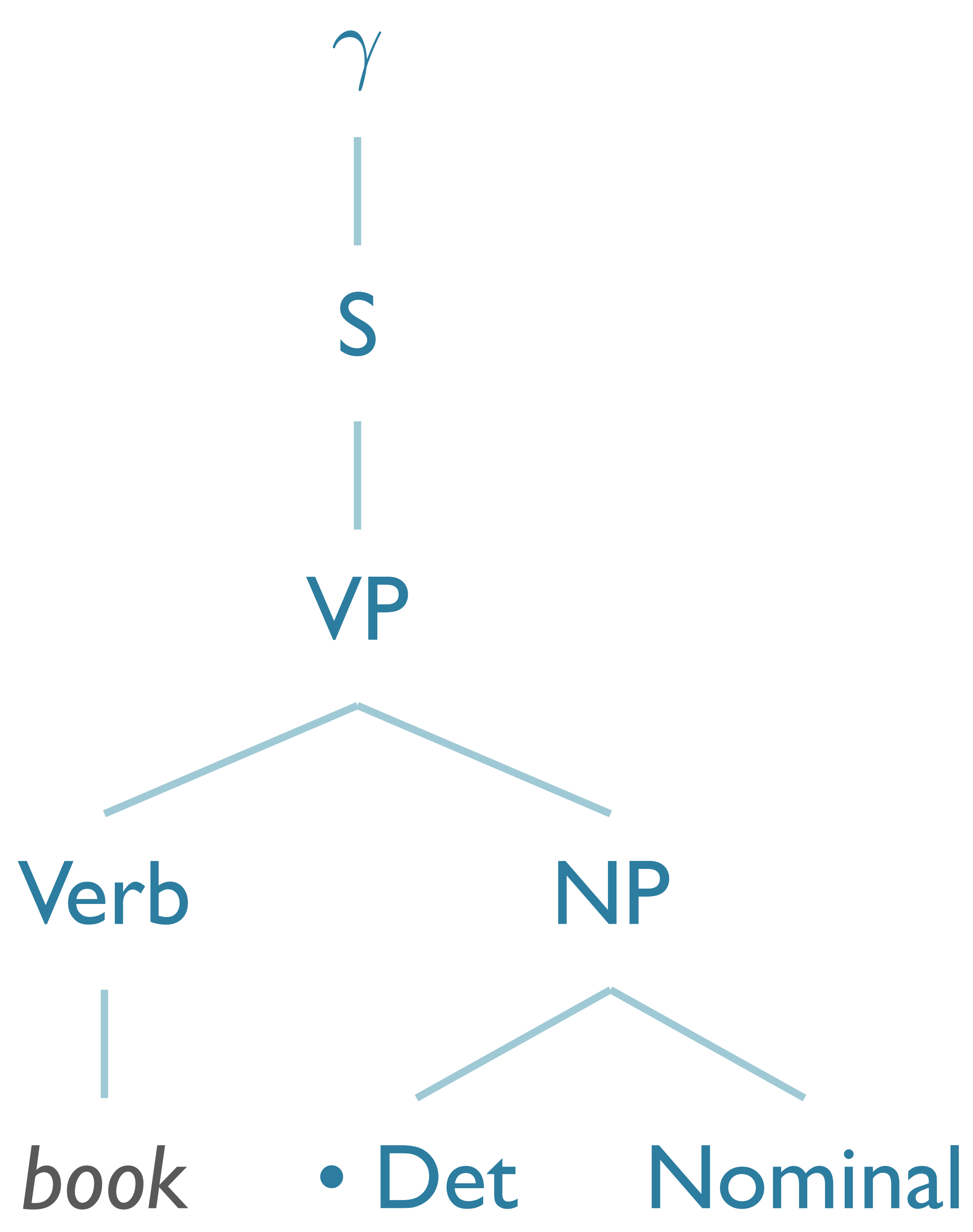


Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]

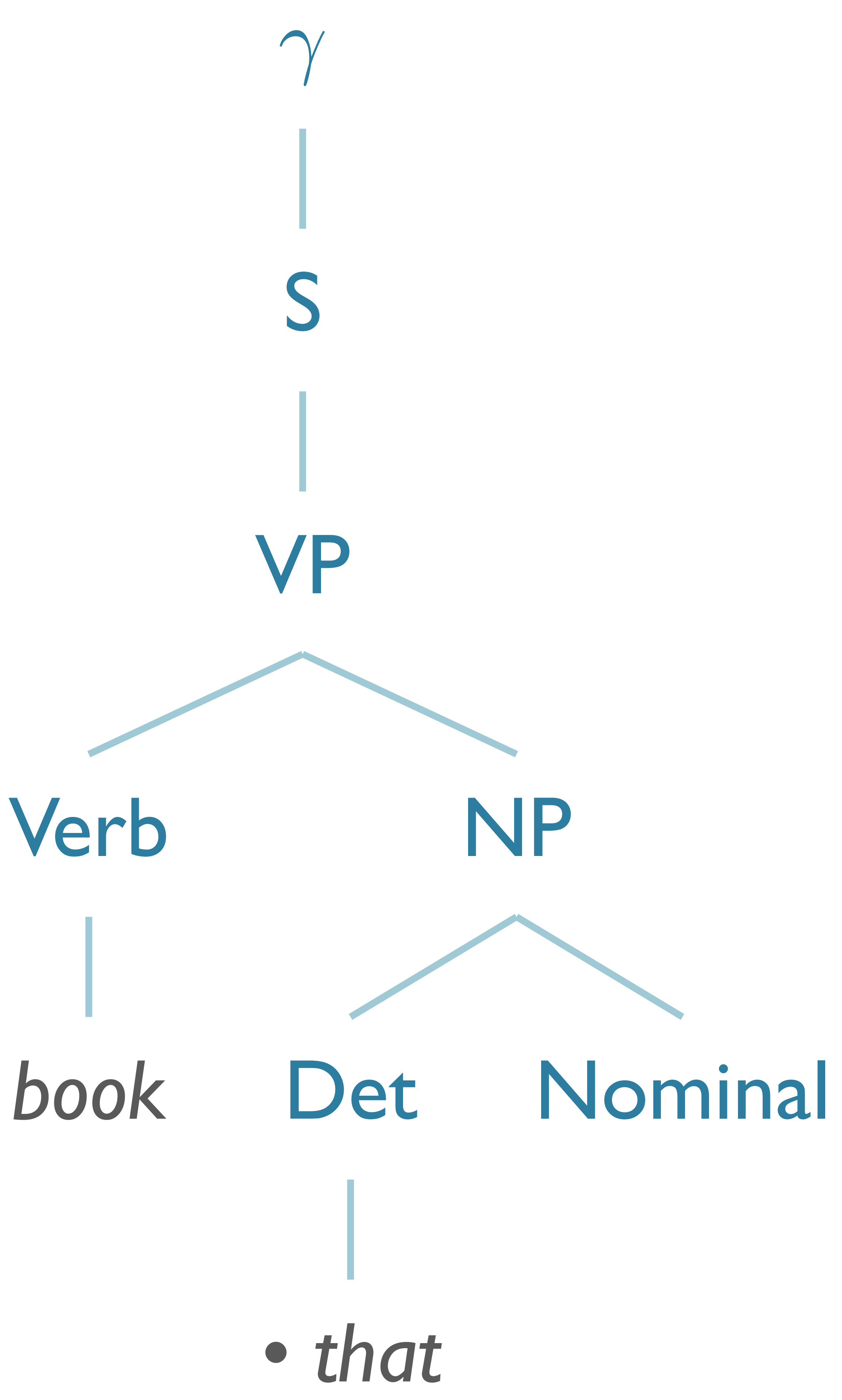
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow \bullet\ Det\ Nominal$ [1,1]



