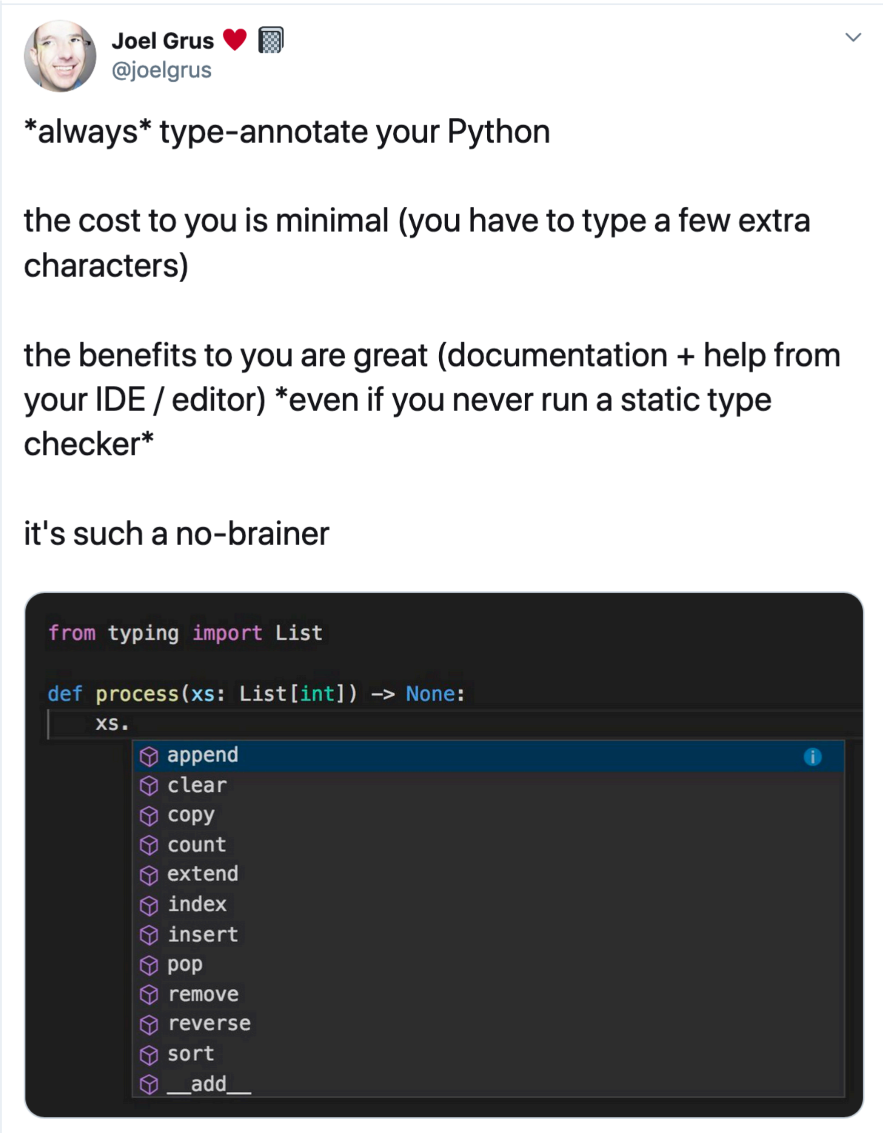
# CKY Parsing & CNF Conversion

LING 571 — Deep Processing Techniques for NLP
Shane Steinert-Threlkeld

# Announcements

- **HW #1** due tonight at **11:59pm**.
- Use full paths (for python binary and for files)!
  - Condor broken symlink:
    - `/mnt/dropbox` instead of `/dropbox` will always work
    - Updated example.sh and hw1 spec to reflect this

# Type Hinting in Python



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- Supported in ≥3.6

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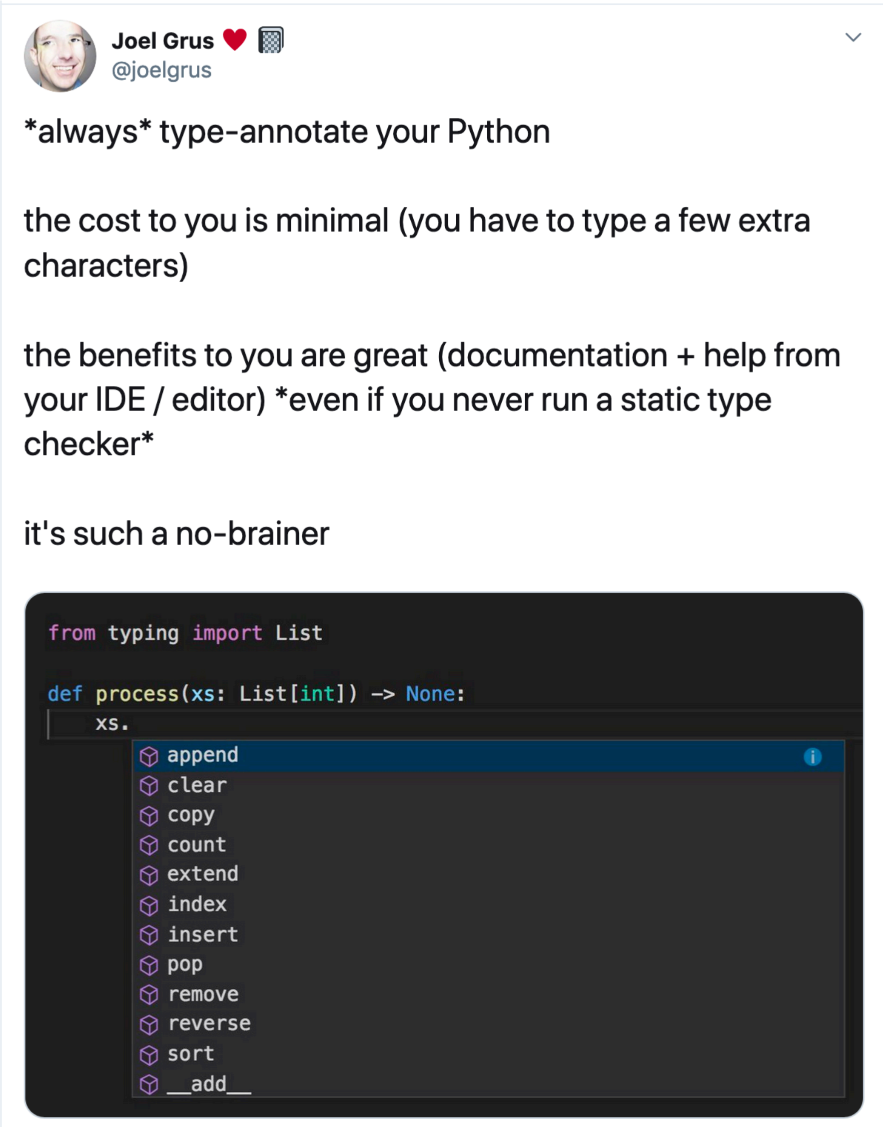

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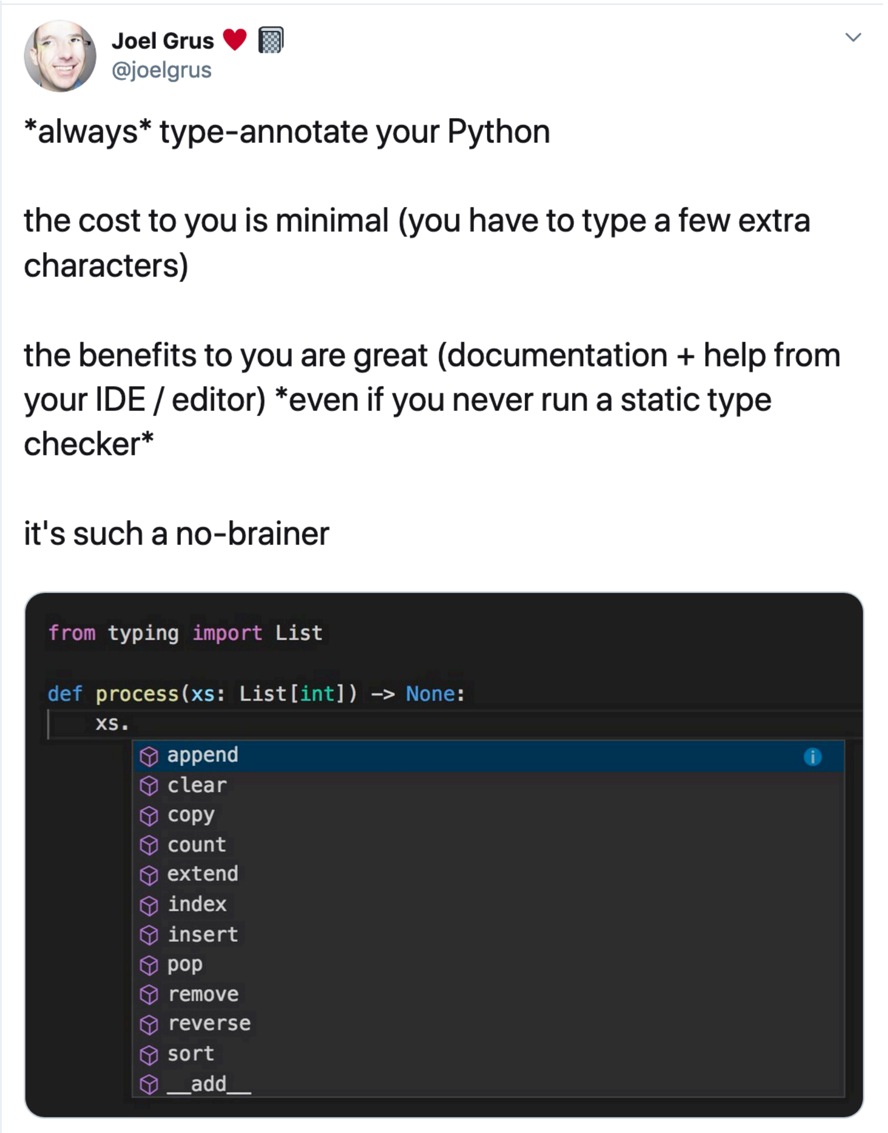

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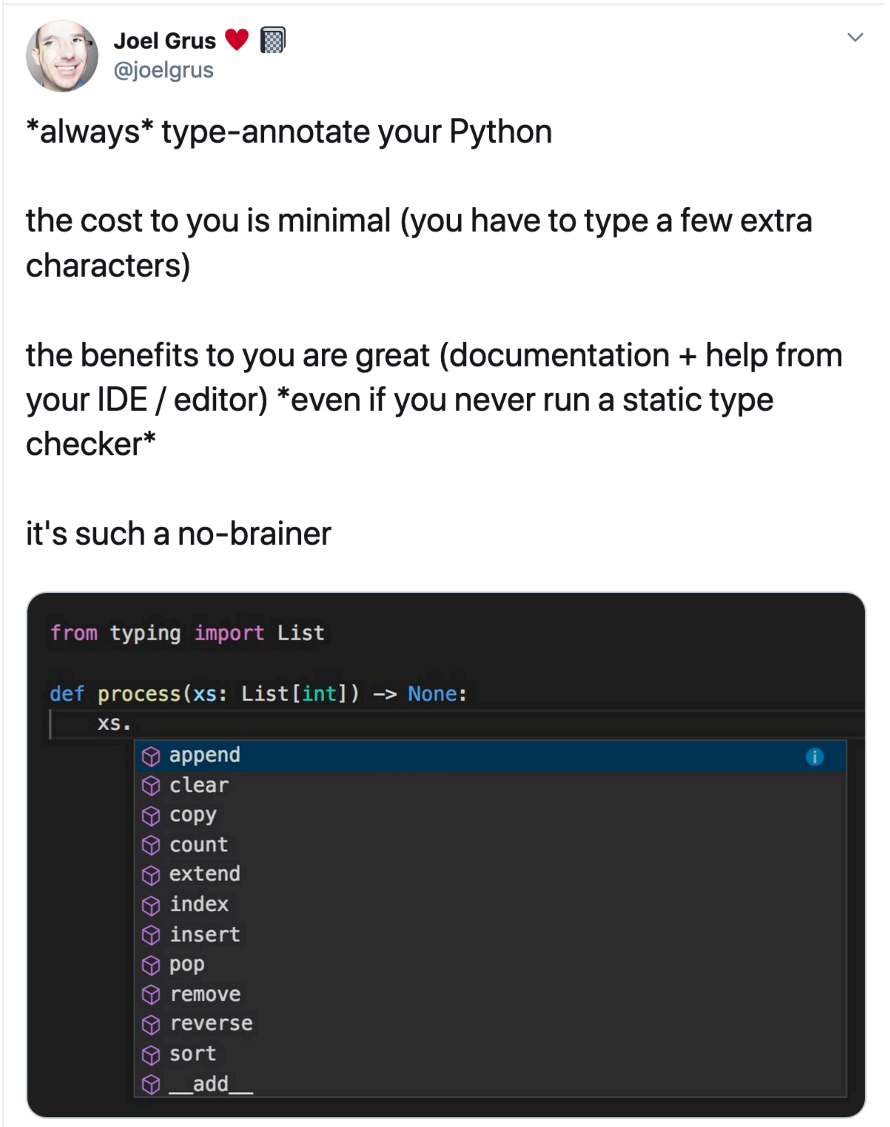

# Recursion in the Wild

**Stephen Boisvert** @srboisvert.bsky.social · 5d
They sometimes surprise with their diets.


3 3 48 •••

**Ed Yong** · 5d •••
@edyong209.bsky.social

A snapping-turtle-snapping heron.

And you: a snapping-turtle-snapping-heron-snapping human.

Sep 28, 2023 at 1:31 PM

**2** reposts **78** likes

# Roadmap

- **Parsing-as-Search**
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

# Computational Parsing

- Given a body of (annotated) text, how can we derive the grammar rules of a language, and employ them in automatic parsing?
  - Treebanks & PCFGs
- Given a grammar, how can we derive the analysis of an input sentence?
  - Parsing as search
  - CKY parsing
    - Conversion to CNF

# What is Parsing?

- CFG parsing is the task of assigning trees to input strings
  - For any input $A$ and grammar $G$
    - …assign ≥0 parse trees $T$ that represent its syntactic structure, and…
    - Cover all and only the elements of $A$
    - Have, as root, the start symbol $S$ of $G$
    - …do not necessarily pick one single (or correct) analysis
- Subtask: Recognition
  - Given input $A$, $G$ – is $A$ in language defined by $G$ or not?

# Motivation

- Is this sentence in the language — i.e. is it “grammatical?”
  - ** I prefer United has the earliest flight.*
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).

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  - ** I prefer United has the earliest flight.*
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).
- What is the syntactic structure of this sentence?
  - *What airline has the cheapest flight?*
  - *What airport does Southwest fly from near Boston?*
  - Syntactic parse provides framework for semantic analysis
    - What is the subject? Direct object?

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- Syntactic parsing searches through possible trees to find one or more trees that derive input
- Formally, search problems are defined by:
  - Start state $S$
  - Goal state $G$ (with a test)
  - Set of actions that transition from one state to another
    - "Successor function"
  - A path cost function

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- Path cost:
  - …ignored for now.

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- Search start node (initial state):
  - Input string
  - Start symbol of CFG
- Goal node:
  - Full parse tree: covering all of, and only the input, rooted at $S$

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- Depth First
  - Keep expanding nonterminals until they reach words
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  - Keep expanding nonterminals until they reach words
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- Breadth First
  - Consider all parses that expand a single nonterminal…
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- Other alternatives, if have associated path costs.

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- Two constraints on parsing:
  - Must start with the start symbol
  - Must cover exactly the input string
- Correspond to main parsing search strategies
  - Top-down search (Goal-directed)
  - Bottom-up search (Data-driven search)

# A Grammar

| Grammar | Lexicon |
|---|---|
| $S \rightarrow NP\ VP$ | $Det \rightarrow that \mid this \mid a$ |
| $S \rightarrow Aux\ NP\ VP$ | $Noun \rightarrow book \mid flight \mid meal \mid money$ |
| $S \rightarrow VP$ | $Verb \rightarrow book \mid include \mid prefer$ |

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| $NP \rightarrow Proper\text{-}Noun$ | $Proper\text{-}Noun \rightarrow Houston \mid NWA$ |
| $NP \rightarrow Det\ Nominal$ | $Aux \rightarrow does$ |
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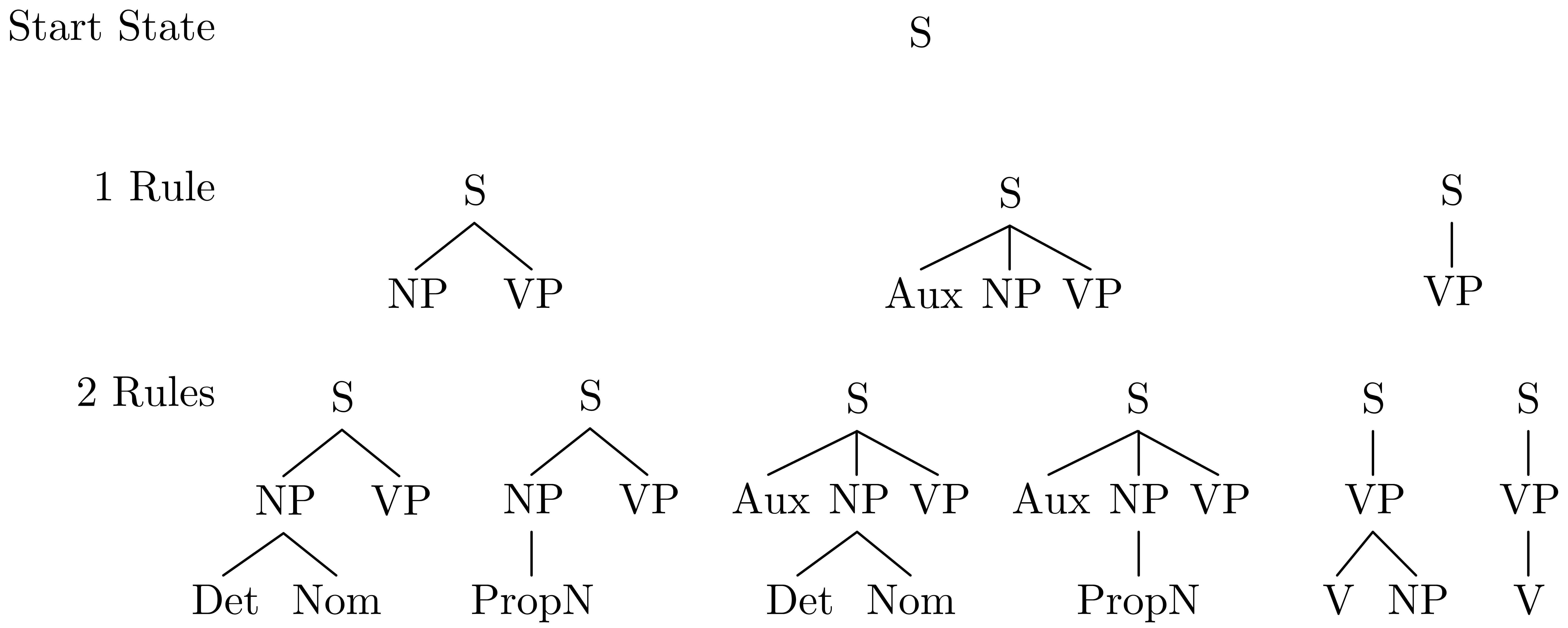
  - e.g. $NP \rightarrow Det\ Nominal;\ VP \rightarrow V\ NP$
- Terminate when all leaves are terminals

# Depth-First Search




Book that flight.

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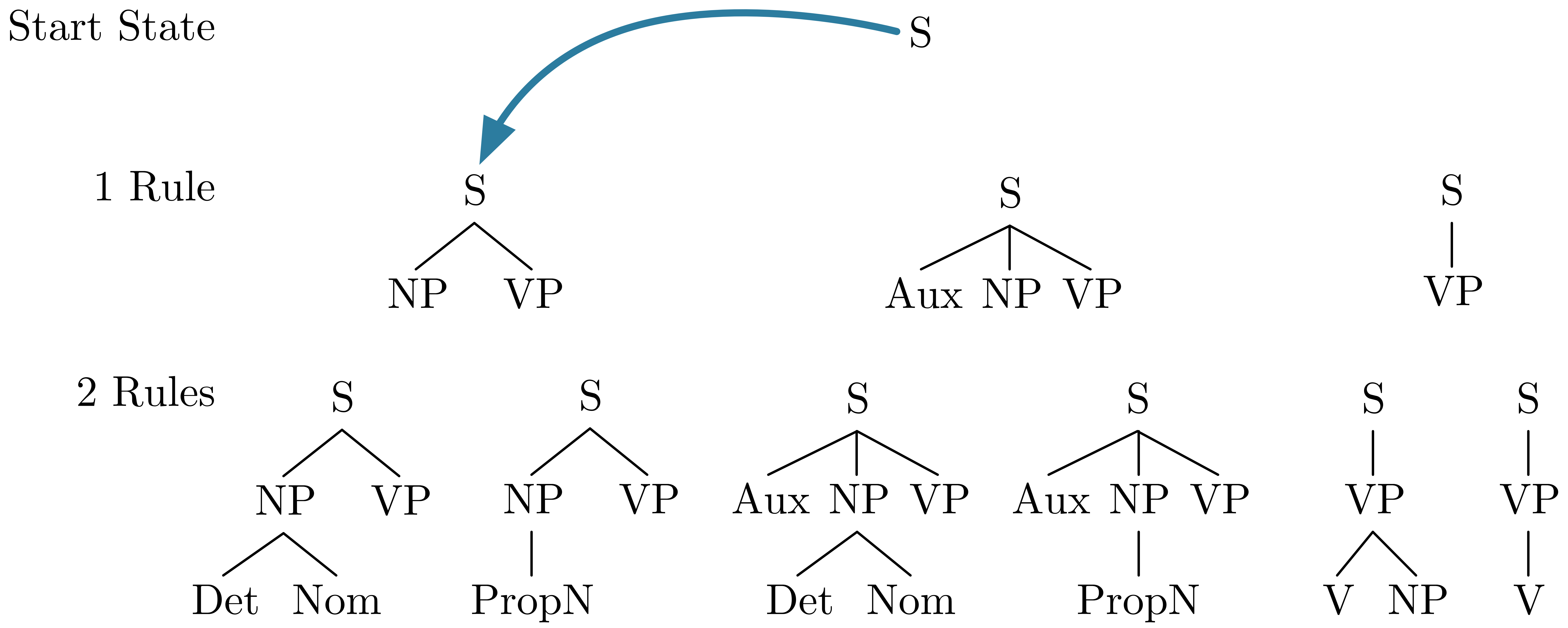


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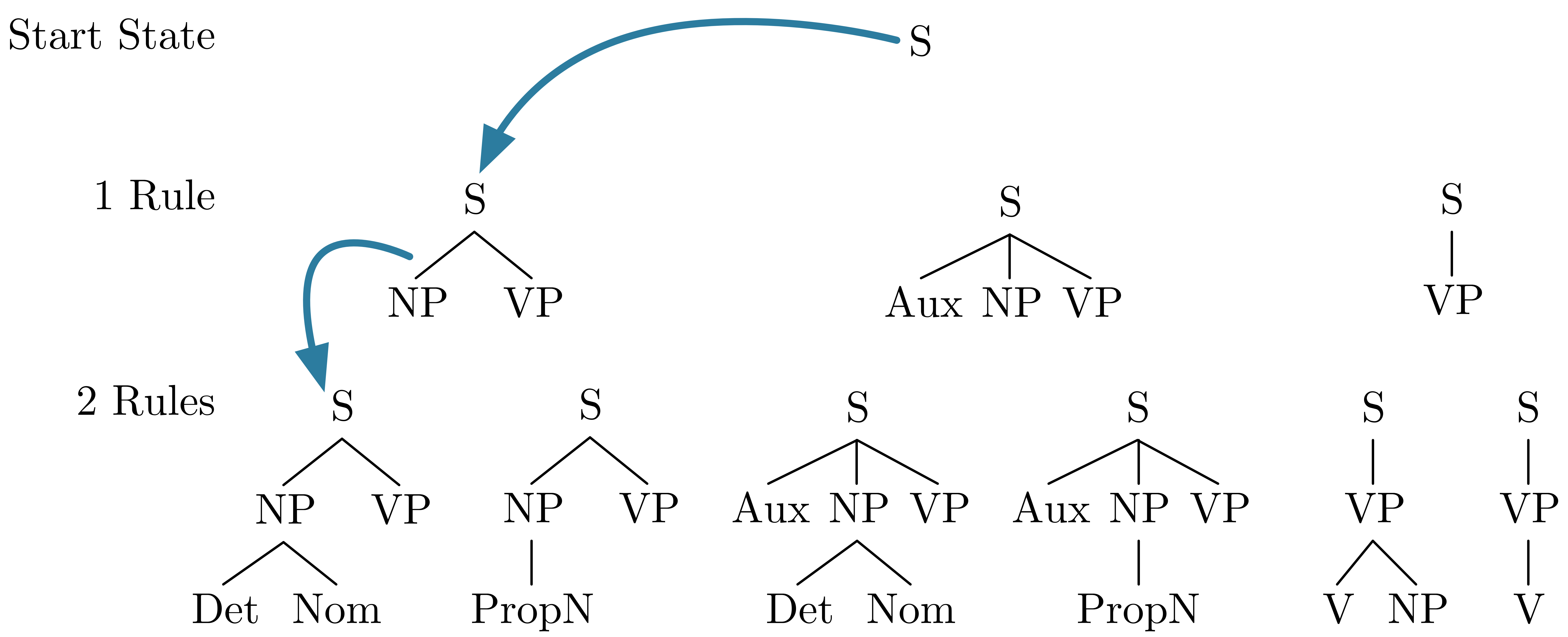


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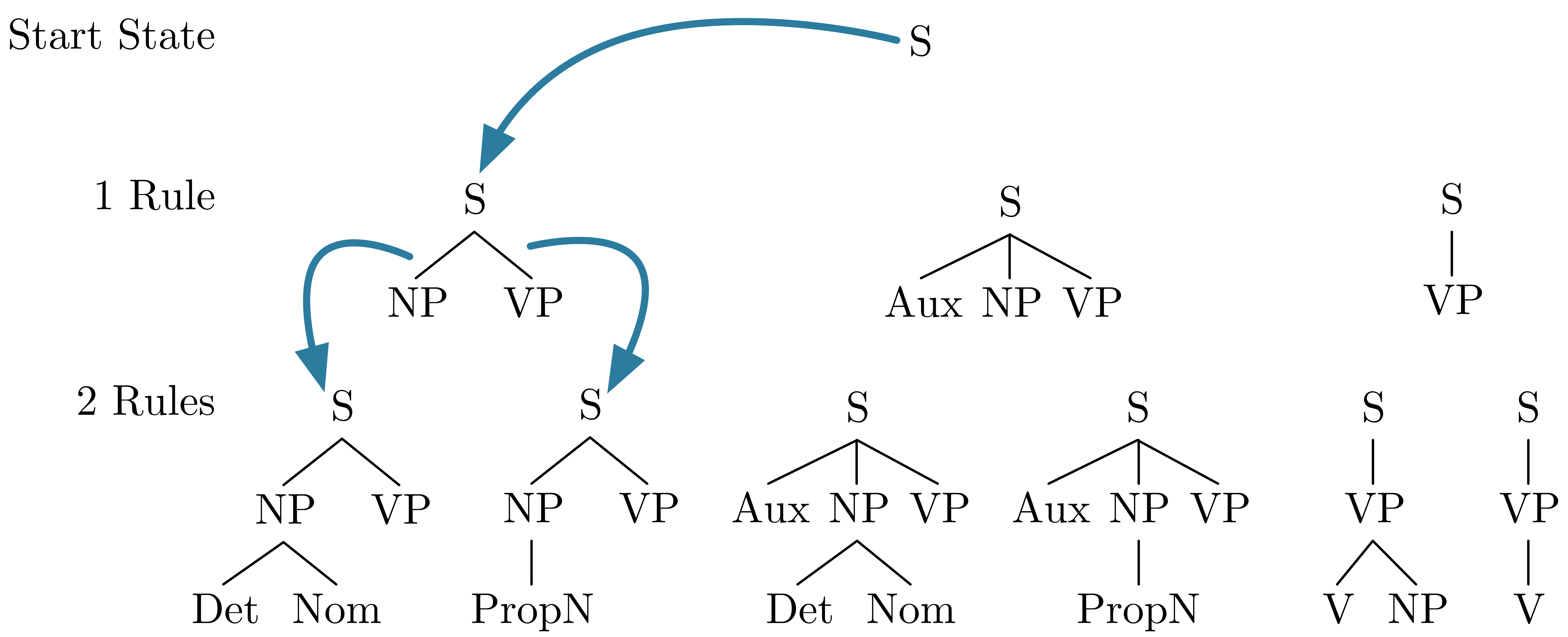


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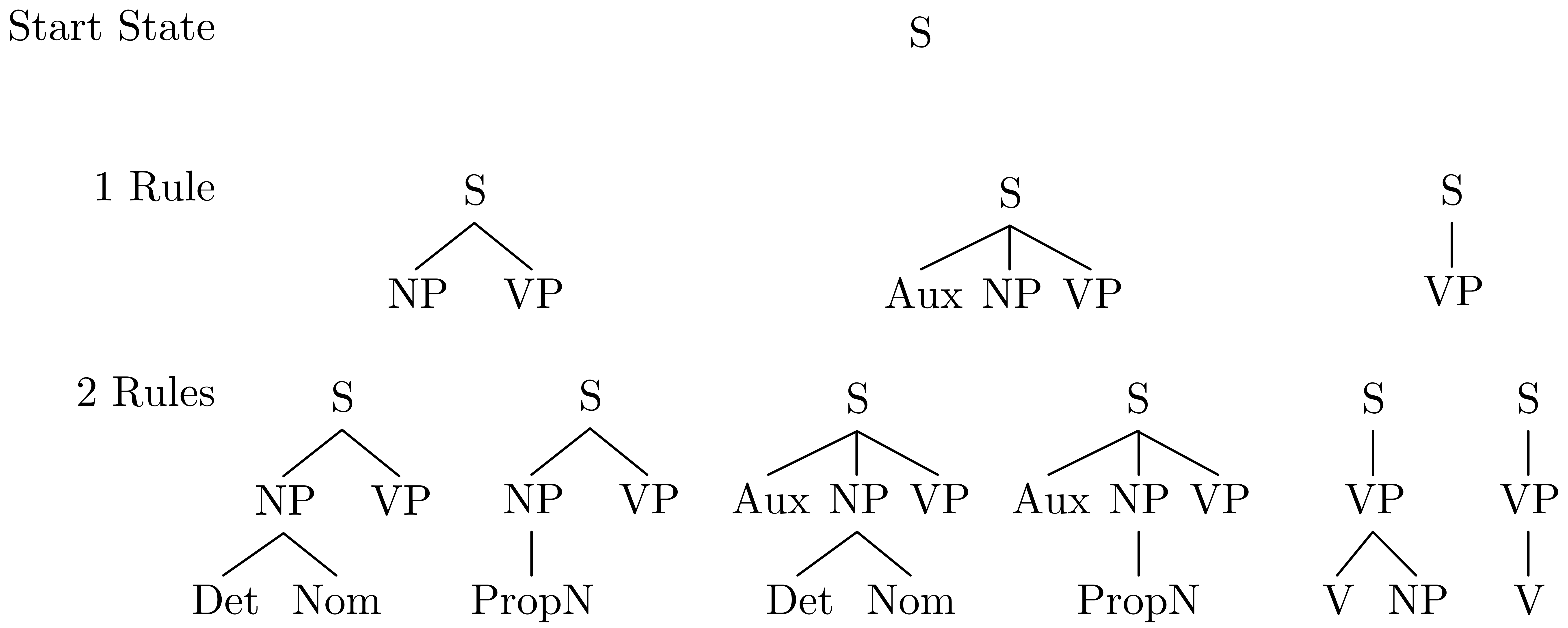


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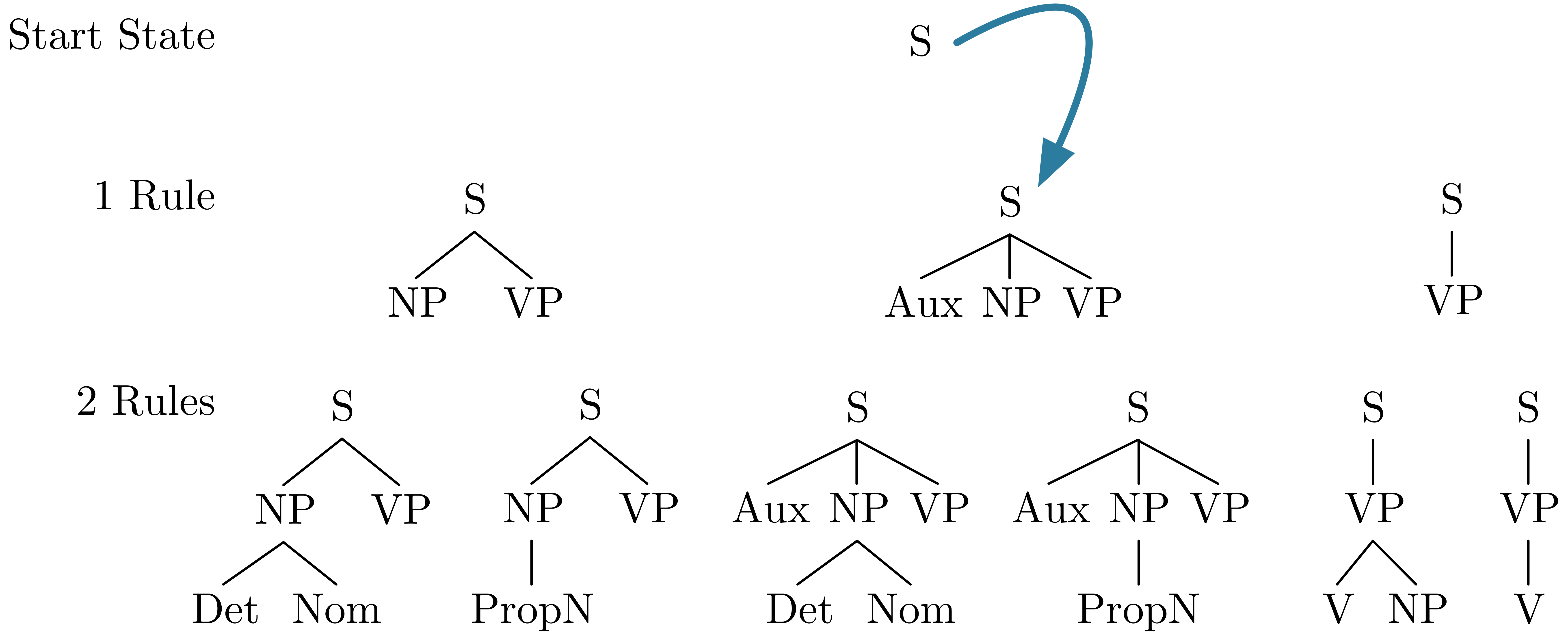


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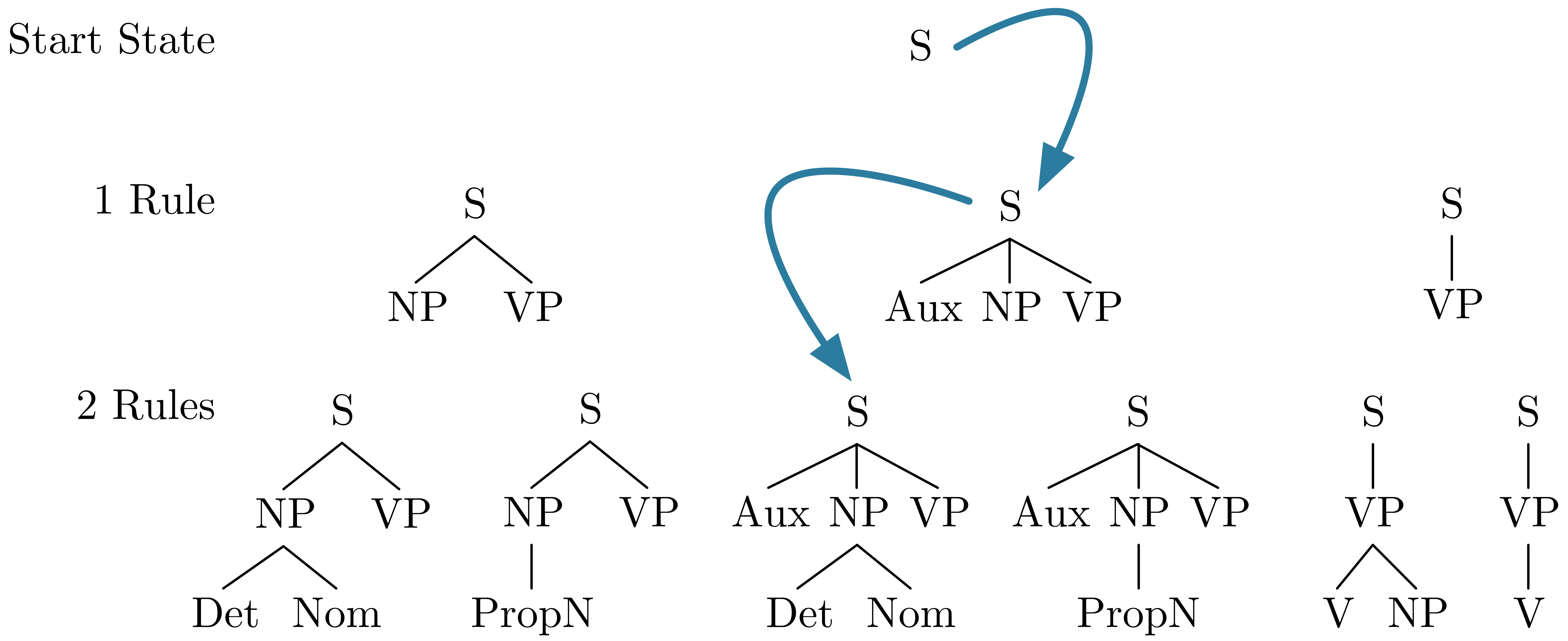


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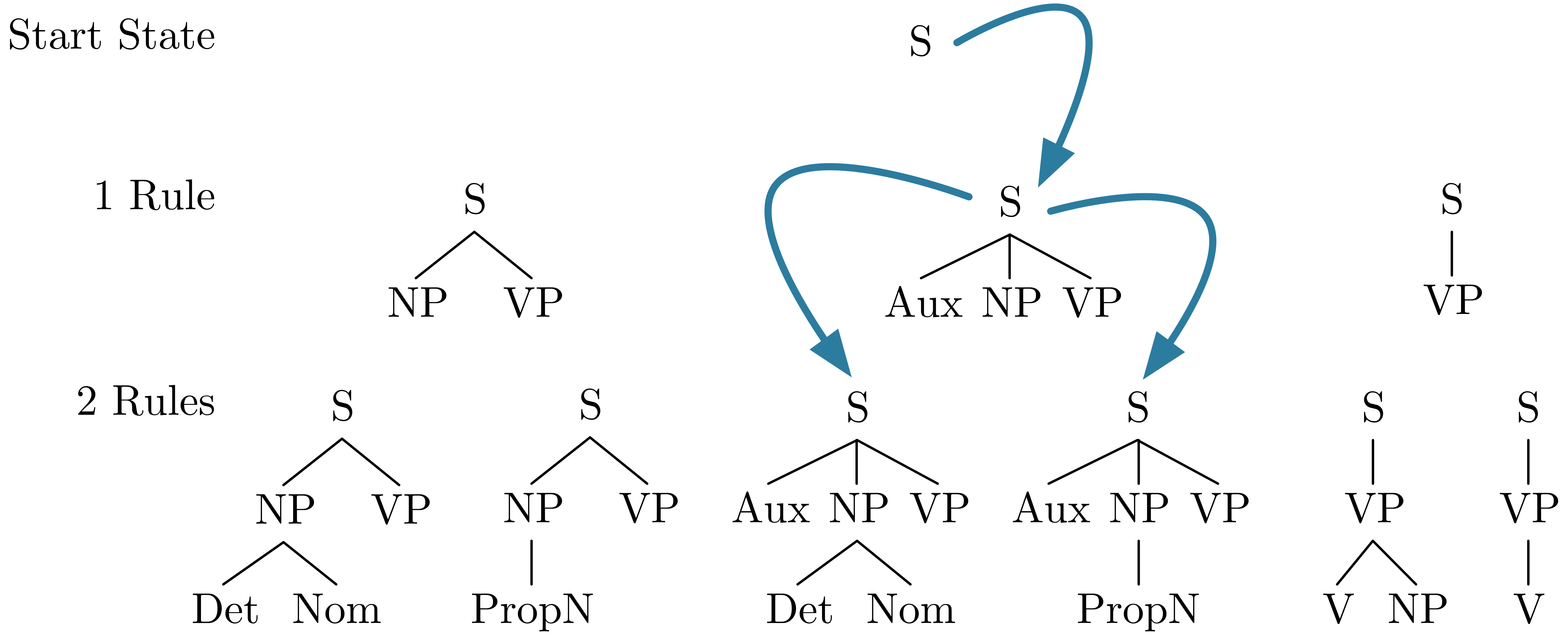