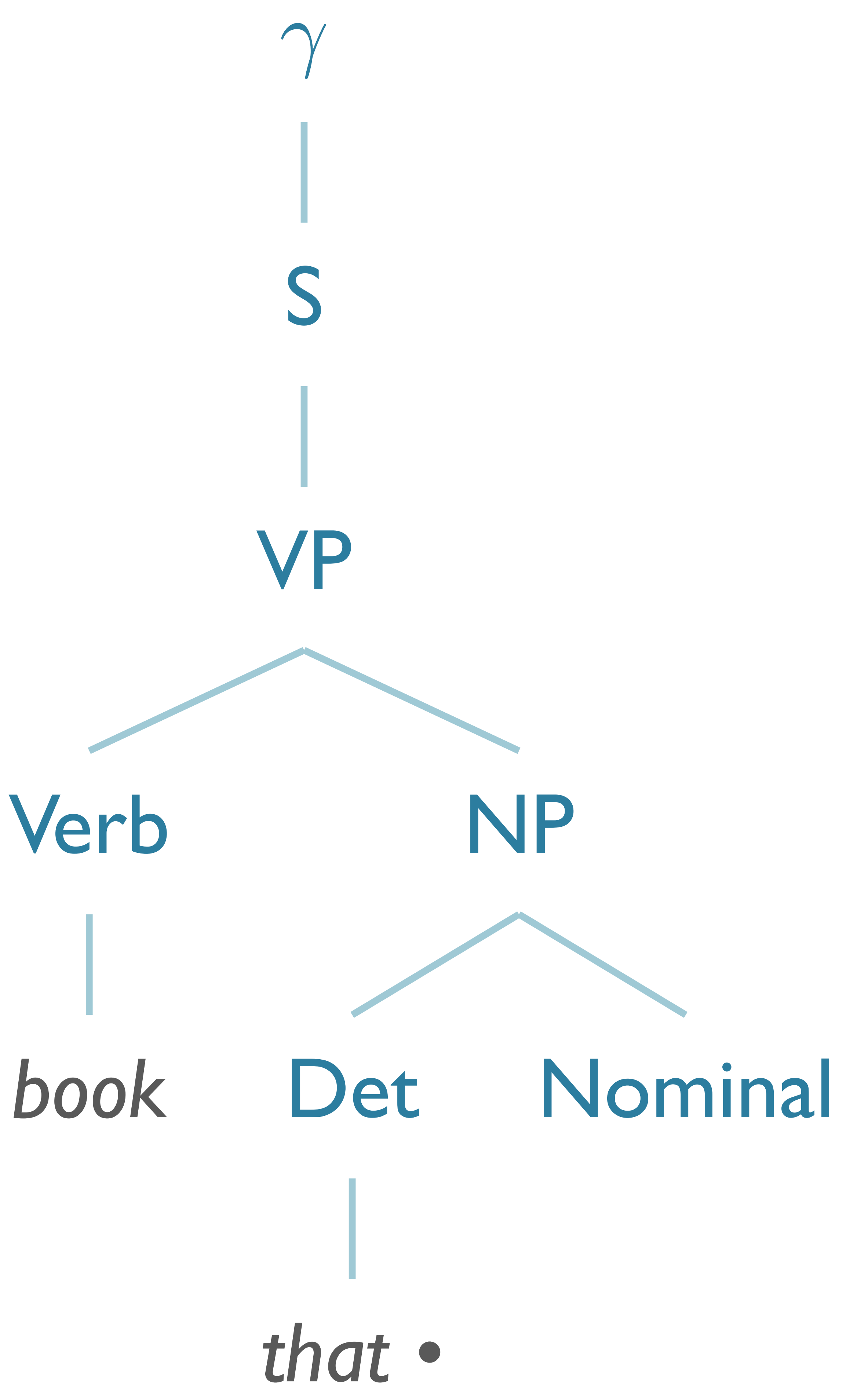
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow \bullet\ Det\ Nominal$ [1,1]
S23: $Det \rightarrow \bullet$ *"that"* [1,1]



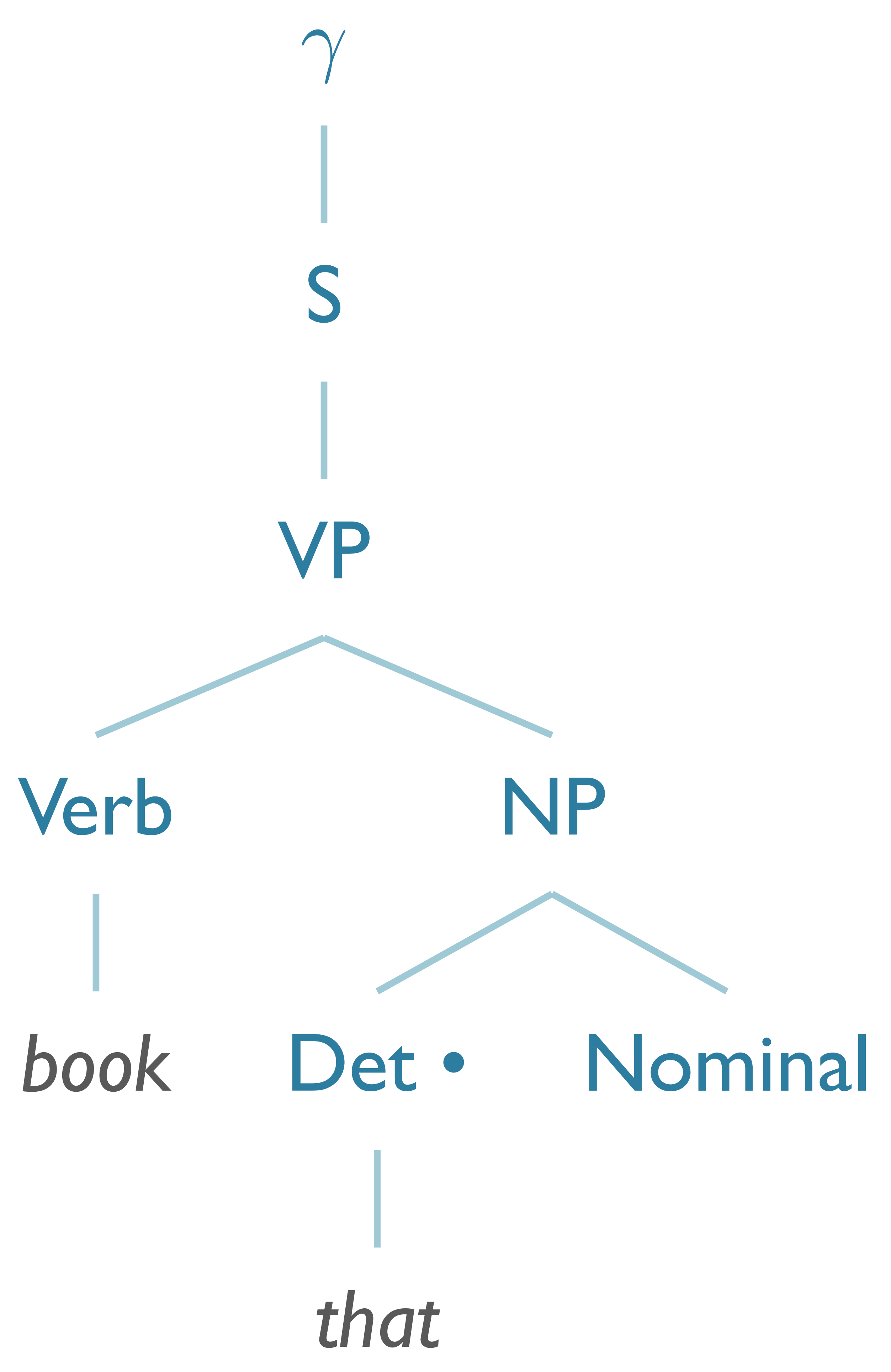
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow \bullet\ Det\ Nominal$ [1,1]
S23: $Det \rightarrow$ *"that"* • [1,2]