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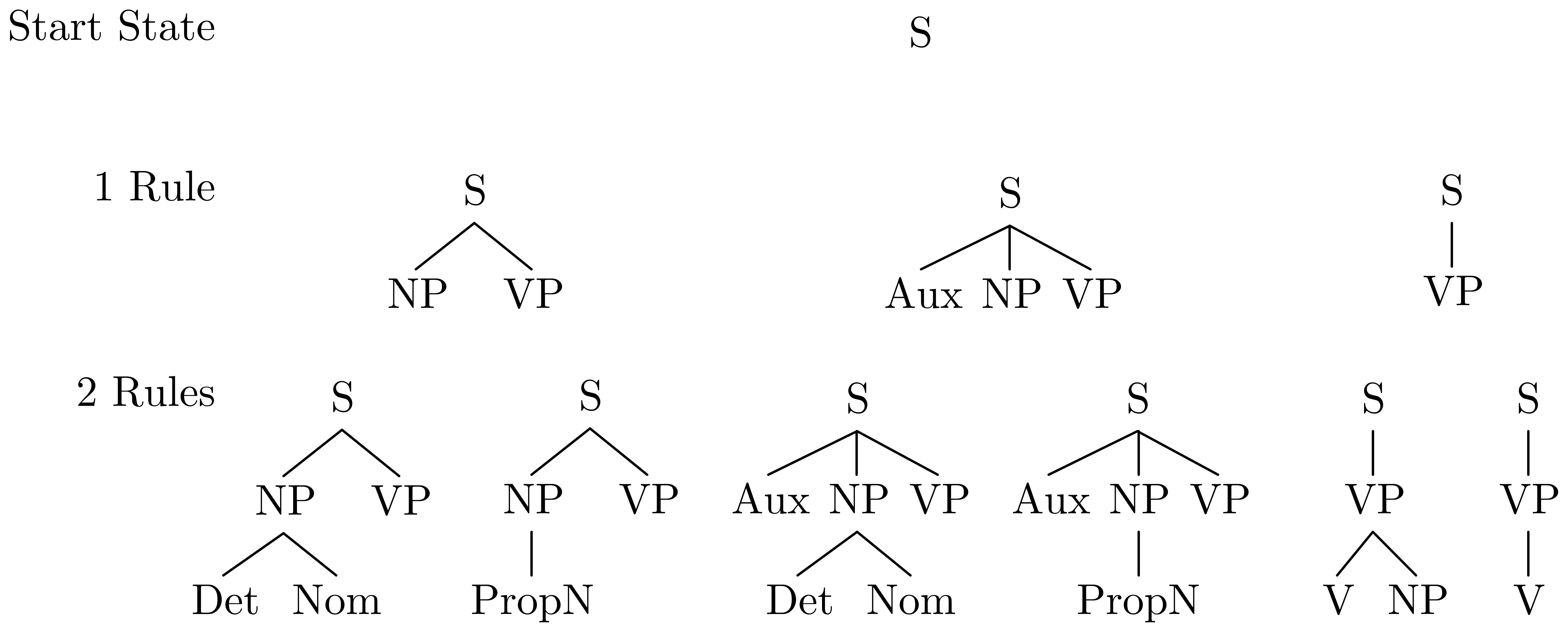


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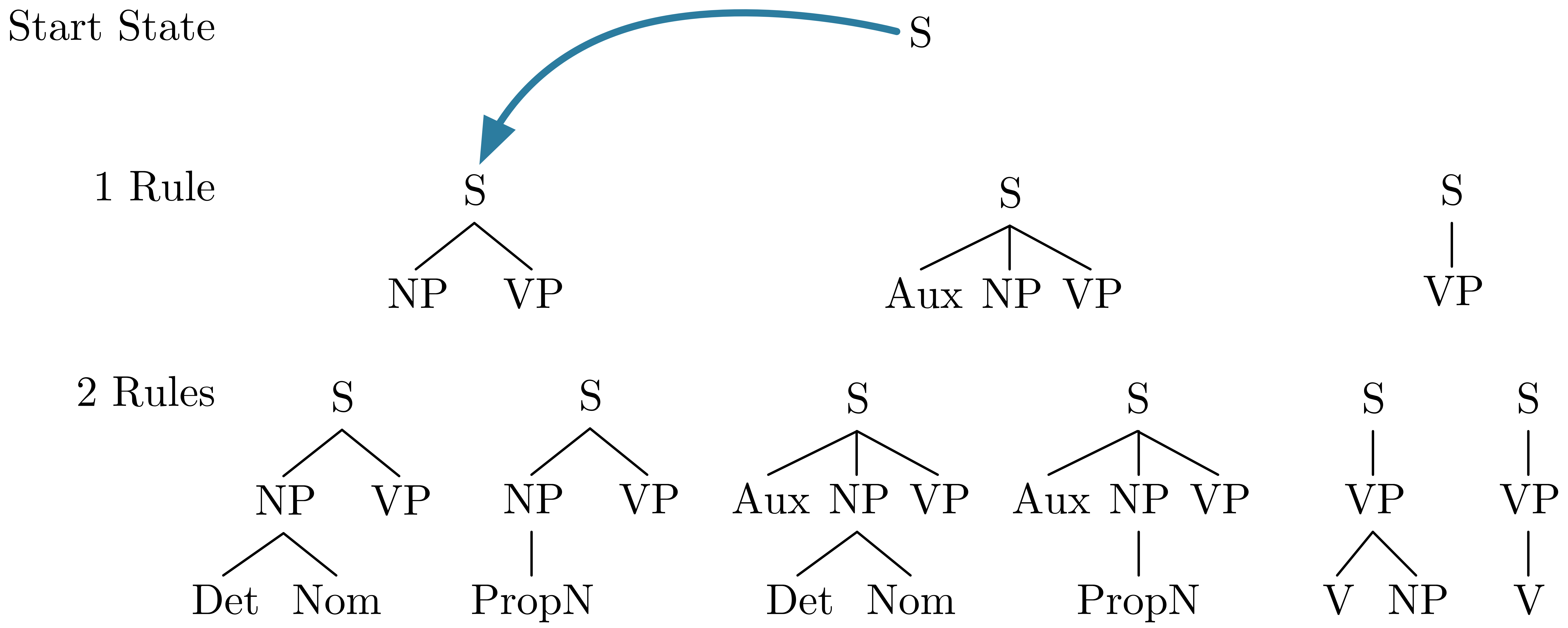


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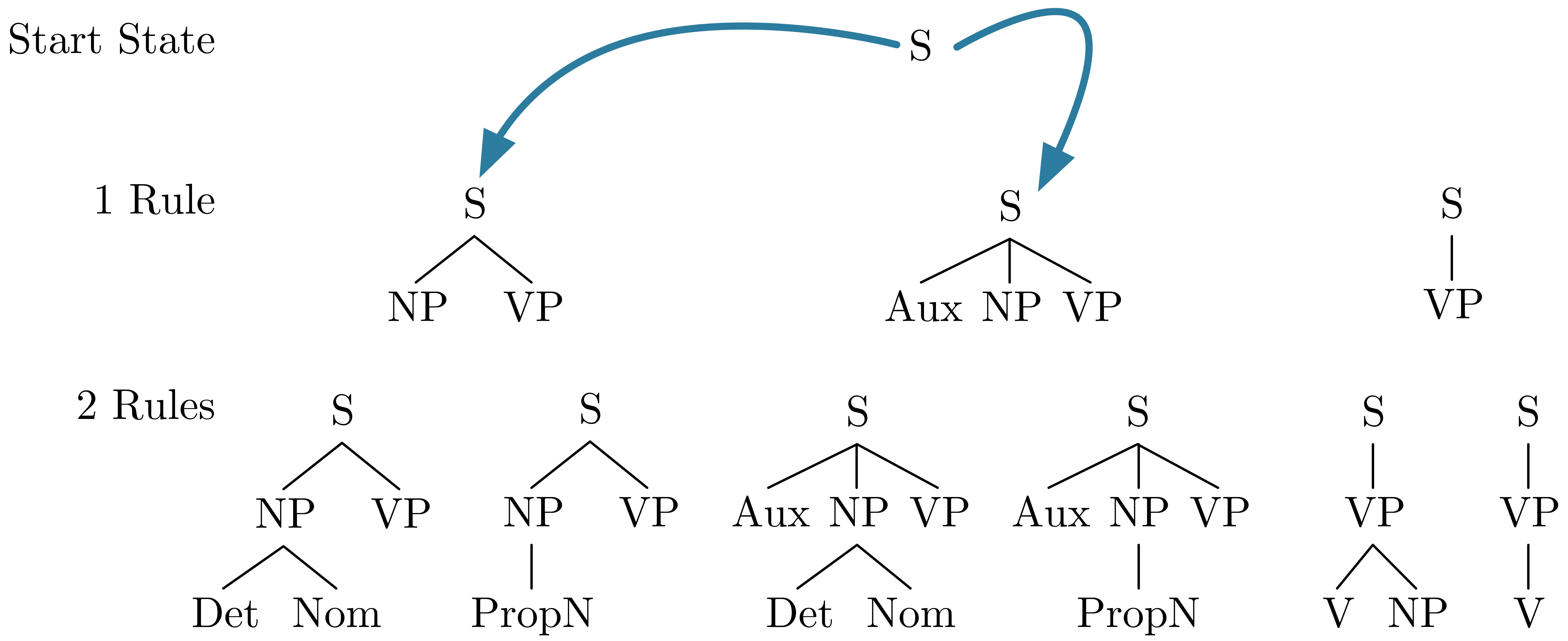


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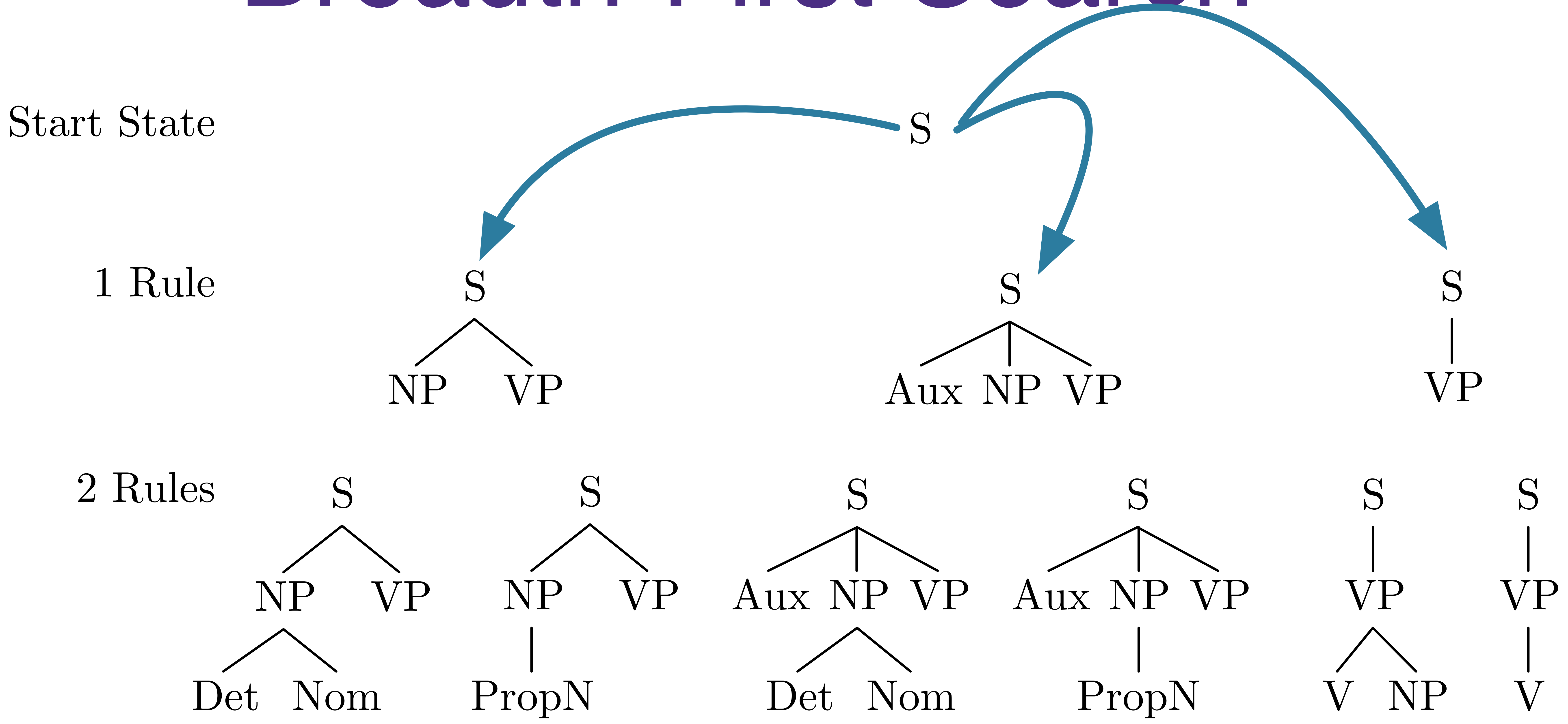


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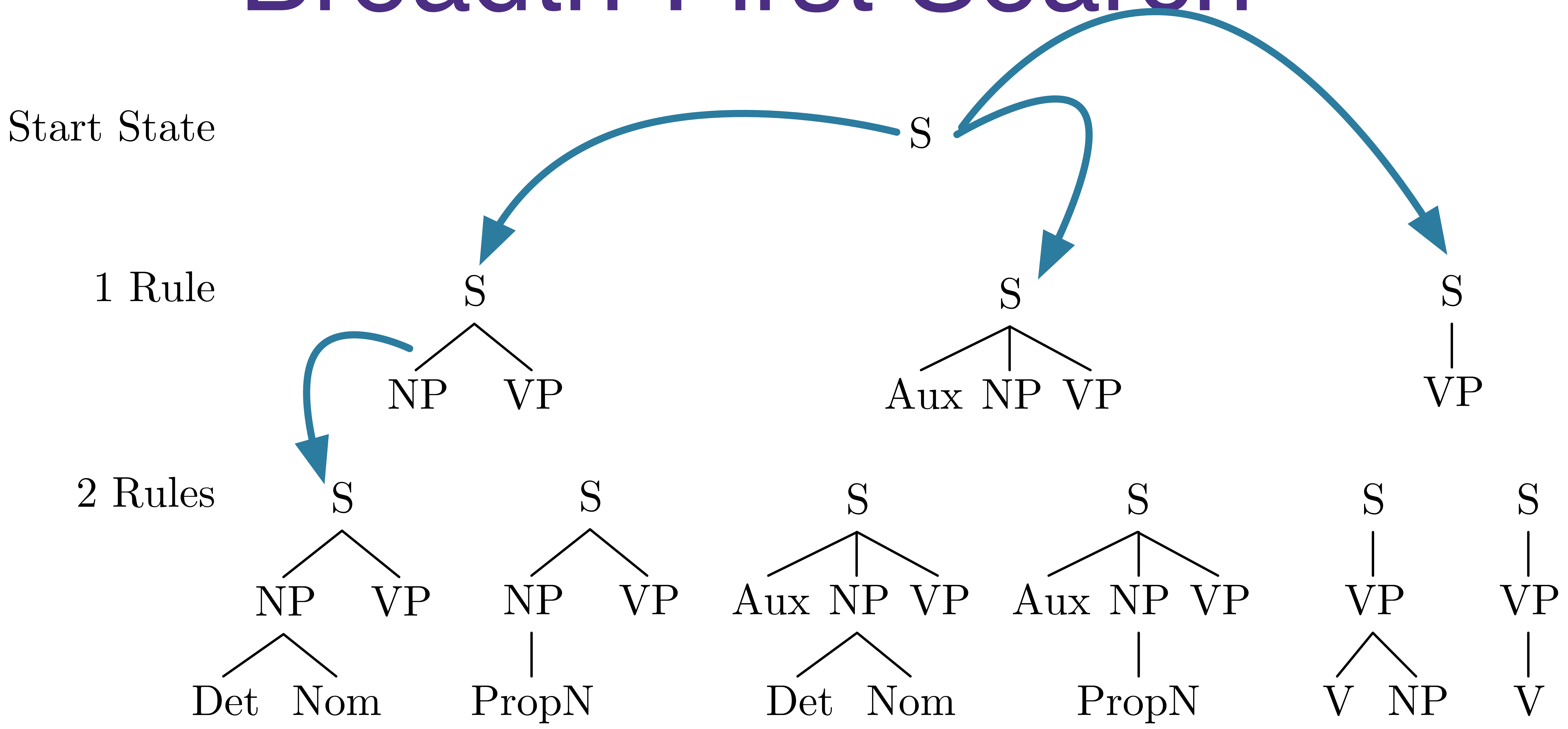


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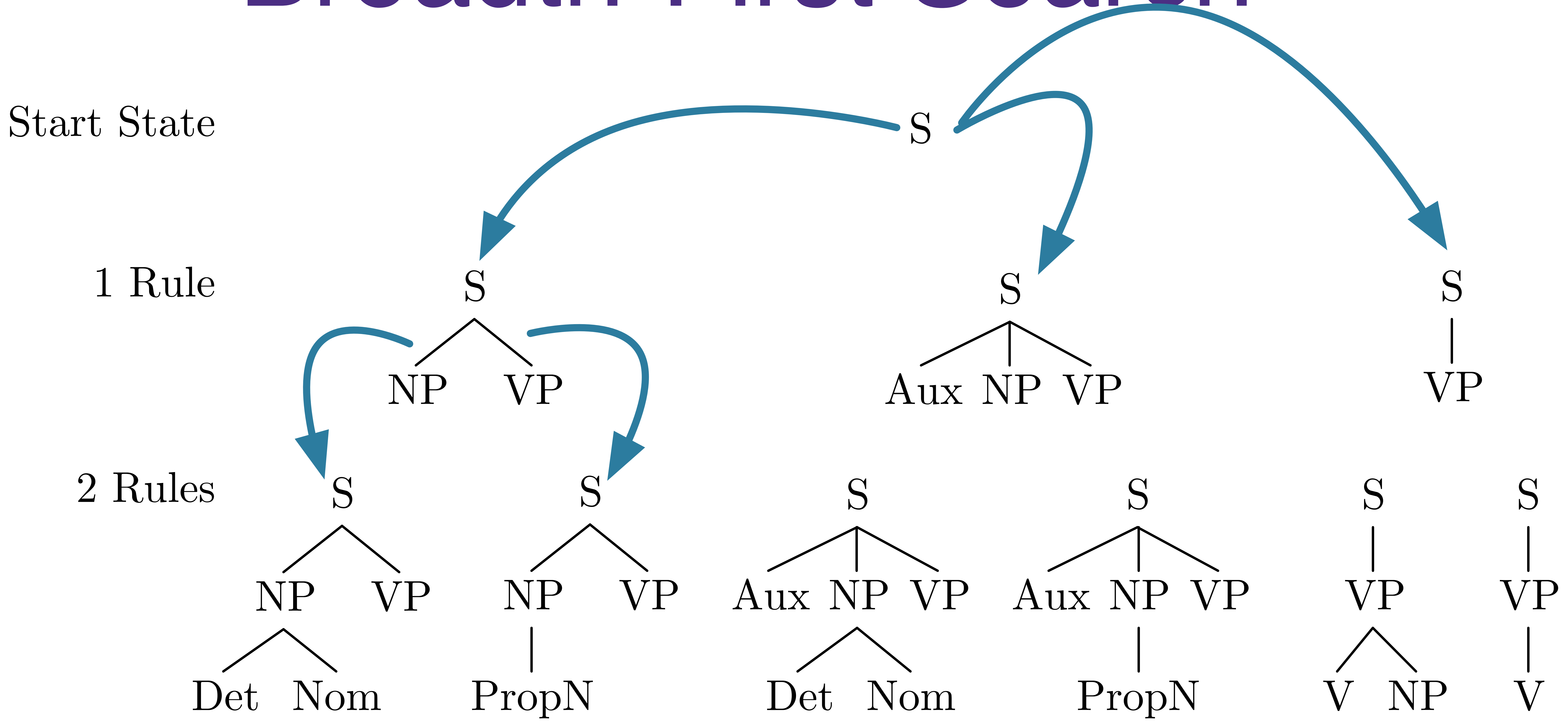