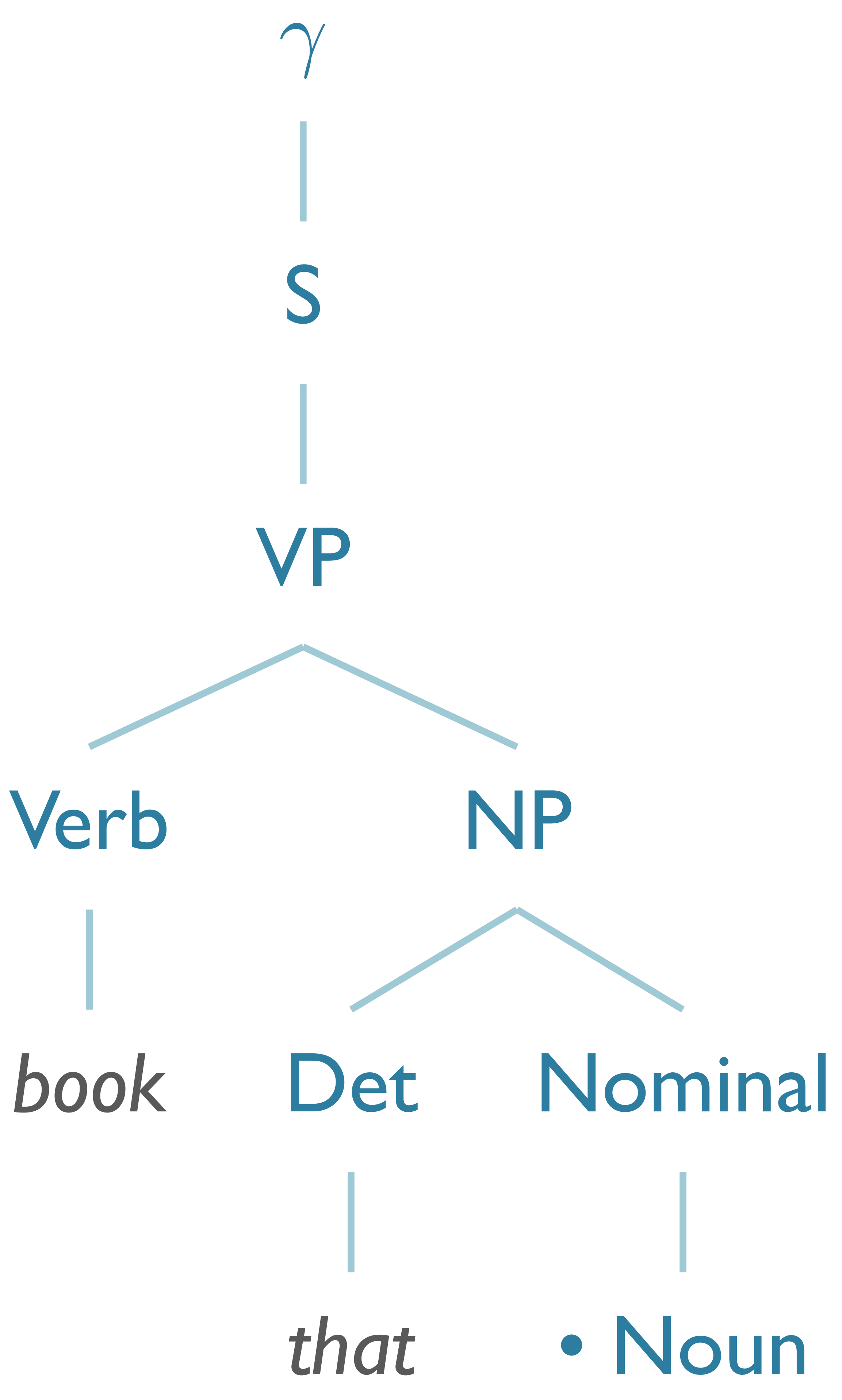
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]



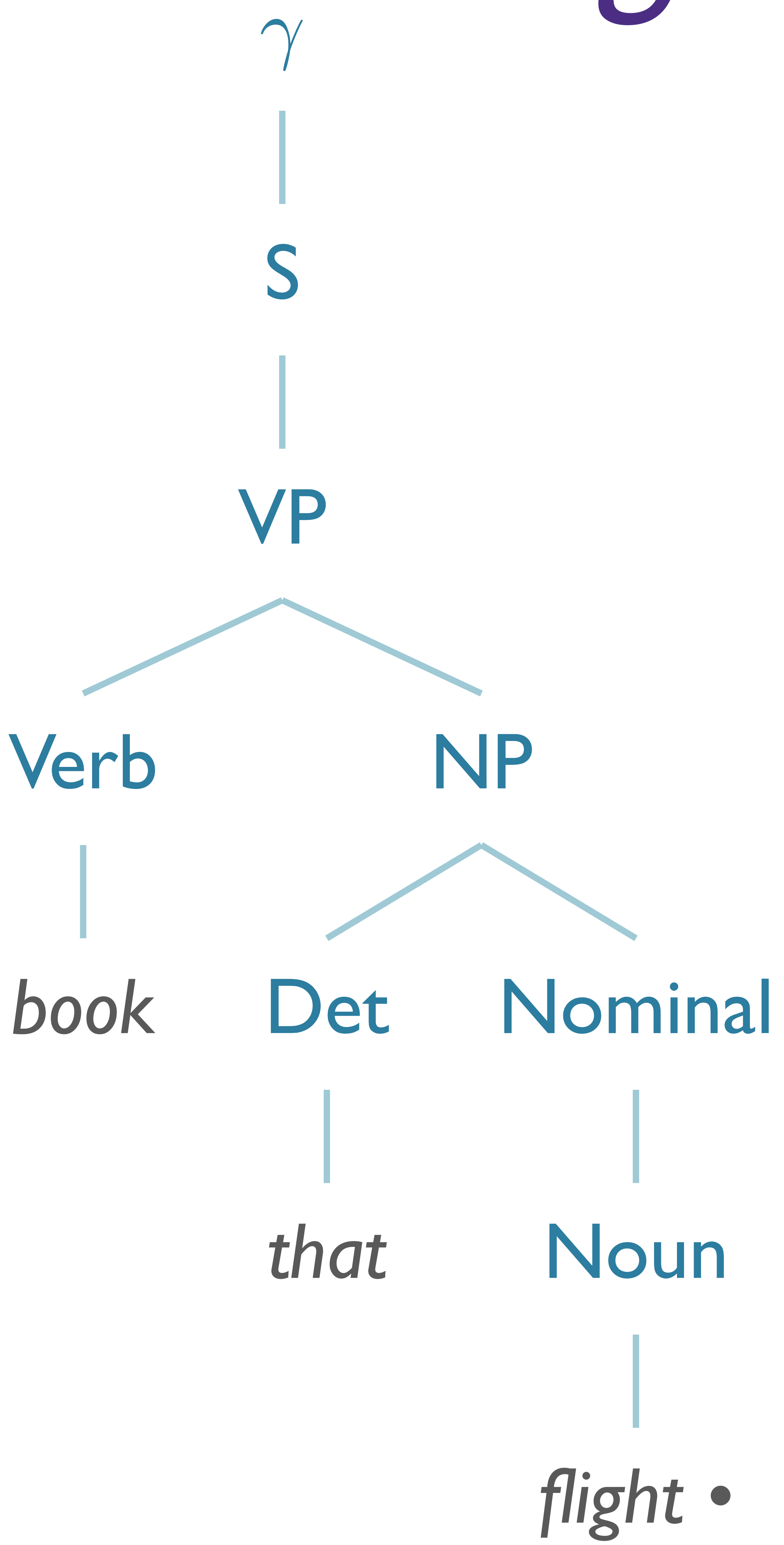
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP\ \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]
S25: $Nominal \rightarrow \bullet\ Noun$ [2,2]



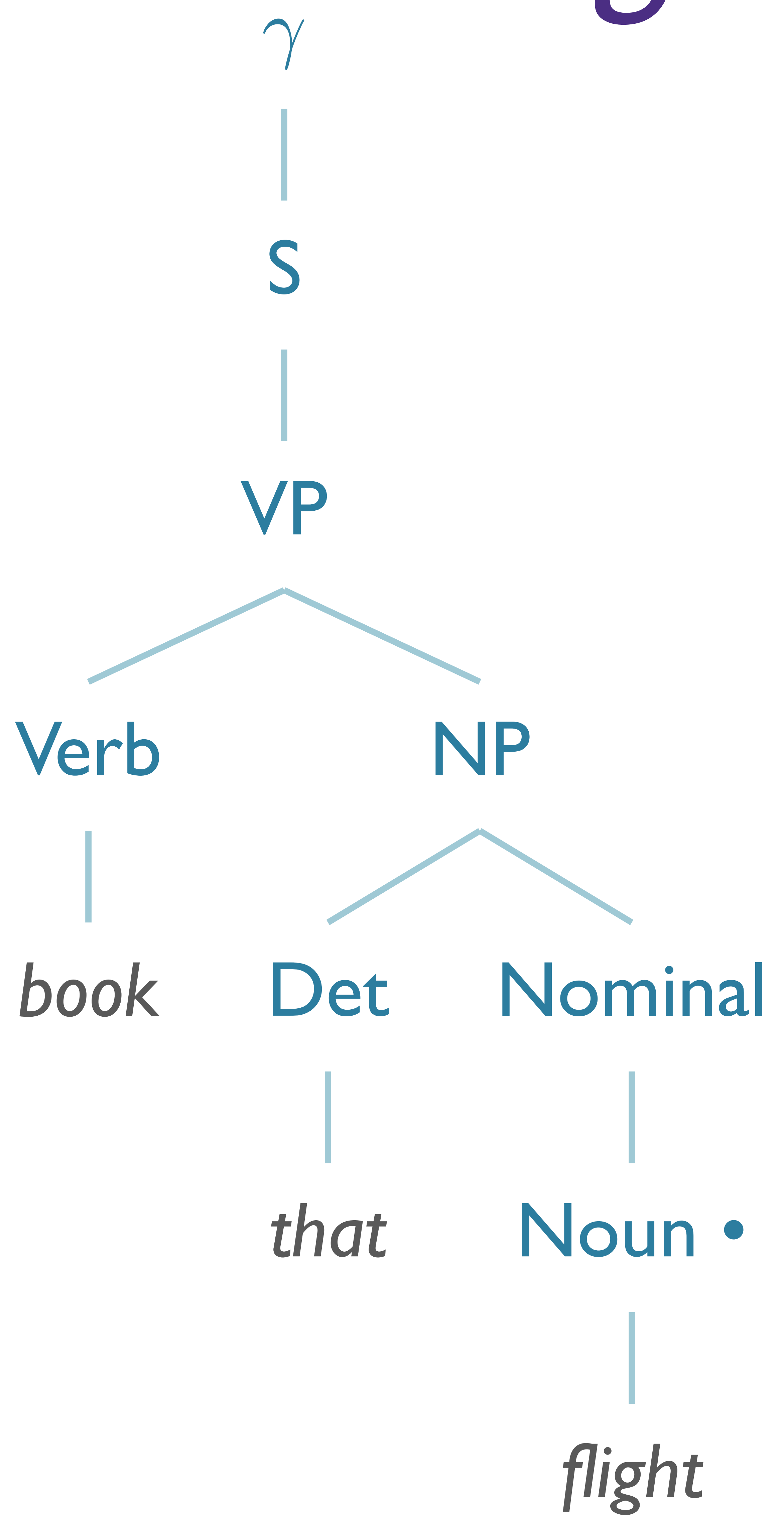
Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]
S25: $Nominal \rightarrow \bullet\ Noun$ [2,2]
S28: $Noun \rightarrow$ *"flight"* $\bullet$ [2,3]



Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb \bullet NP$ [0,1]
S21: $NP \rightarrow Det \bullet Nominal$ [1,2]
S25: $Nominal \rightarrow Noun \bullet$ [2,3]