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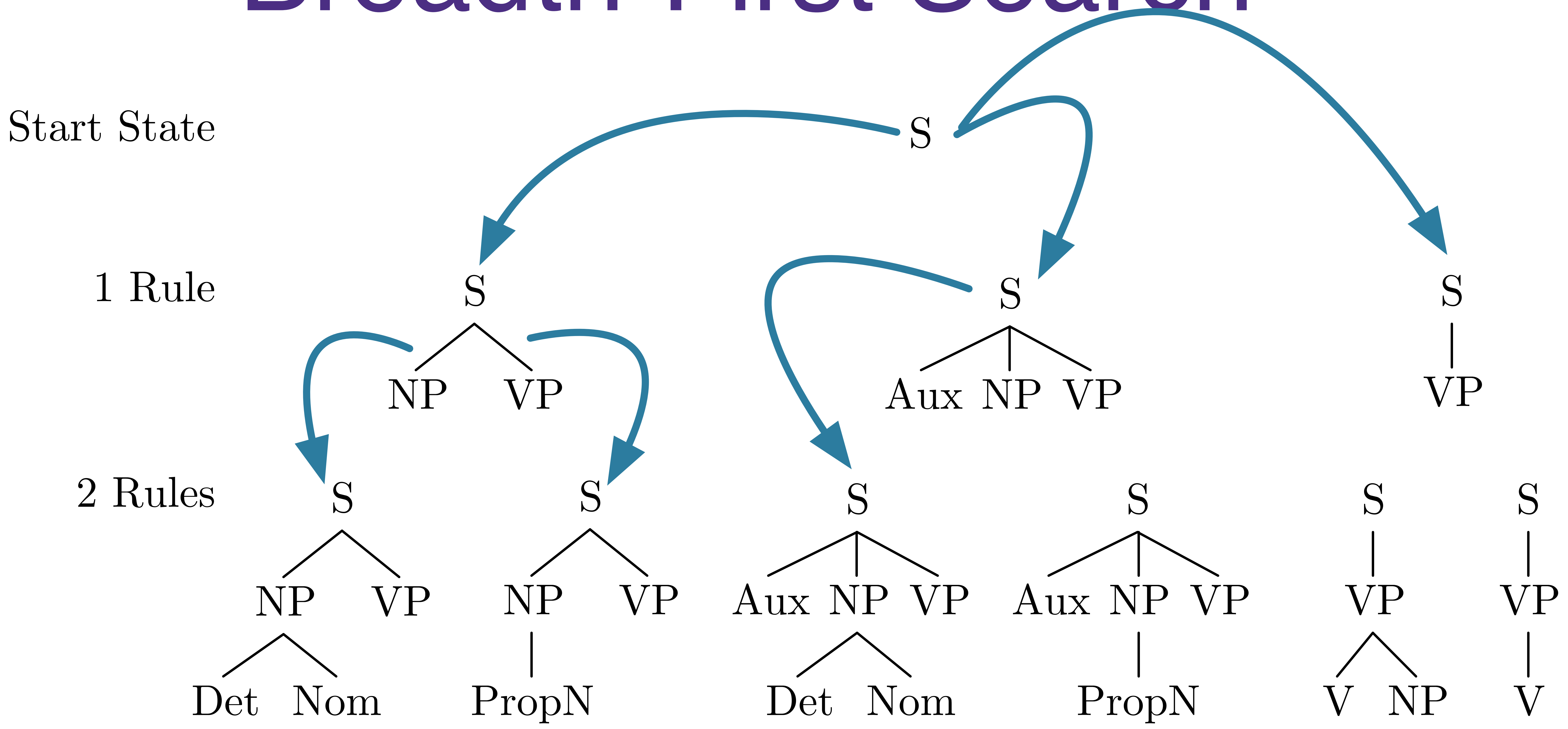


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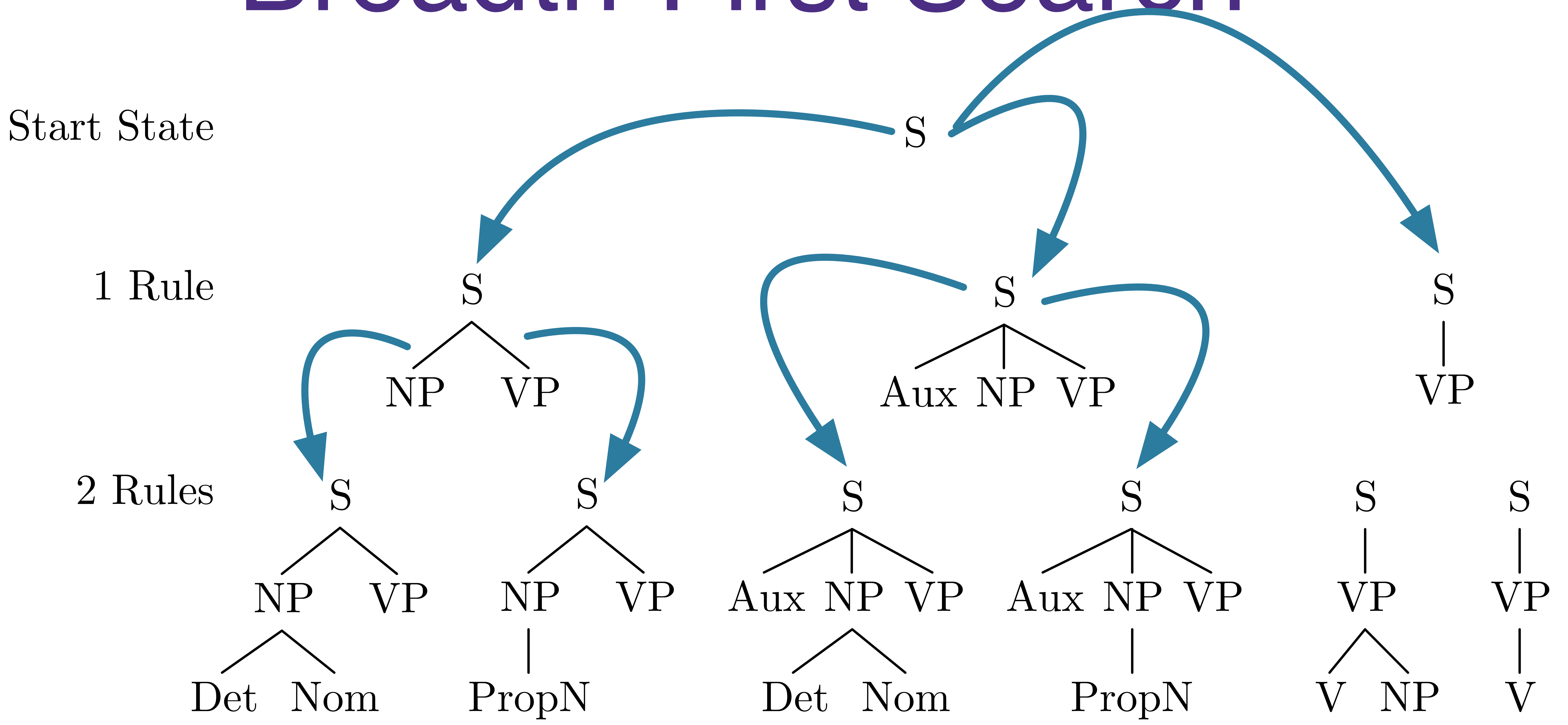


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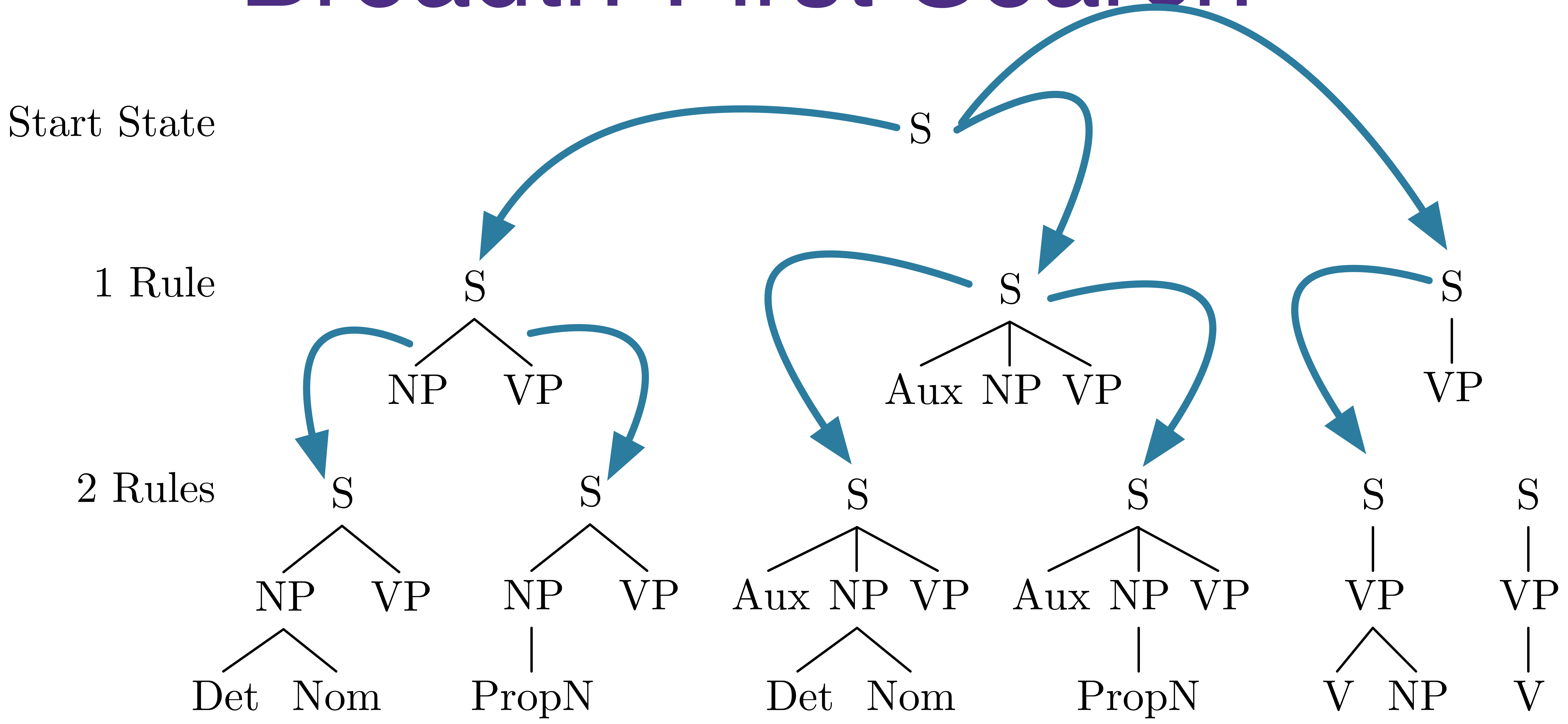


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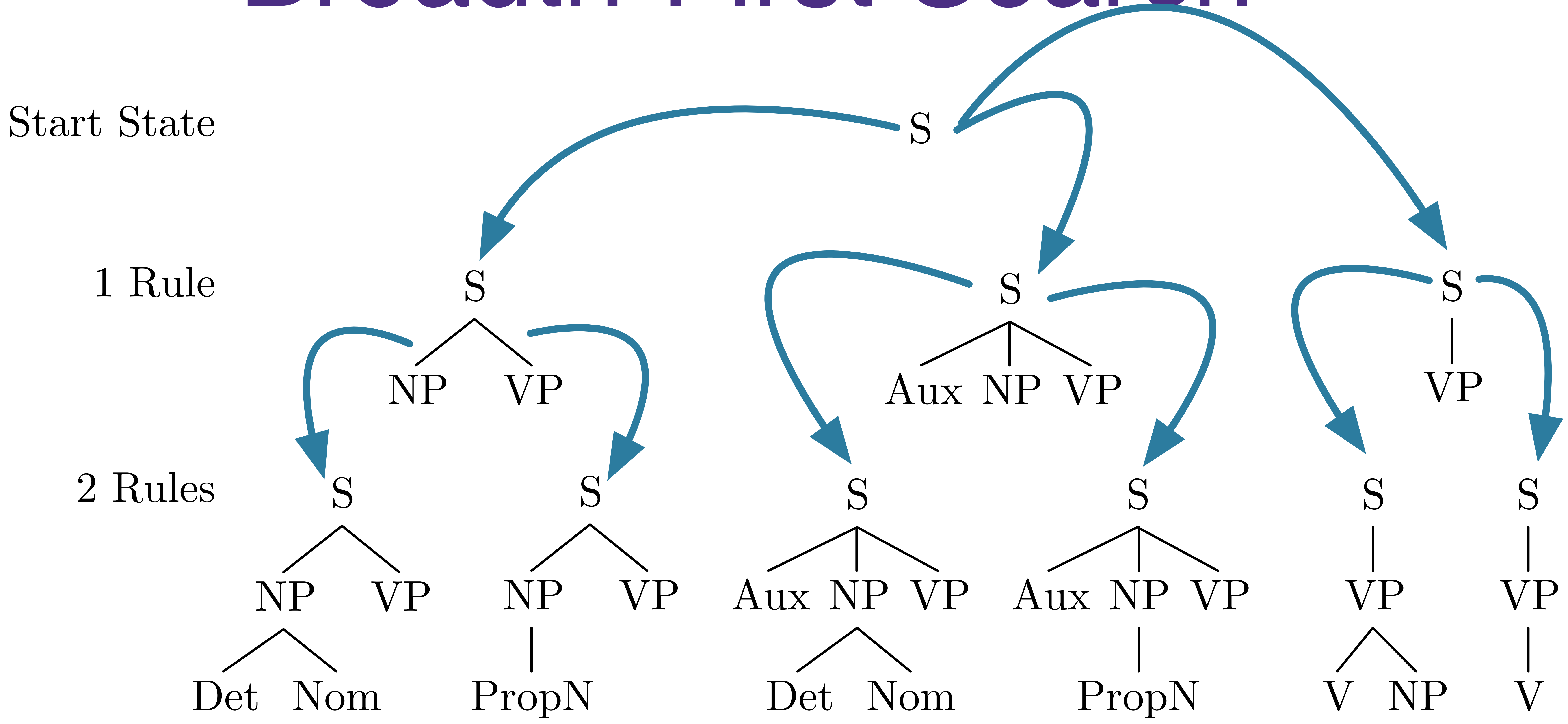


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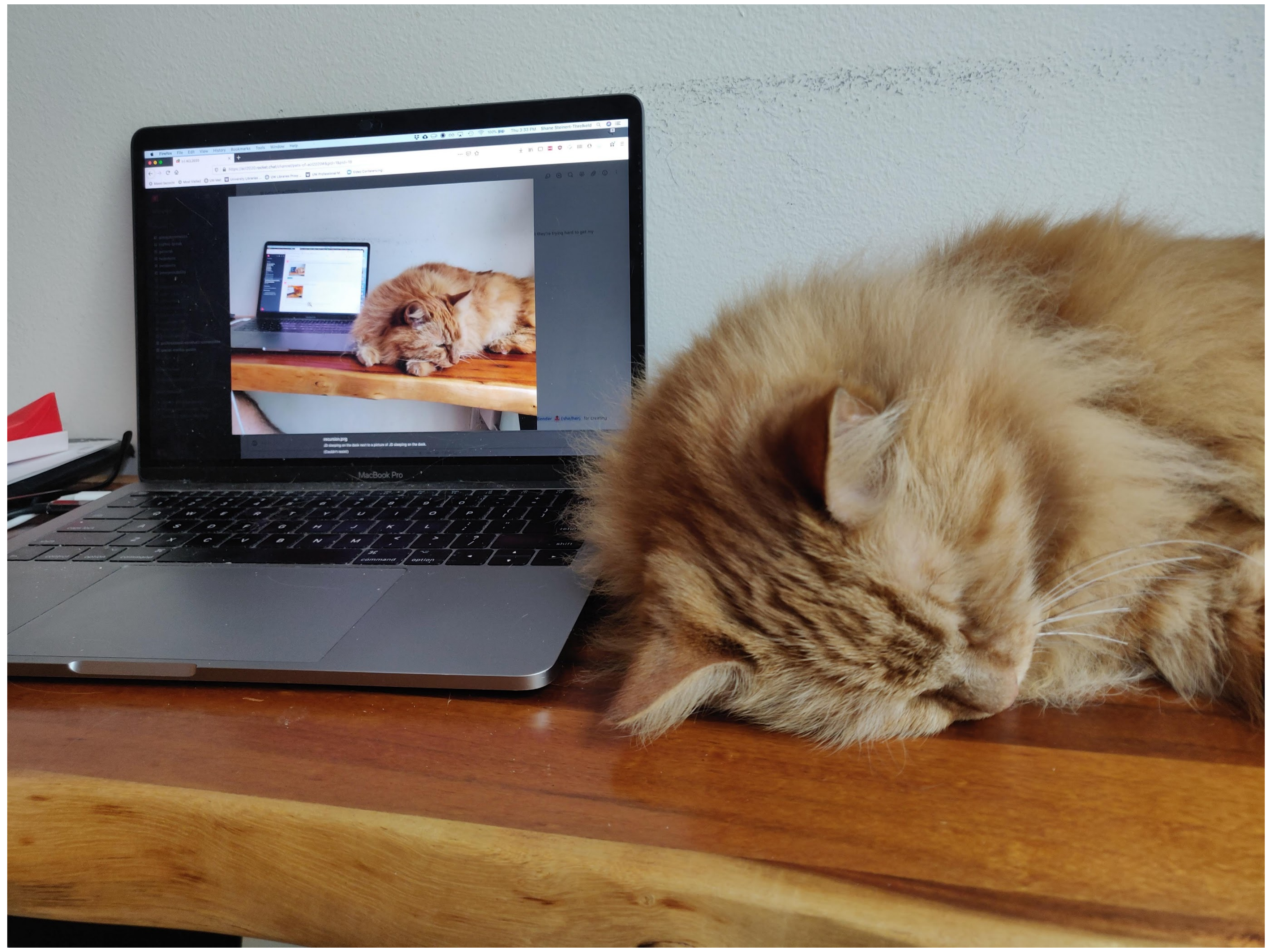

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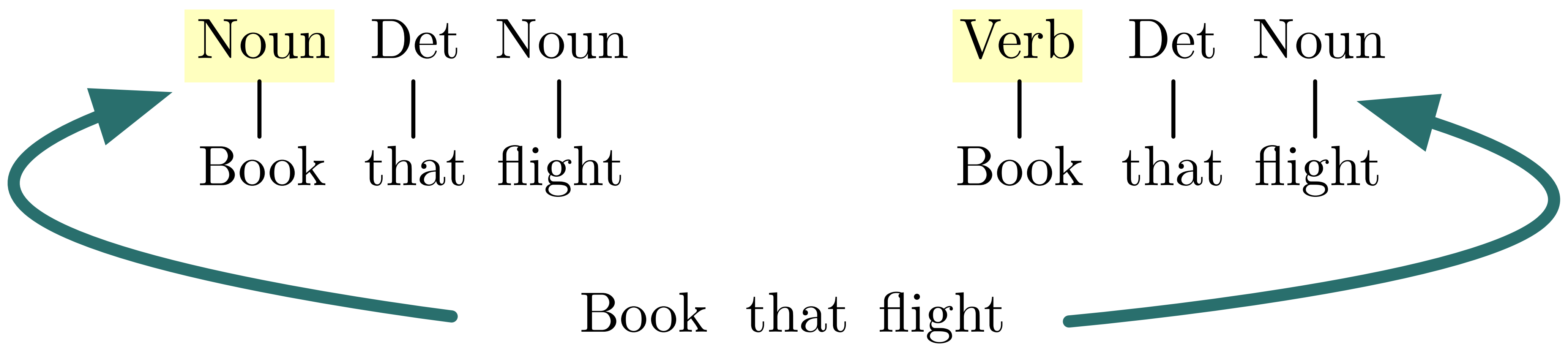
- Stop when spanned by S, or no more rules apply

Book that flight


Noun Det Noun
Book that flight
Verb Det Noun
Book that flight
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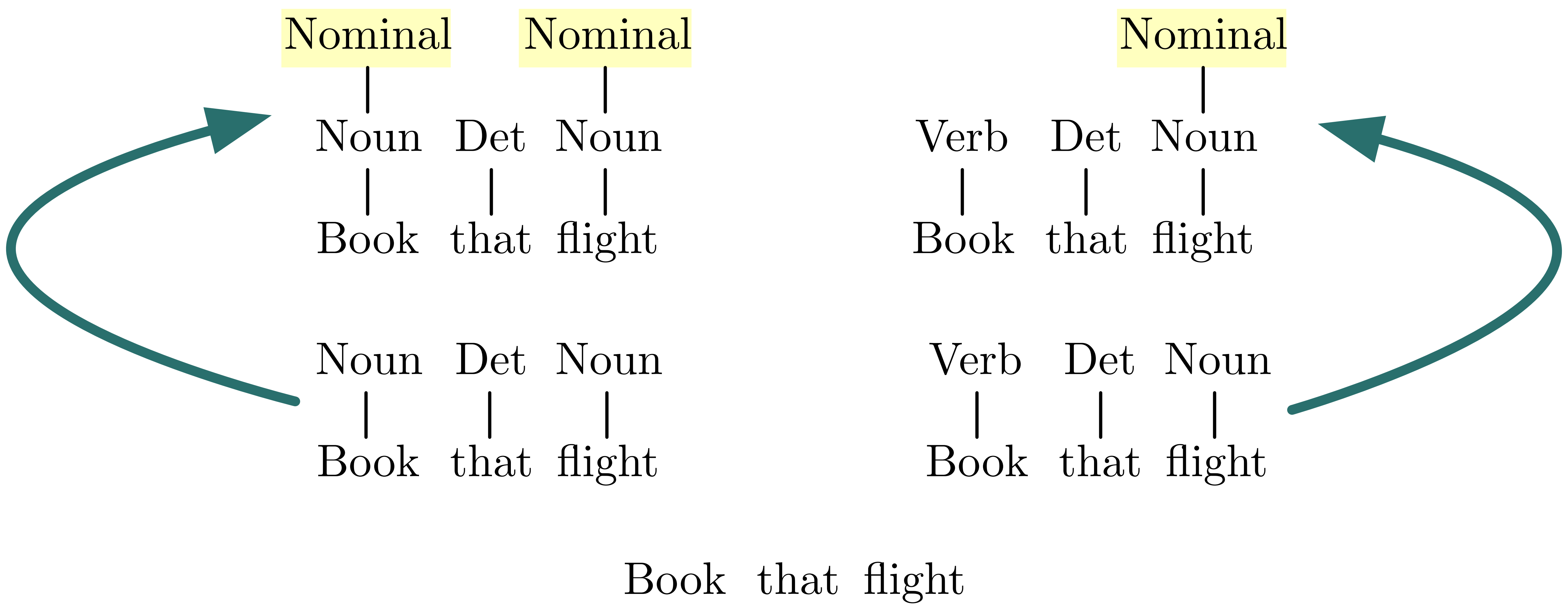
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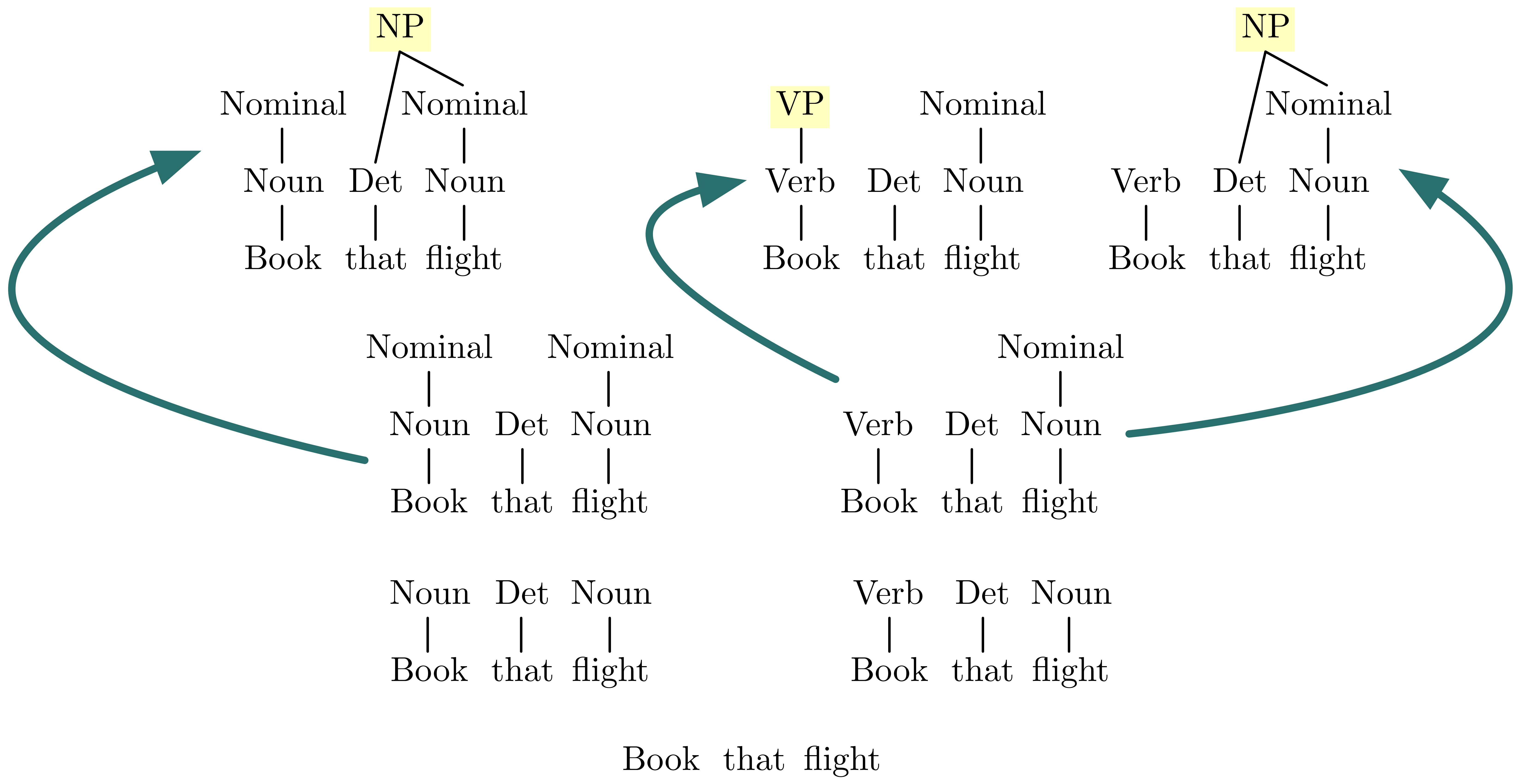
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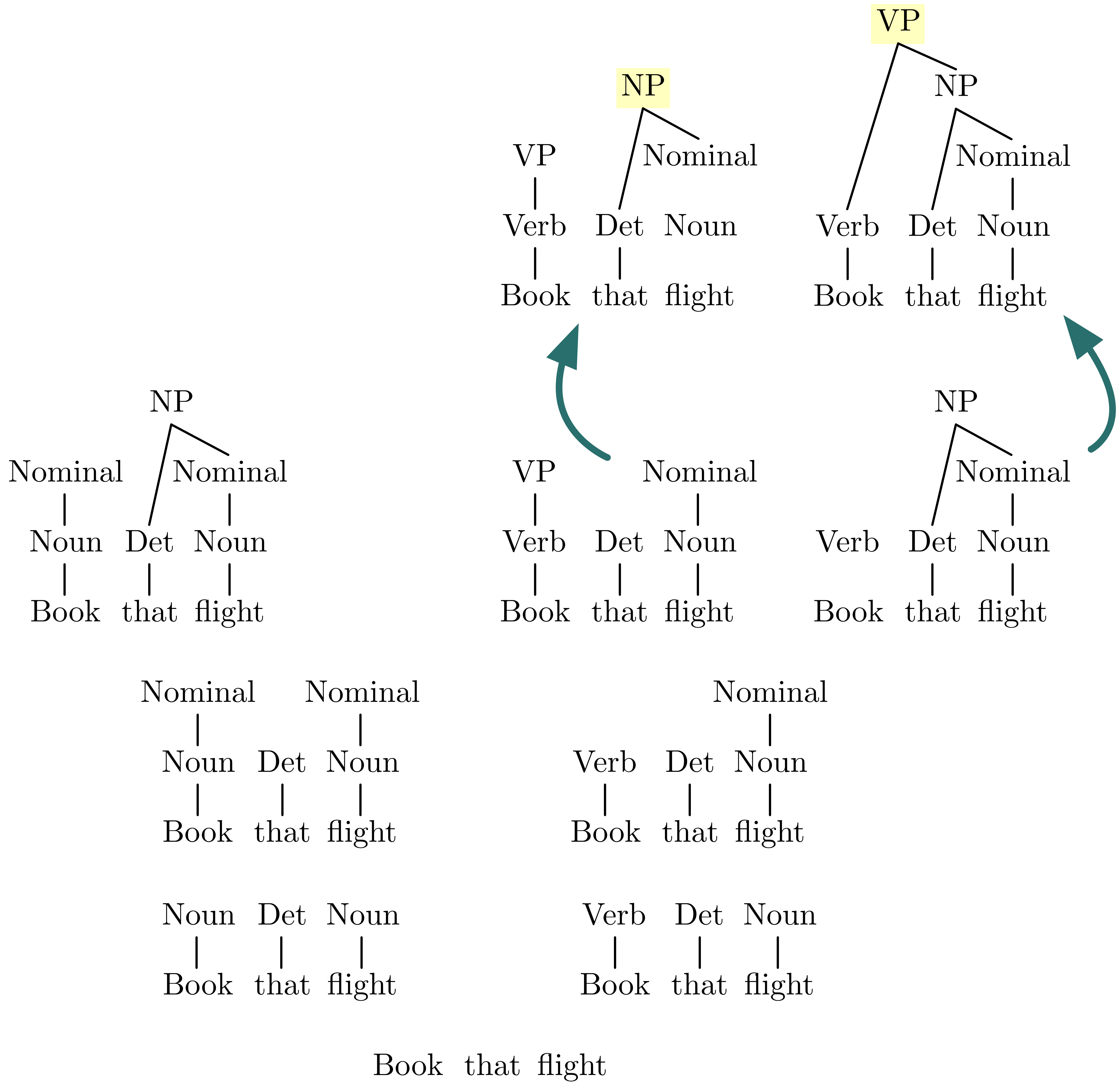
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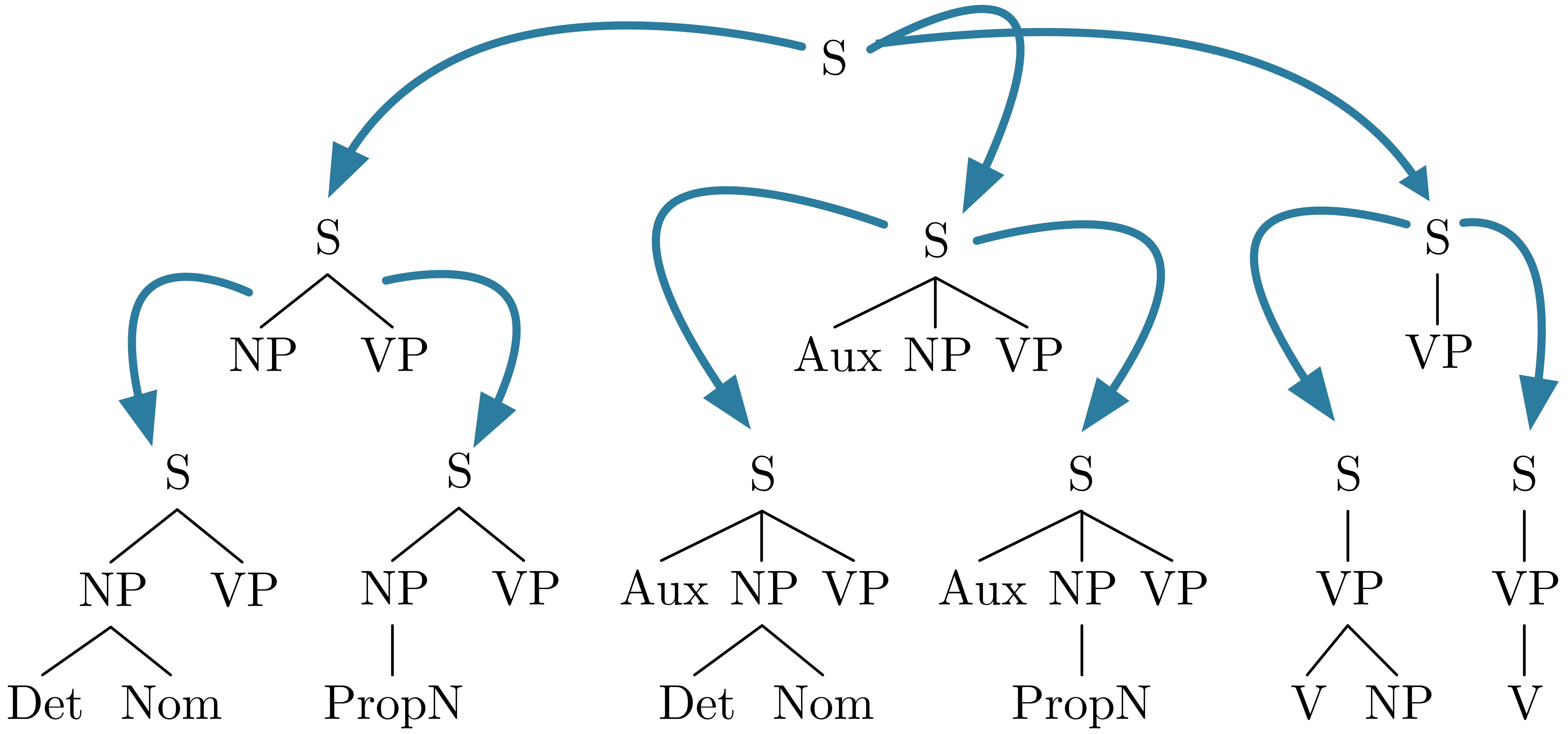
- Cons:
  - Explore subtrees that will not fit full input

# Recap: Parsing as Search



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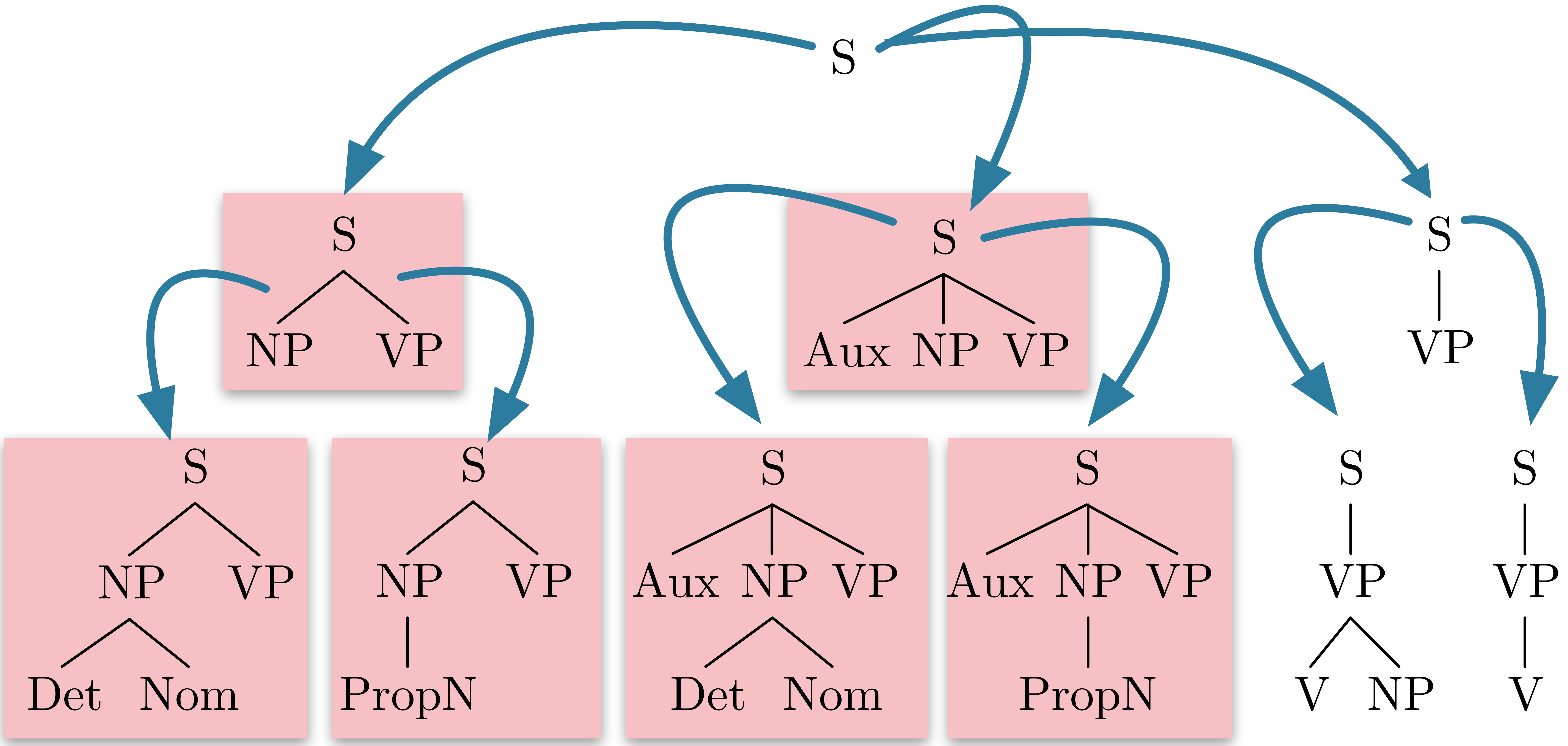
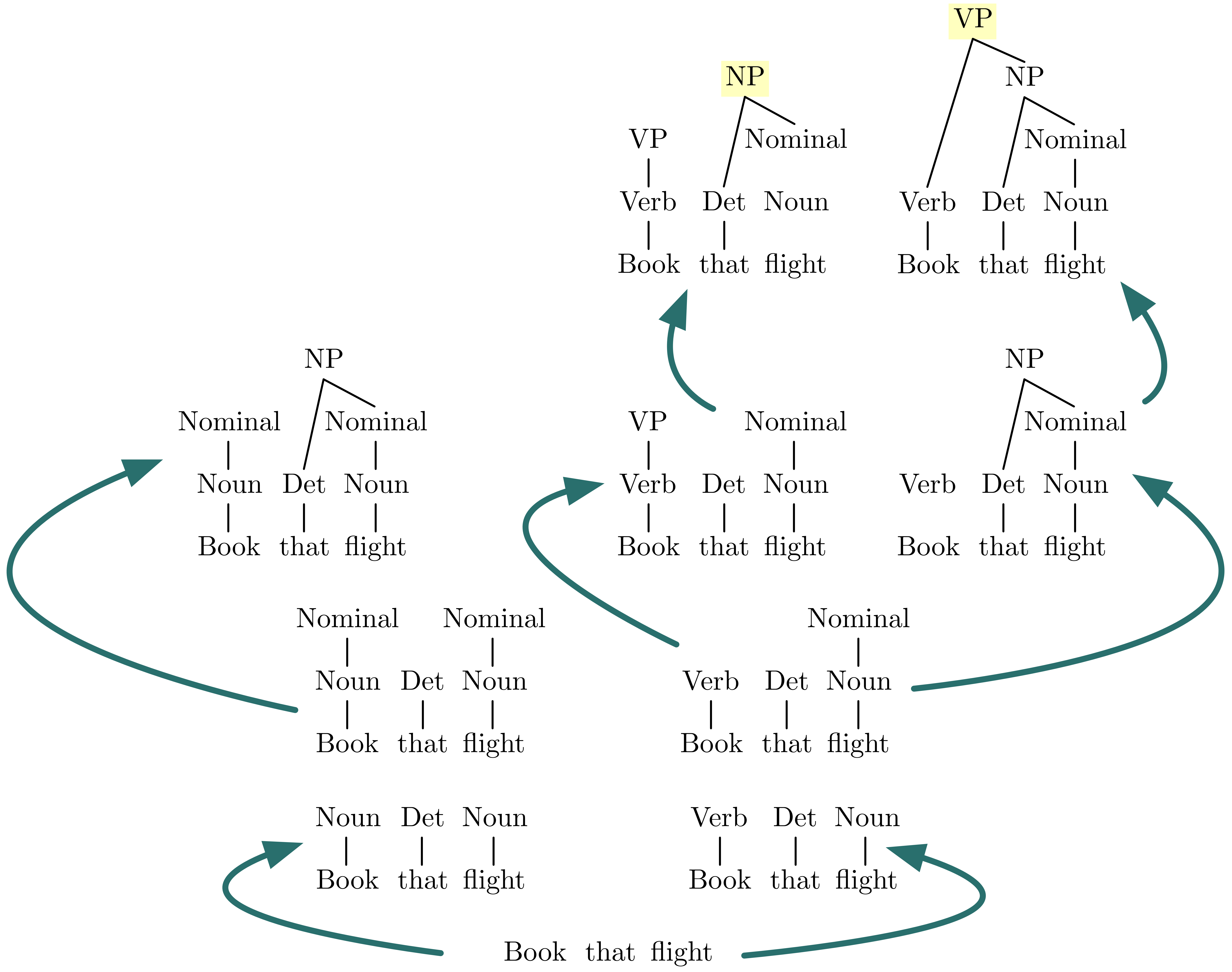