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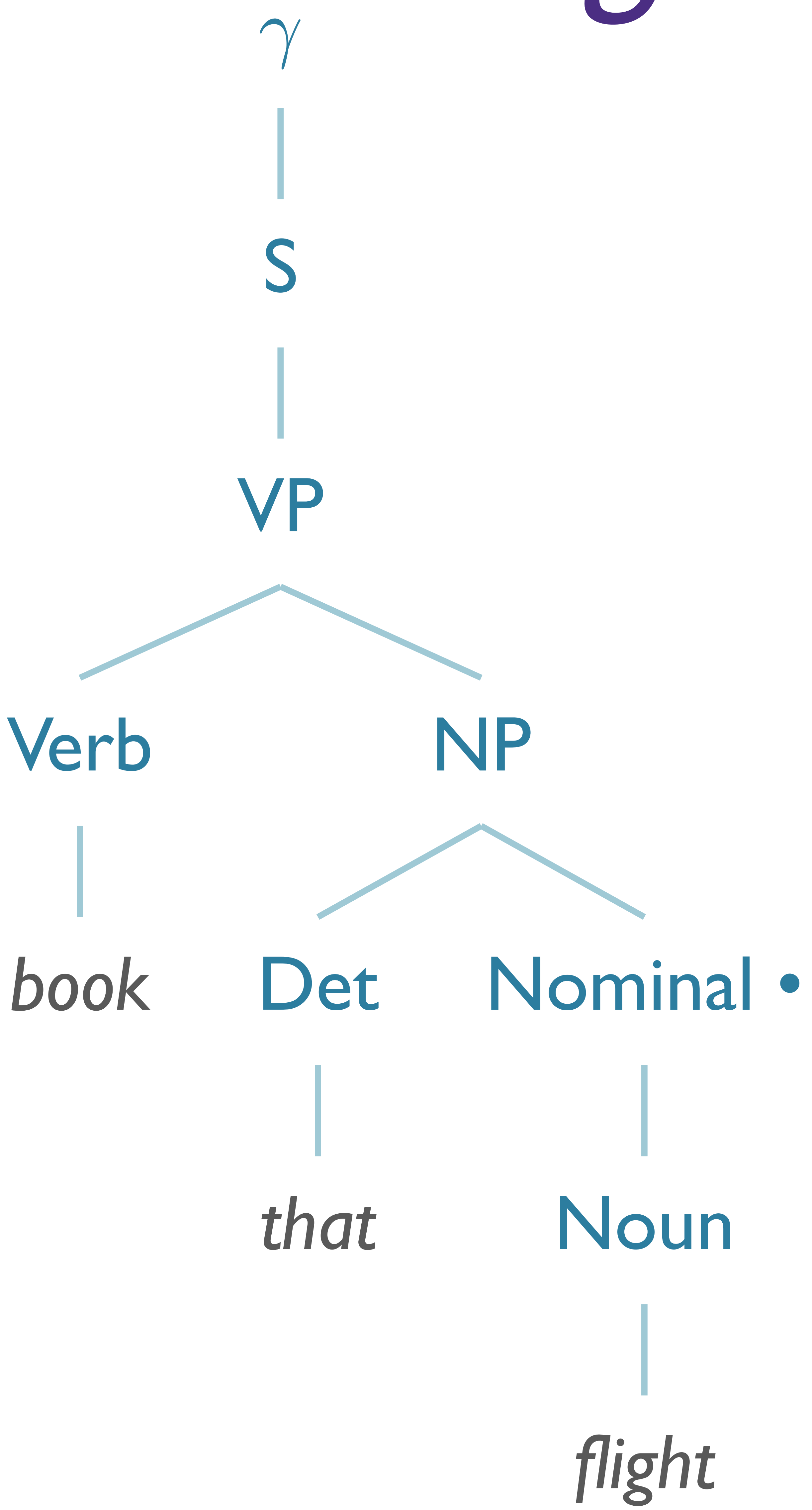

Book that flight

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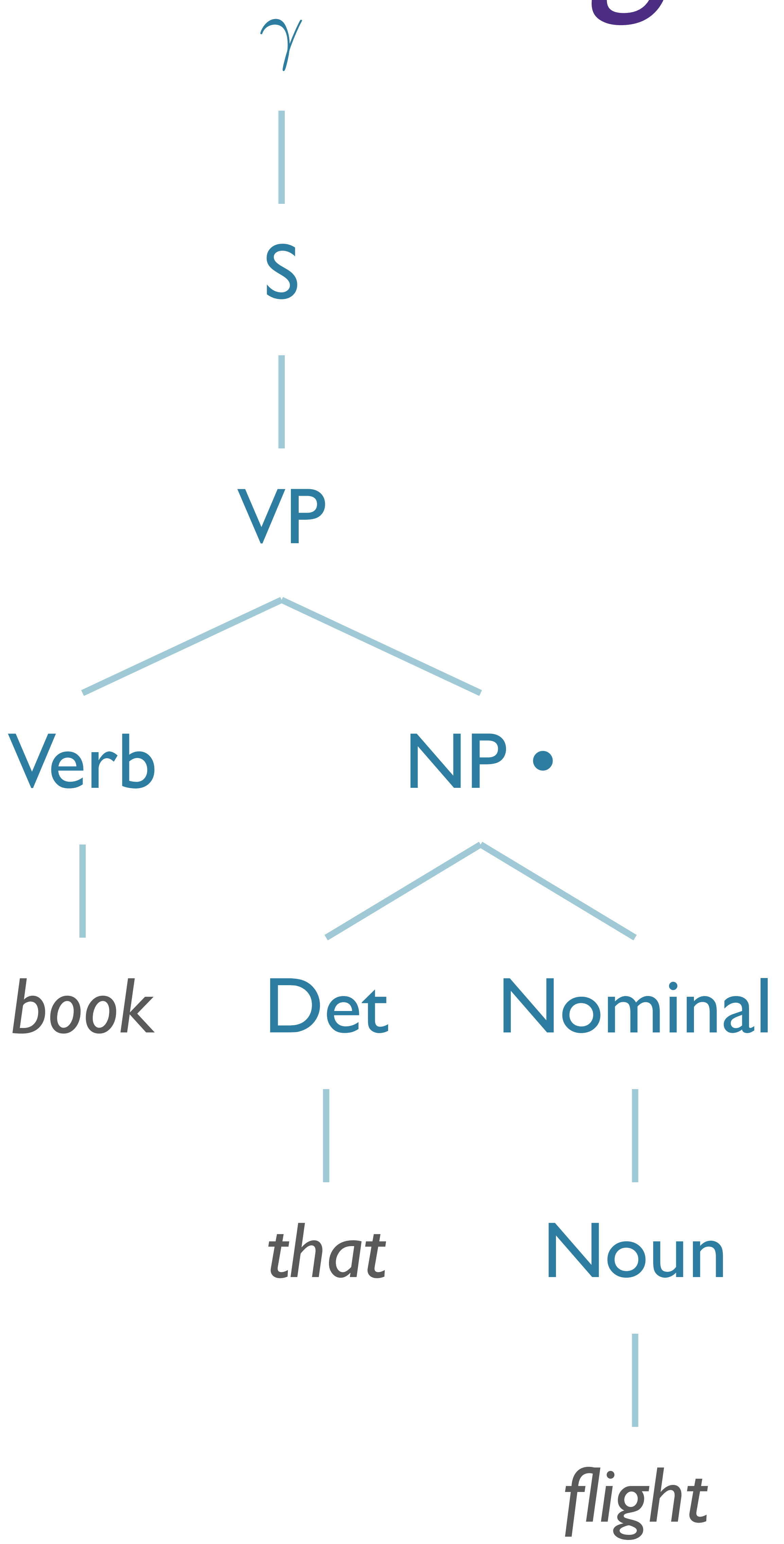
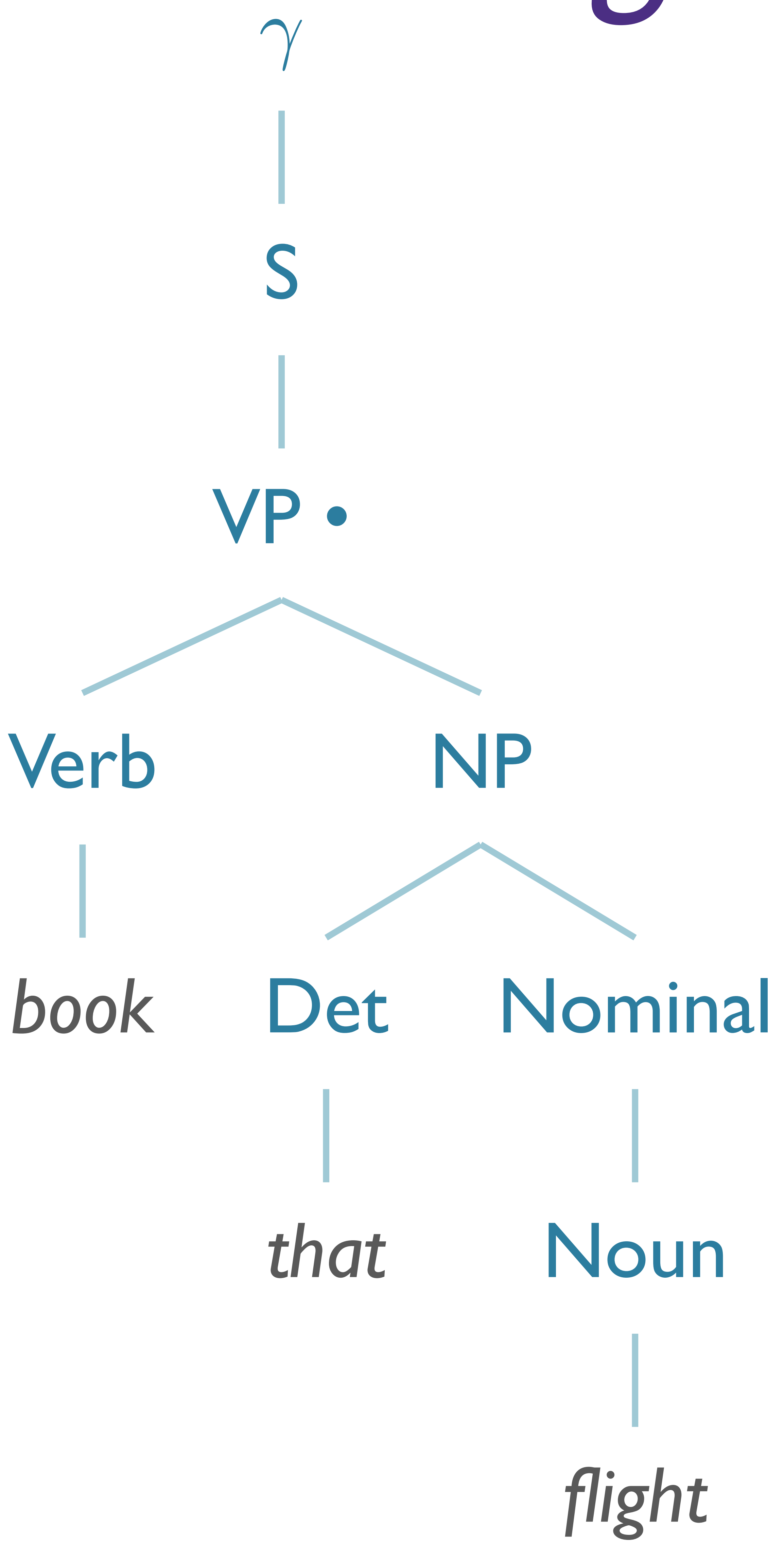
S3: $S \rightarrow VP \bullet$ [0,1]
S8: $VP \rightarrow Verb\ NP \bullet$ [0,3]



Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S3: $S \rightarrow VP \bullet$ [0,3]