None of these nodes can produce *book* as first terminal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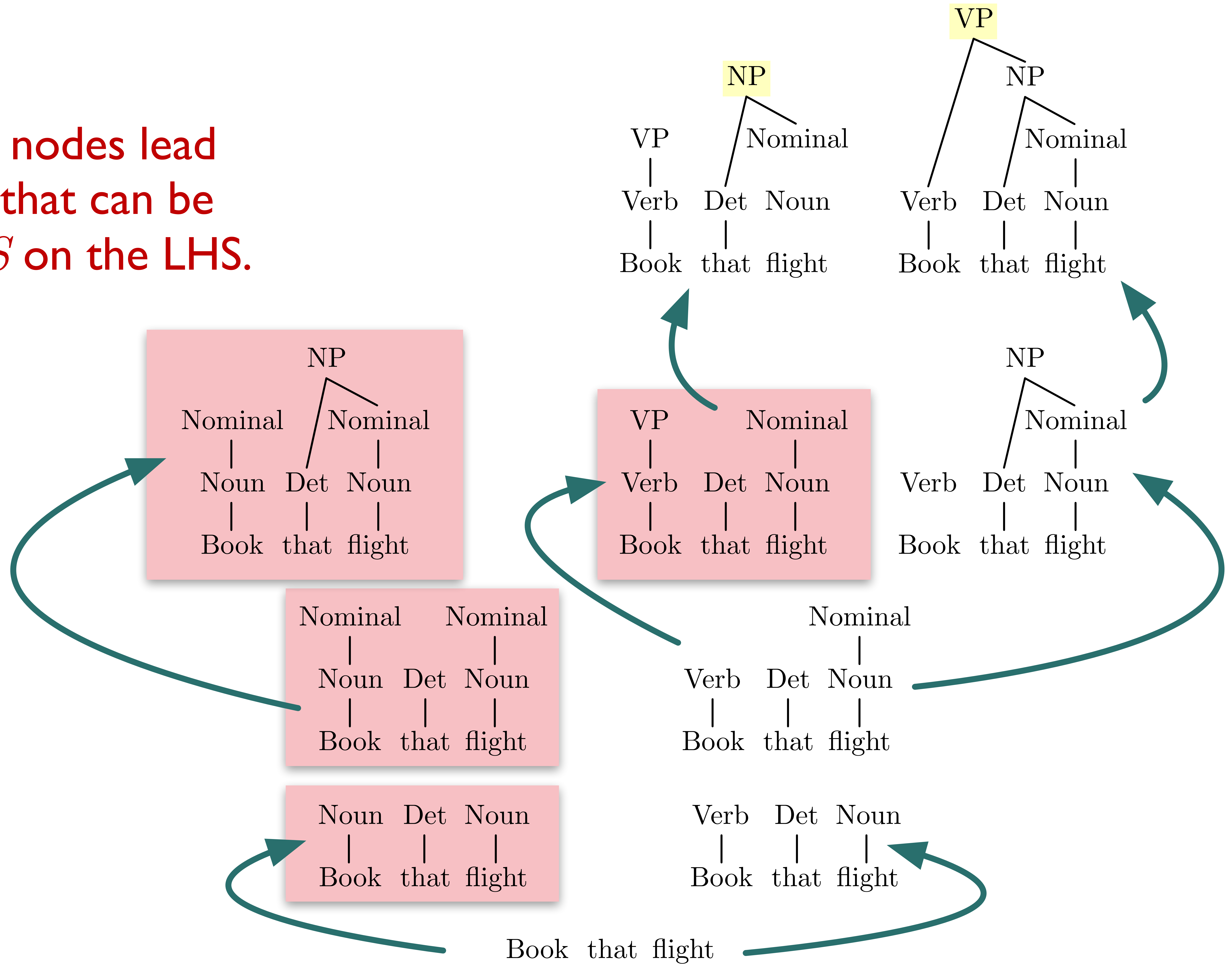
None of these nodes lead lead to a RHS that can be combined with $S$ on the LHS.



# Parsing Challenges

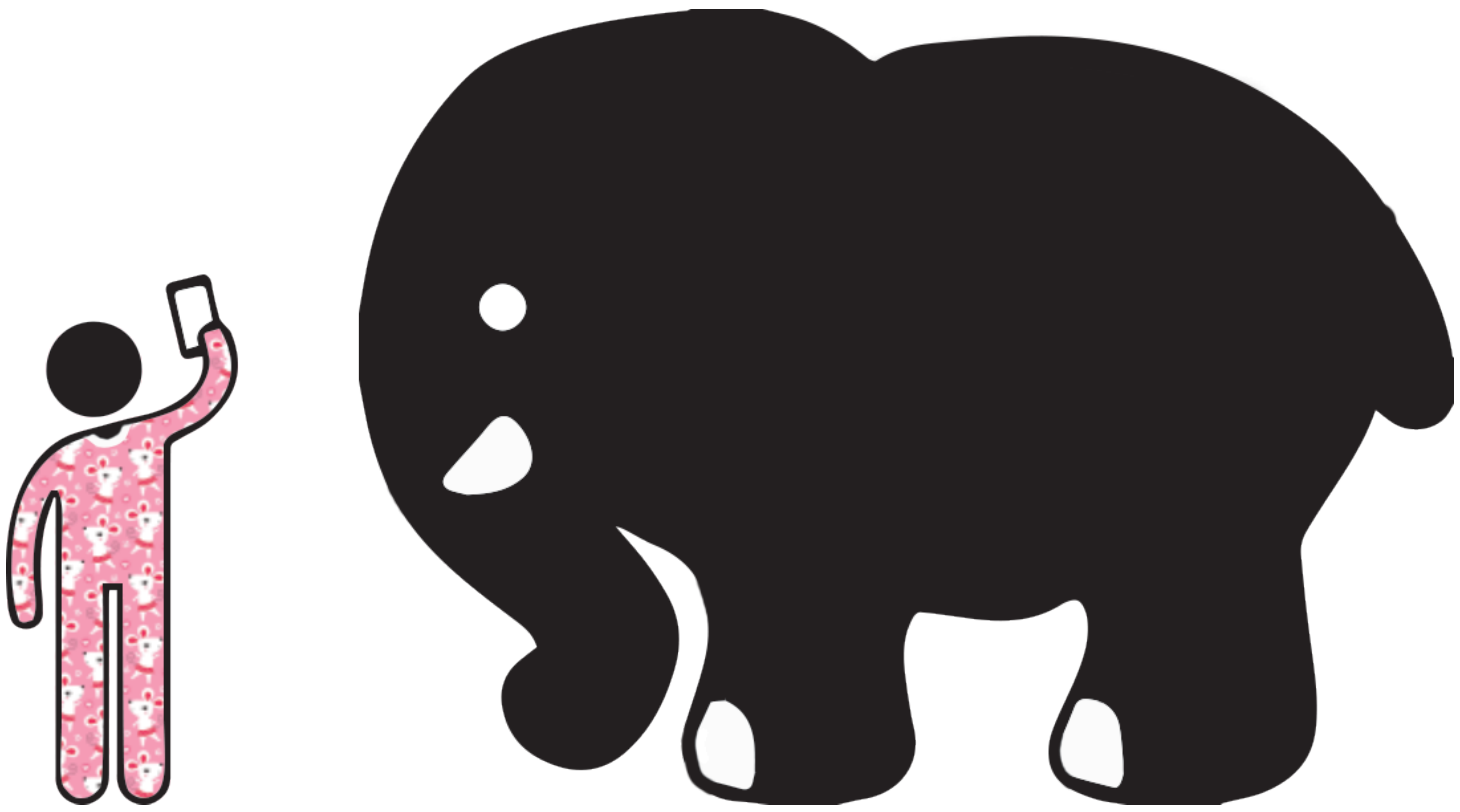
- Parsing-as-Search
- **Parsing Challenges**
  - **Ambiguity**
  - Repeated Substructure
  - Recursion
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

# Parsing Ambiguity

- **Lexical Ambiguity**:
  - Book/NN → *I left a **book** on the table.*
  - Book/VB → ***Book** that flight.*
- Structural Ambiguity

# Attachment Ambiguity

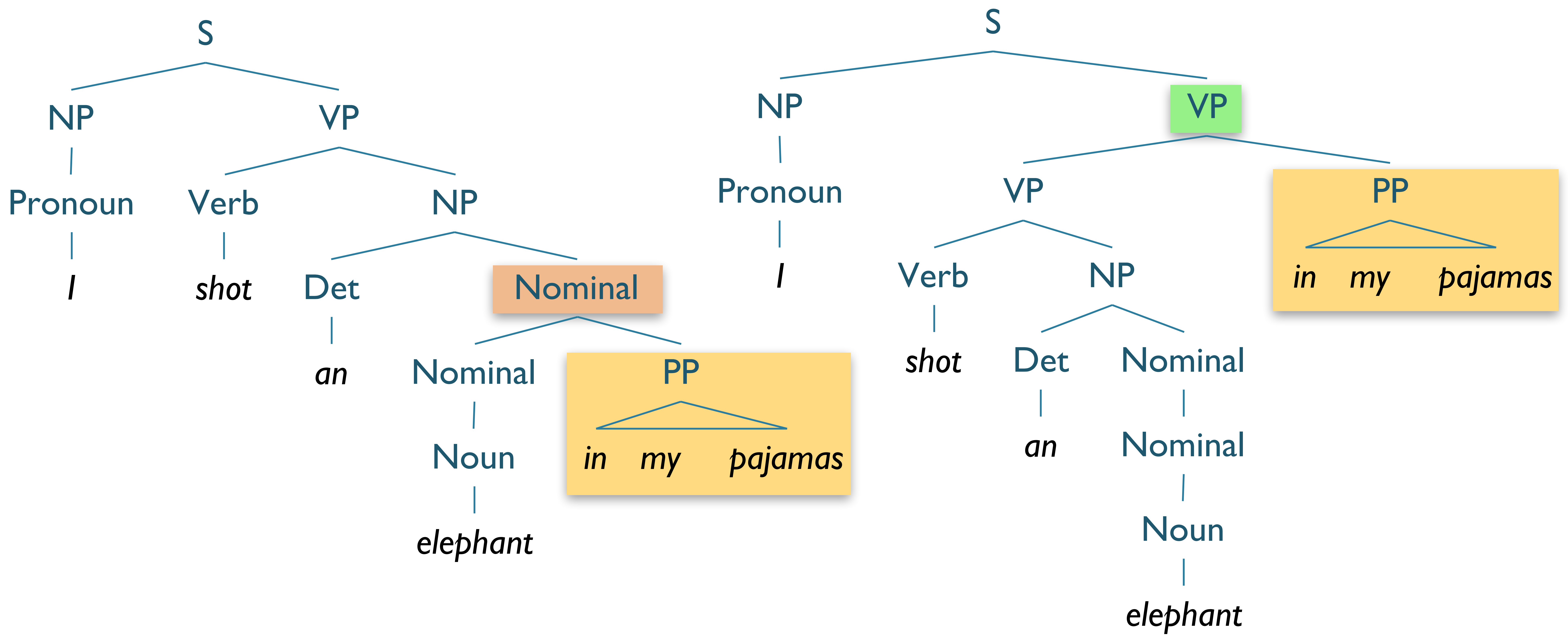
“One morning, I shot an elephant in my pajamas.



# Attachment Ambiguity

"One morning, I shot an elephant in my pajamas.
How he got into my pajamas, I'll never know." — *Groucho Marx*

# Attachment Ambiguity



# “We saw the Eiffel Tower flying to Paris”

# *“We saw the Eiffel Tower flying to Paris”*

# Coordination Ambiguity:

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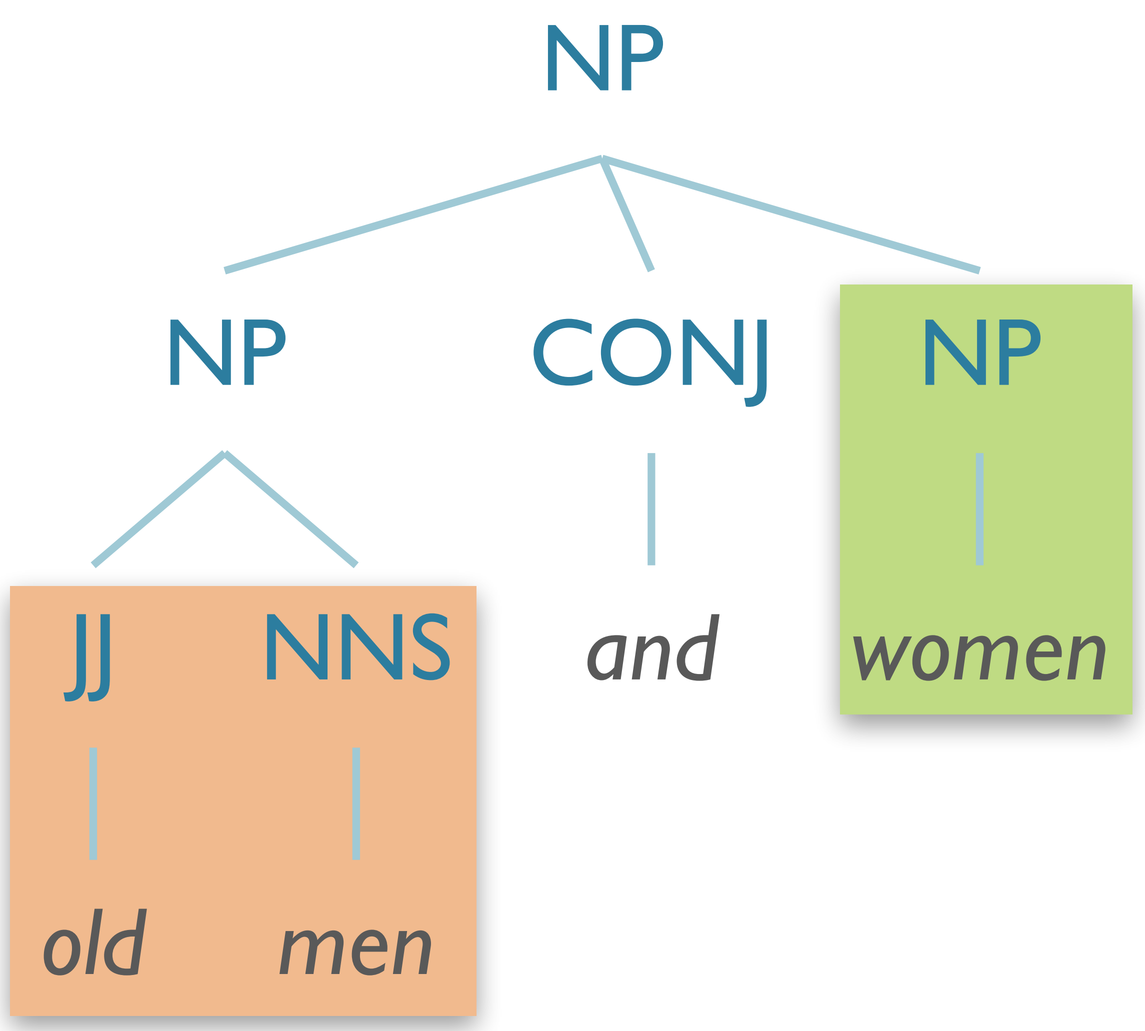
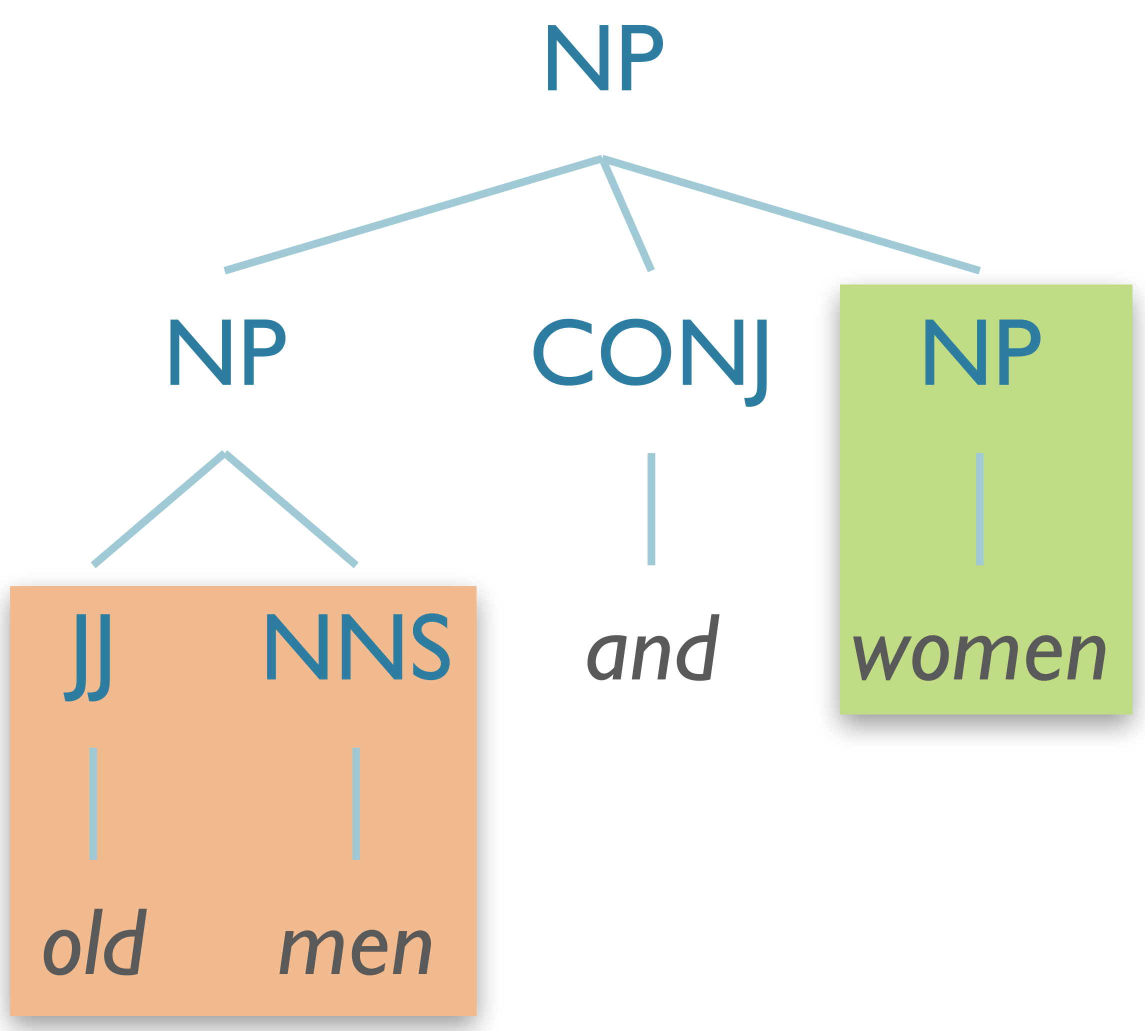
**[old men]** *and* **[women]**