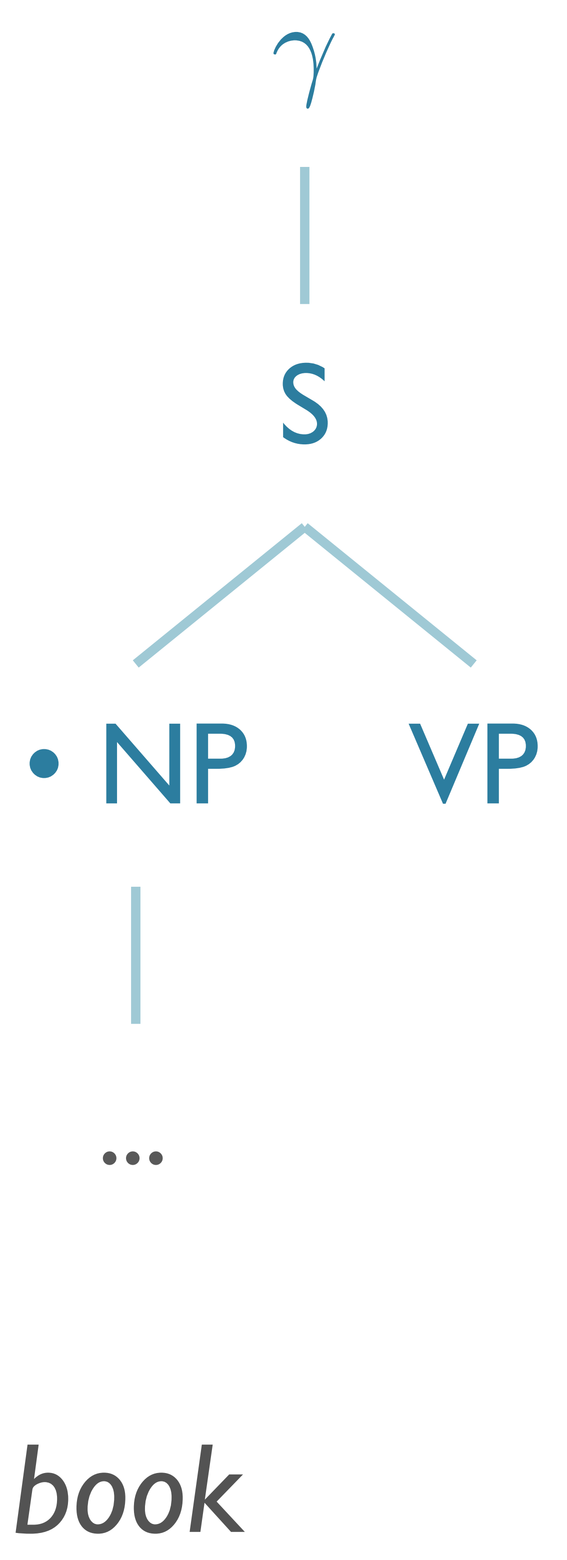


What About Dead Ends?

Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S1: $S \rightarrow \bullet\ NP\ VP$ [0,0]

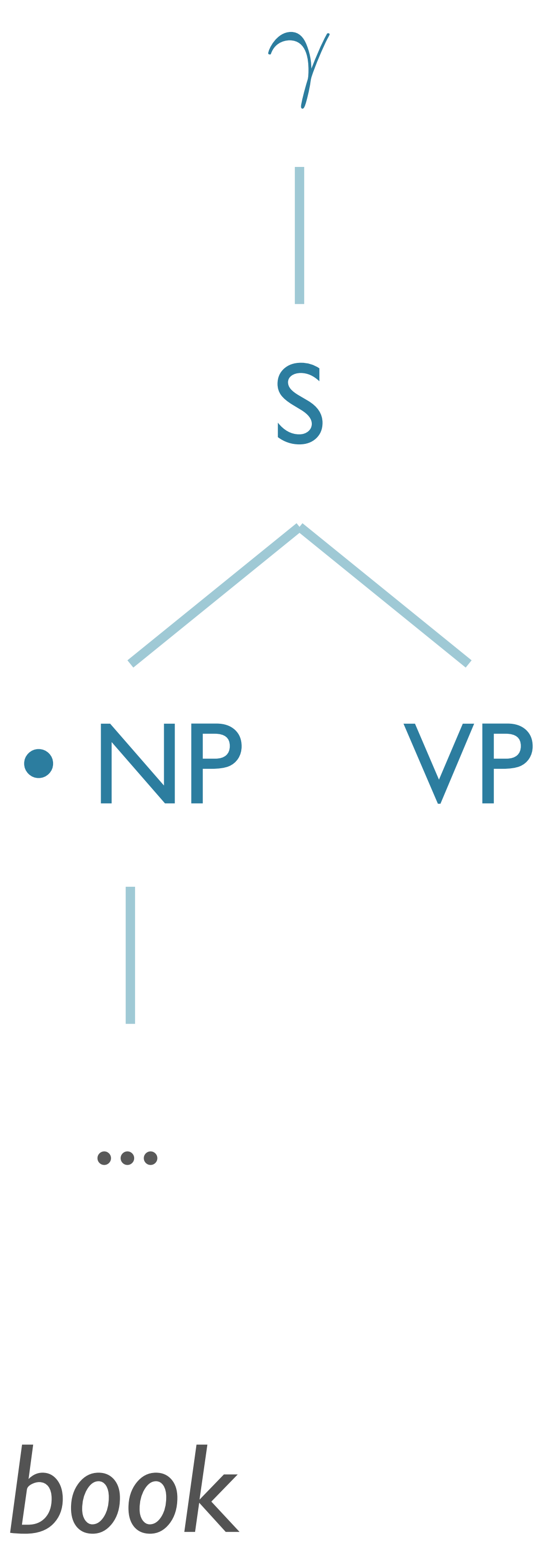
$NP \rightarrow \bullet\ Pronoun$
$NP \rightarrow \bullet\ Proper\text{-}Noun$
$NP \rightarrow \bullet\ Det\ Nominal$



Book that flight

S0: $\gamma \rightarrow \bullet\ S$ [0,0]
S1: $S \rightarrow \bullet\ NP\ VP$ [0,0]

~~$NP \rightarrow \bullet\ Pronoun$~~
~~$NP \rightarrow \bullet\ Proper\text{-}Noun$~~
~~$NP \rightarrow \bullet\ Det\ Nominal$~~



What About Recursion?

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- We now have a top-down parser in hand. Does it enter infinite loops on rules like S -> S ‘and’ S?

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- No!

procedure ENQUEUE(*state*, *chart-entry*)
  if *state* is not already in *chart-entry* **then**
    PUSH(*state*, *chart-entry*)
  end

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  end

Exercise: parse 'table and chair' using the very simple grammar
Nom -> Nom 'and' Nom | 'table' | 'chair'