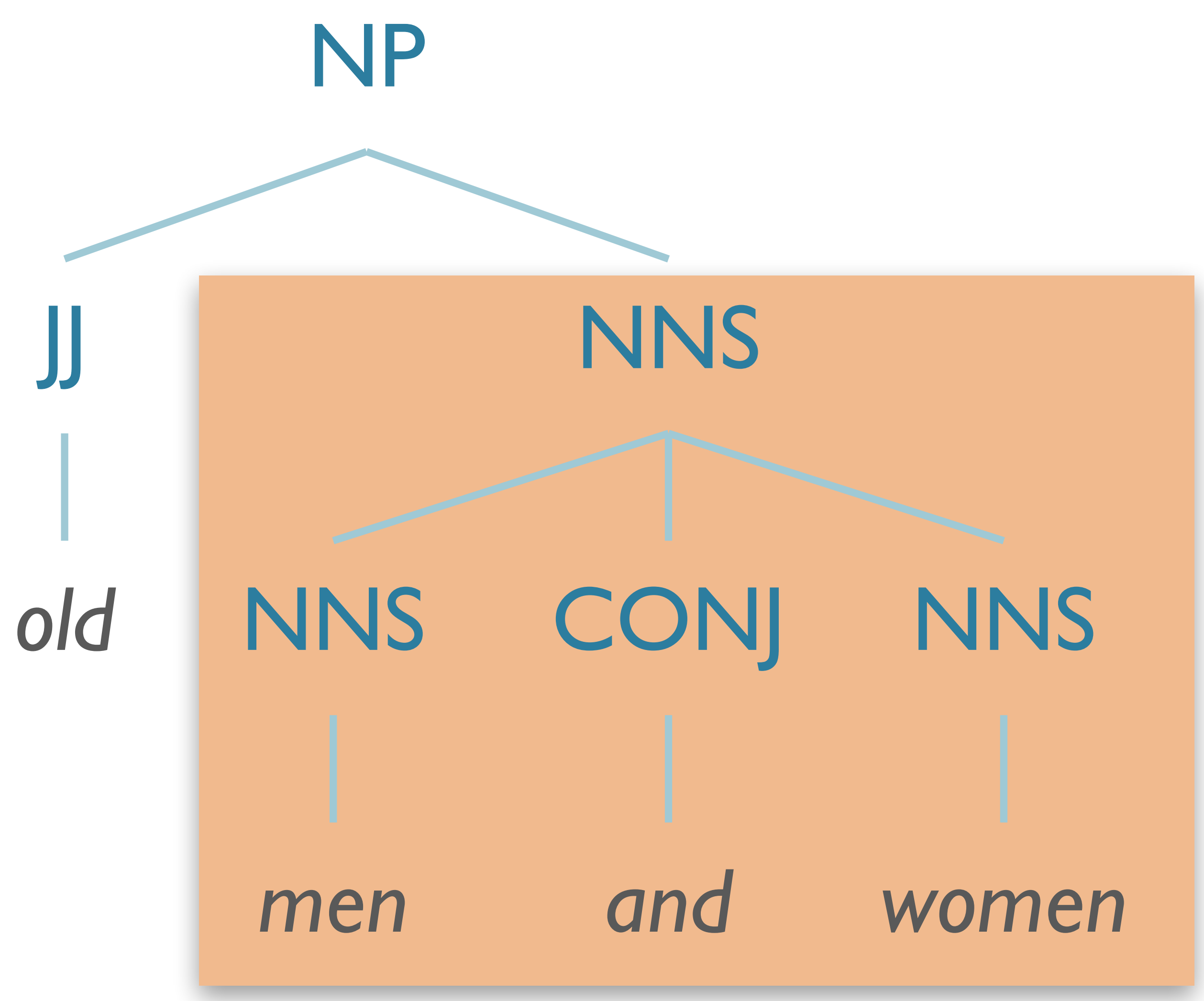


# Coordination Ambiguity:

***[old men]*** *and* ***[women]***

*[old* ***[men and women]****]*



# Local vs. Global Ambiguity

- ***Local*** ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. *Book/NN* in *"Book that flight"*

# Local vs. Global Ambiguity

- *Local* ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. *Book/NN* in *“Book that flight”*
- *Global* ambiguity
  - Multiple valid parses

# Why is Ambiguity a Problem?

- *Local* ambiguity:
  - increased processing time

- *Global* ambiguity:
  - Would like to yield only “reasonable” parses
  - Ideally, the one that was intended*

# Solution to Ambiguity?

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- ***Dis***ambiguation!

# Solution to Ambiguity?

- ***Dis*ambiguation!**
- Different possible strategies to select correct interpretation:

# Disambiguation Strategy:
## Statistical

- Some prepositional structs more likely to attach high/low

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  - *John was thought to have been seen by Mary*
    - Mary could be doing the seeing or thinking — seeing more likely

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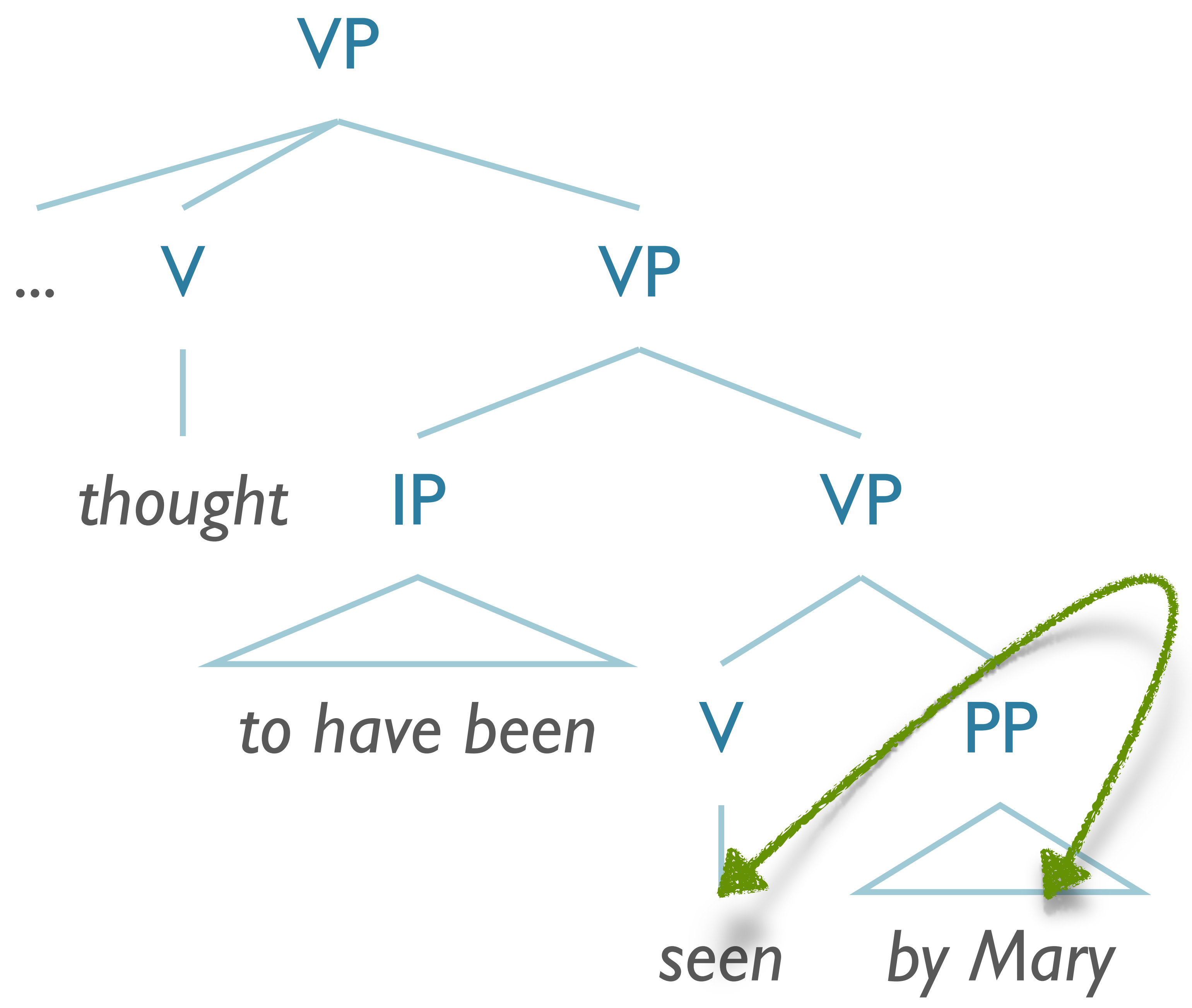

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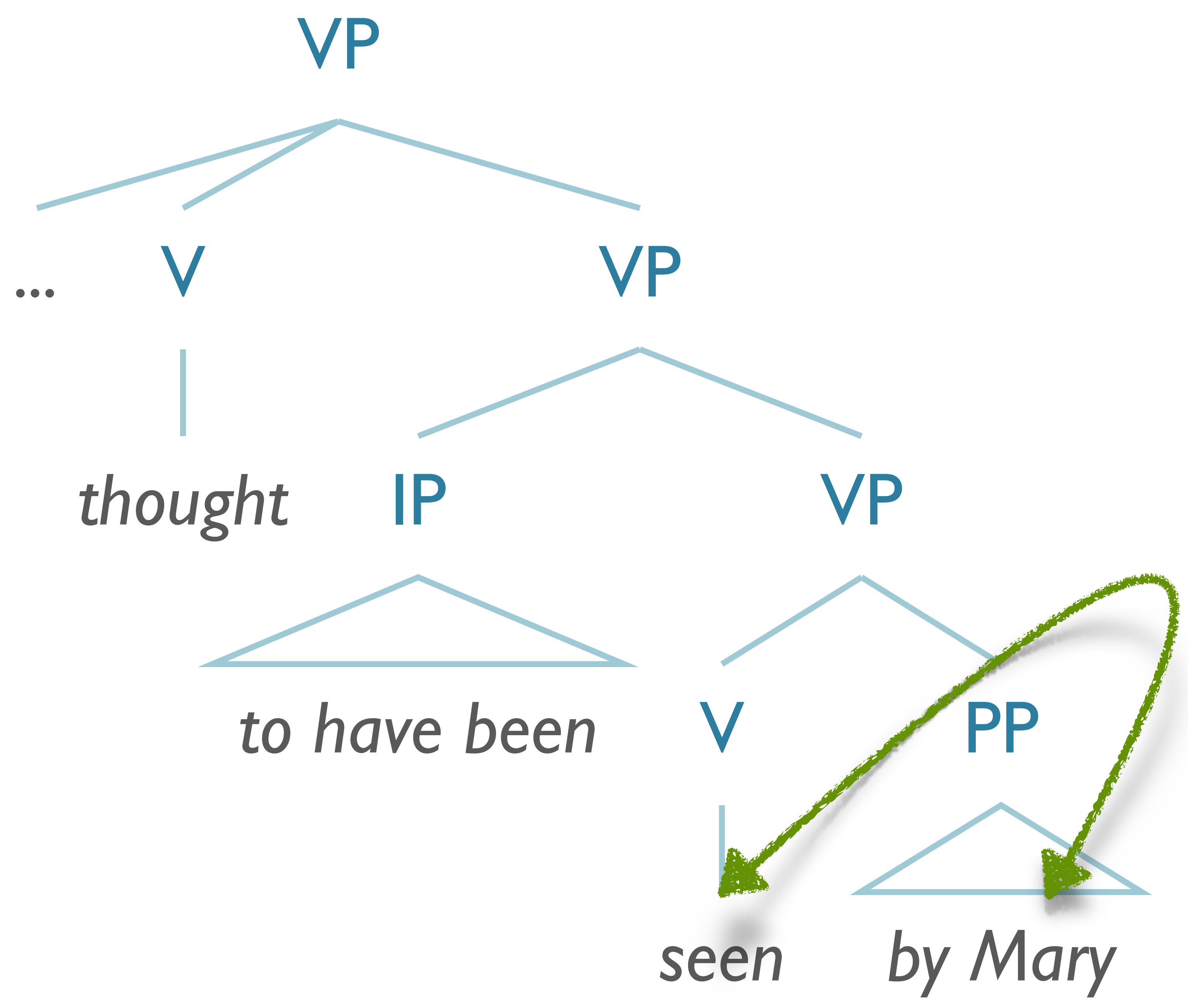
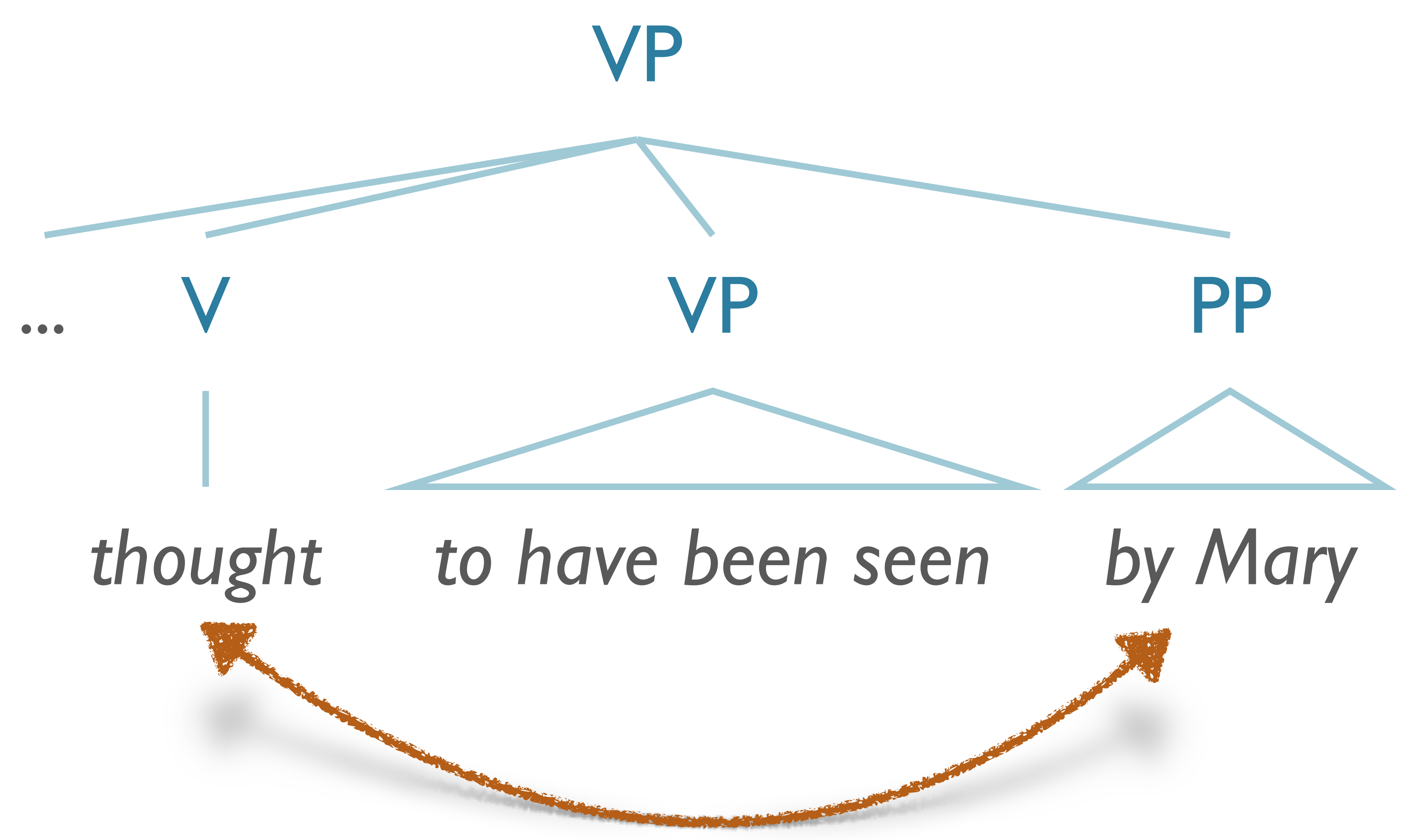


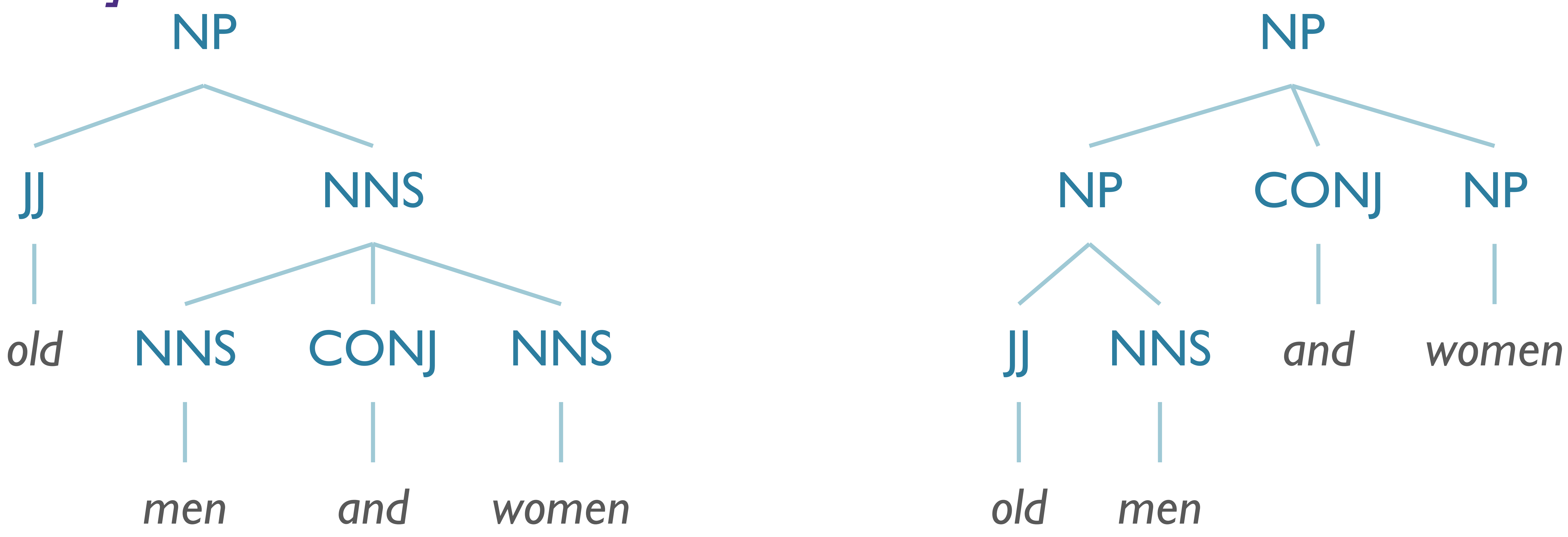



# Disambiguation Strategy: Statistical

- Some phrases more likely overall

# Disambiguation Strategy: Statistical

- Some phrases more likely overall
  - *[old [men and women]] is a more common construction than [old men] and [women]*



# Disambiguation Strategy:
## Semantic

- Some interpretations we know to be semantically impossible

# Disambiguation Strategy: Semantic

- Some interpretations we know to be semantically impossible
  - *Eiffel tower* as subject of *fly*

# Disambiguation Strategy:
## Pragmatic

- Some interpretations are possible, unlikely given world knowledge

# Disambiguation Strategy: **Pragmatic**

- Some interpretations are possible, unlikely given world knowledge
  - e.g. elephants and pajamas

# Incremental Parsing and Garden Paths

- Idea: model *left-to-right* nature of (English) text
- Problem: “garden path” sentences

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# Disambiguation Strategy:

🤷‍♂️

- Alternatively, keep all parses

# Disambiguation Strategy:

🤷‍♂️

- Alternatively, keep all parses
  - *(Might even be the appropriate action for some jokes)*

# Parsing Challenges

- Parsing-as-Search
- **Parsing Challenges**
  - Ambiguity
  - **Repeated Substructure**
  - Recursion
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

# Repeated Work

- Search (top-down/bottom-up) both lead to repeated substructures
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# Repeated Work

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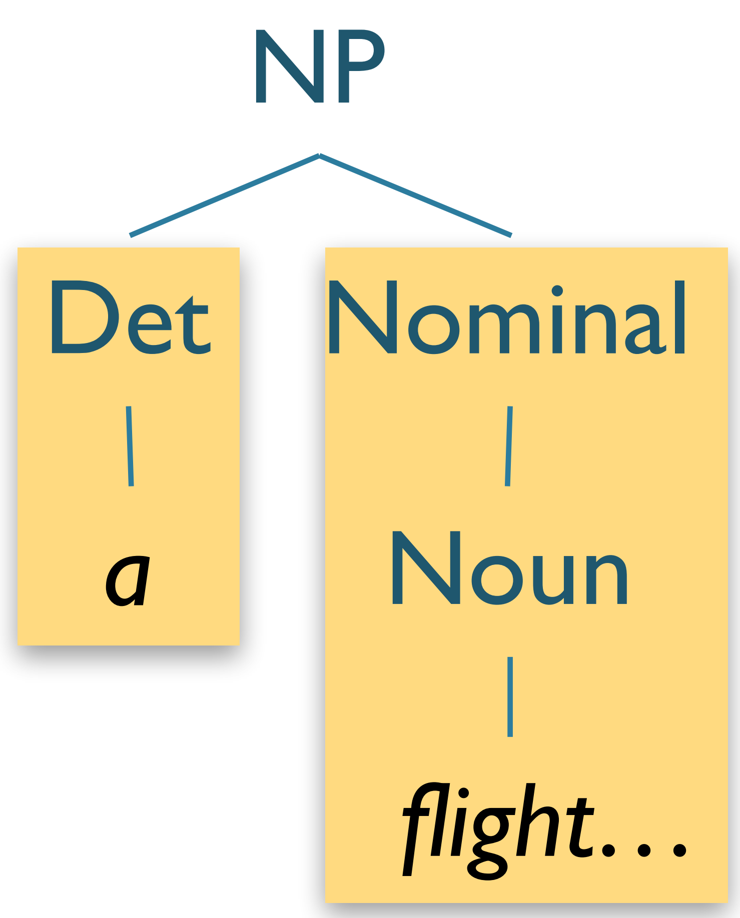
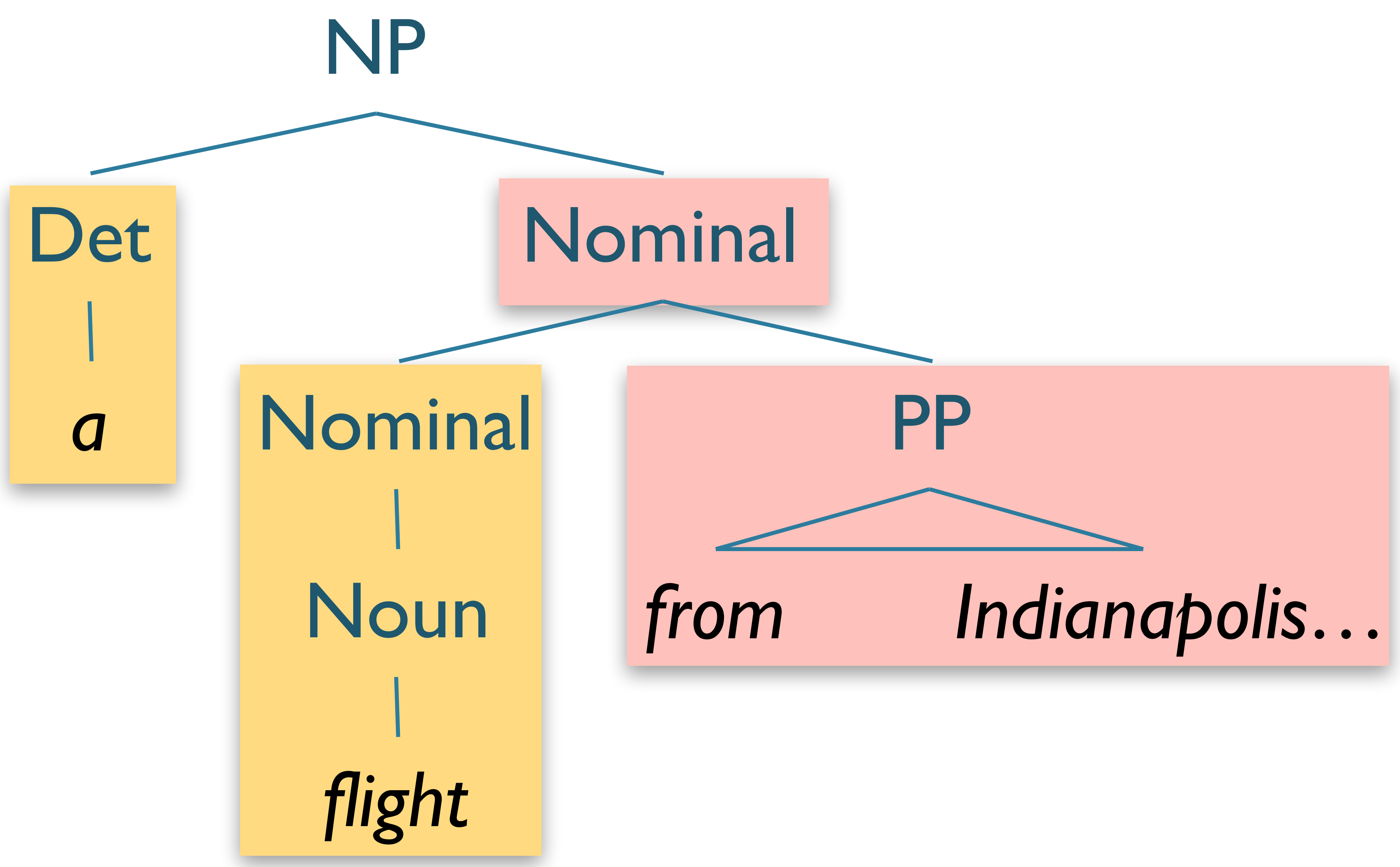
  - No static backtracking can avoid
- Efficient parsing techniques require storage of partial solutions
- Example: *a flight from Indianapolis to Houston on TWA*

# Shared Sub-Problems



# Shared Sub-Problems



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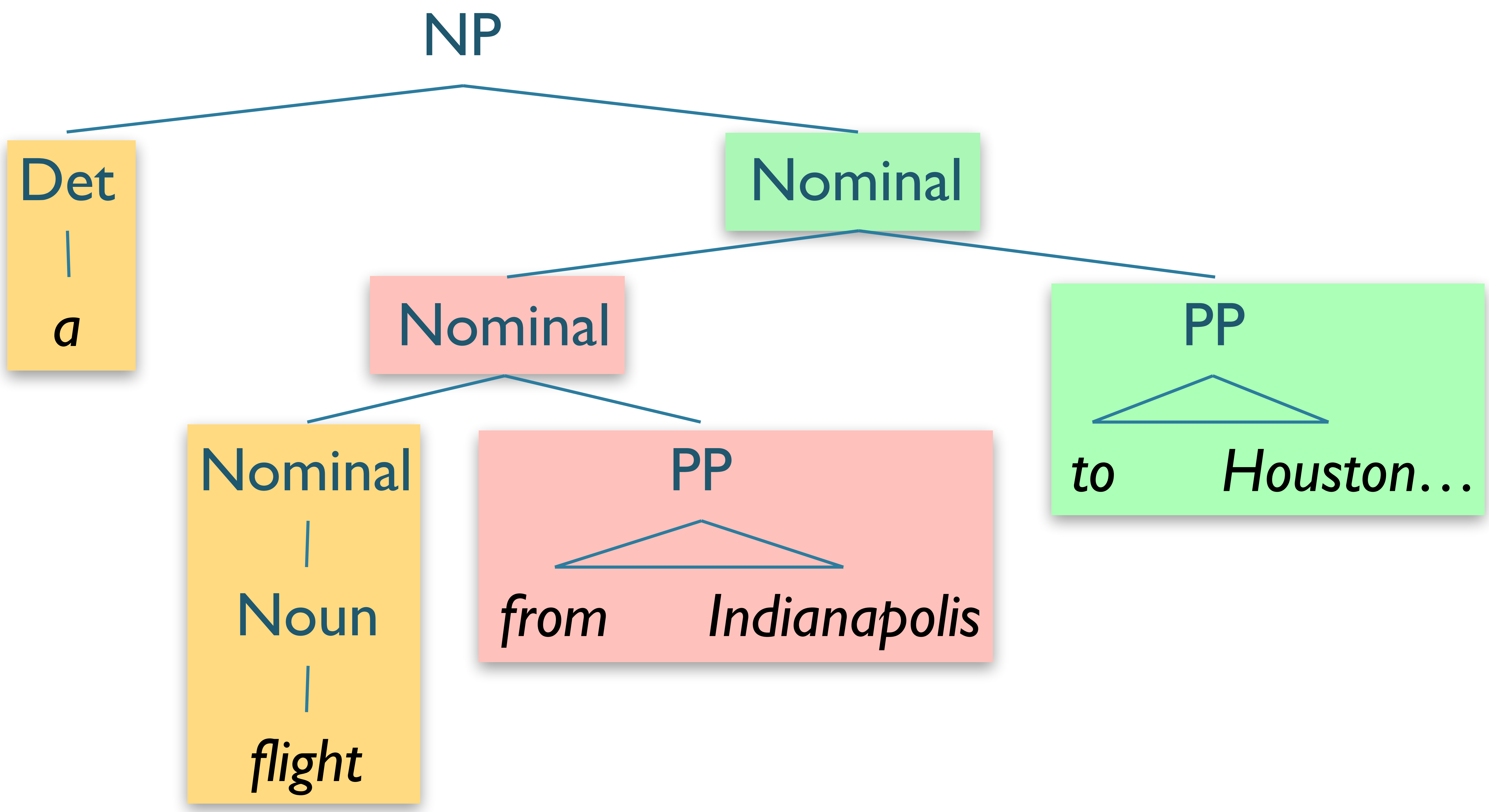

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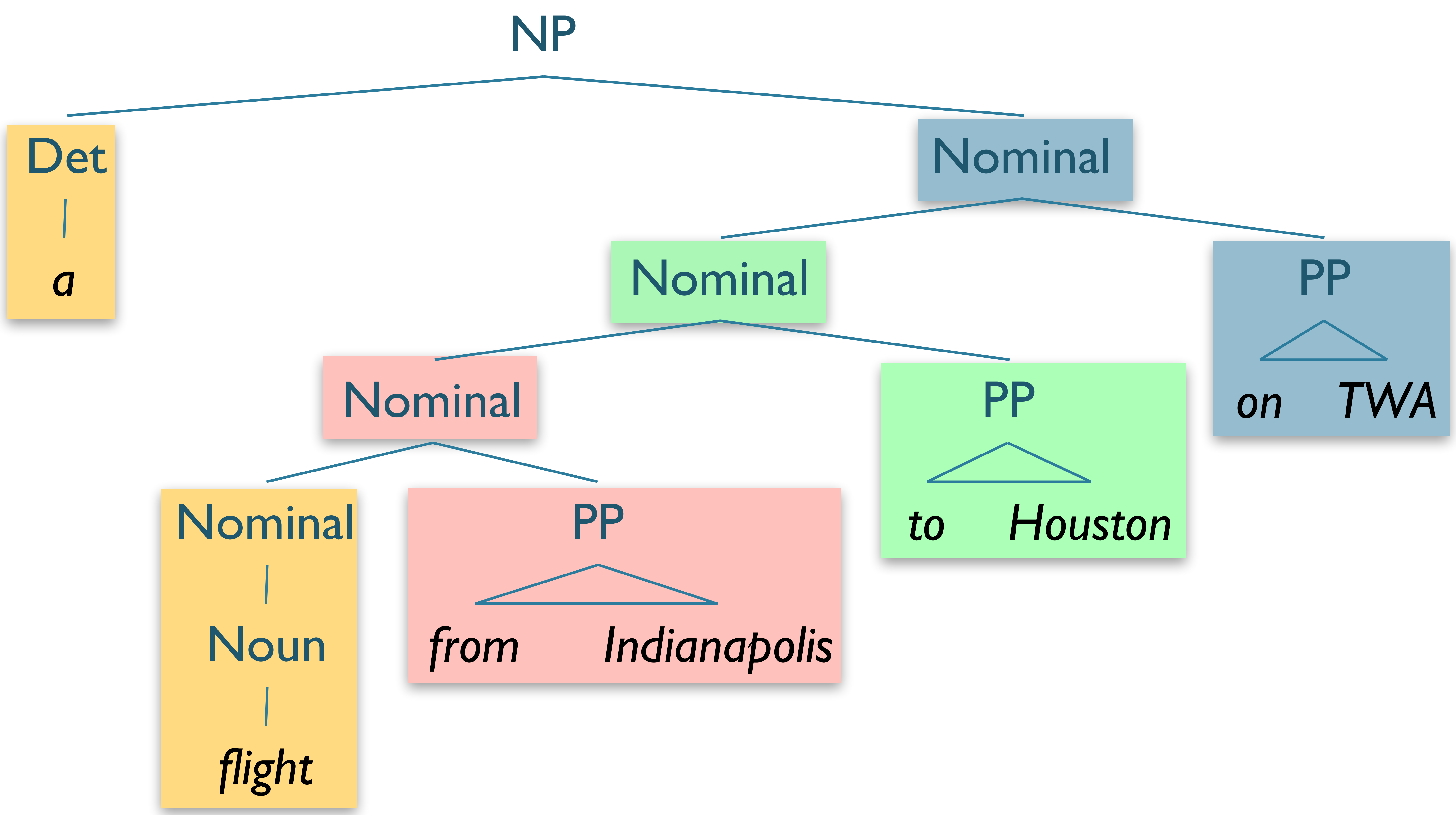

# Parsing Challenges

- Parsing-as-Search
- **Parsing Challenges**
  - Ambiguity
  - Repeated Substructure
  - **Recursion**
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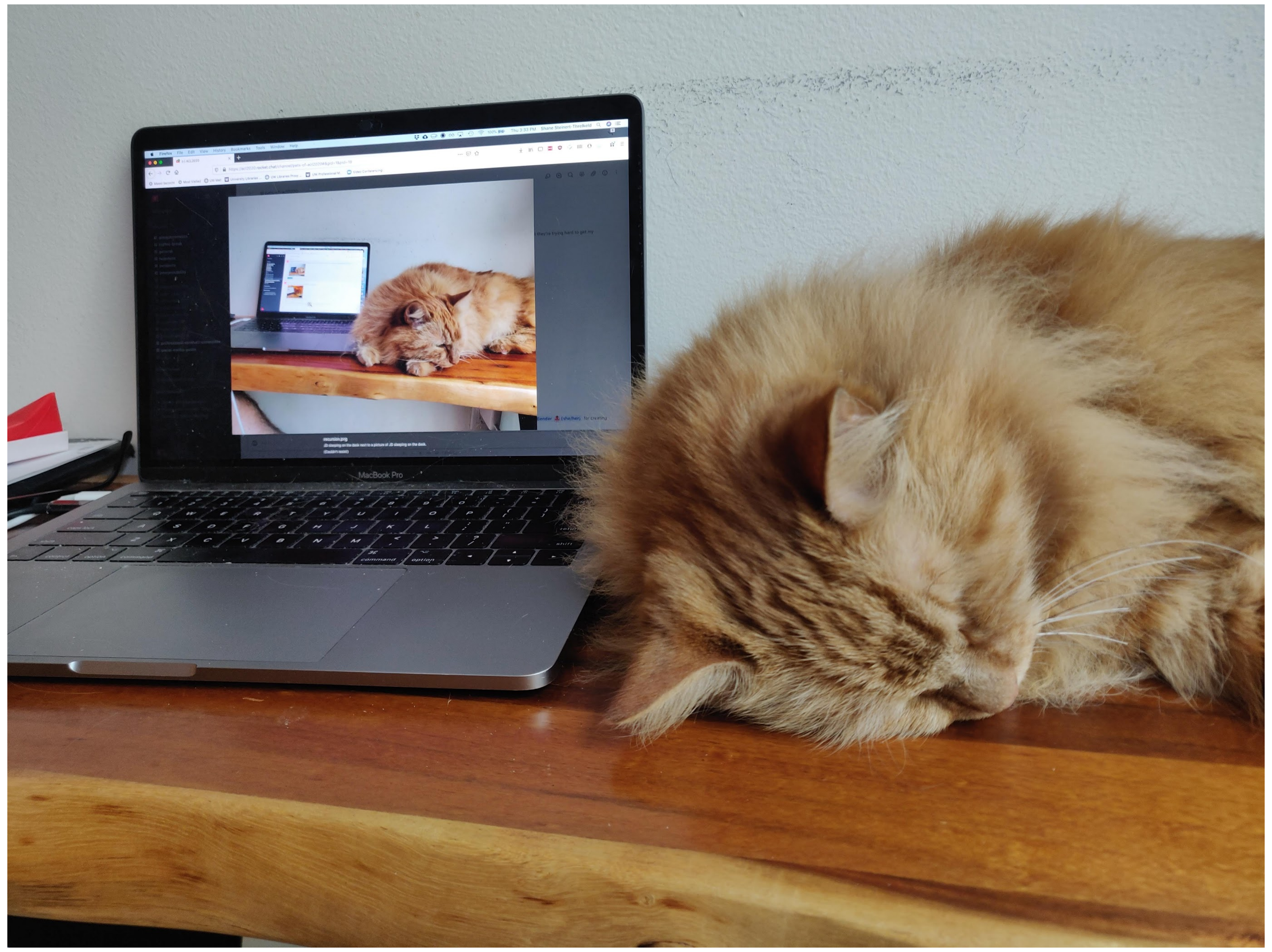
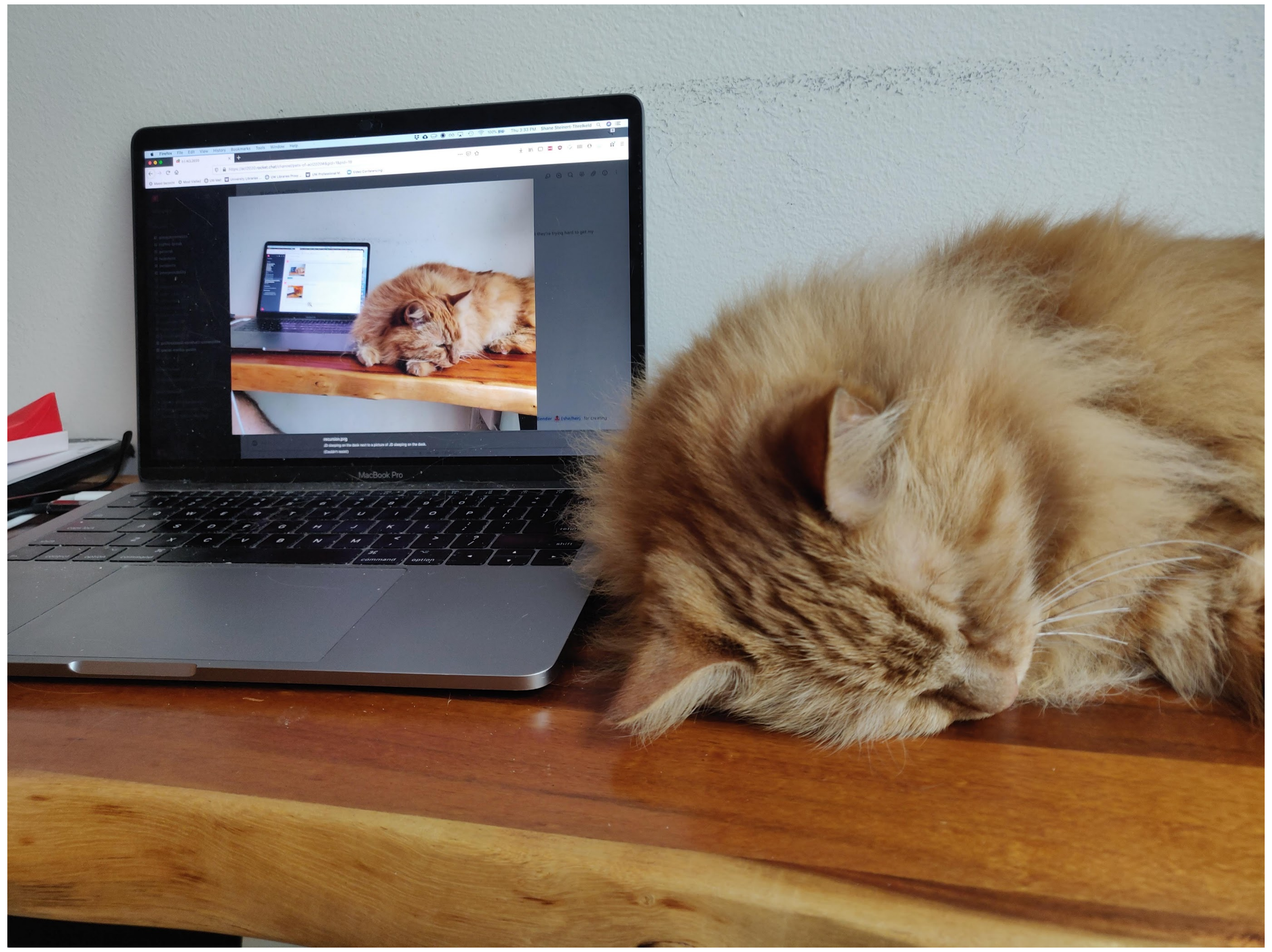
# Recursion

- Many grammars have recursive rules
  - $S \rightarrow S\ Conj\ S$



# Recursion

- Many grammars have recursive rules
  - $S \rightarrow S\ Conj\ S$
- In search approaches, recursion is problematic
  - Can yield infinite searches
  - Top-down especially vulnerable



# Roadmap

- Parsing-as-Search
- Parsing Challenges
- **Strategy: Dynamic Programming**
- Grammar Equivalence
- CKY parsing algorithm

# Dynamic Programming

- Challenge:
  - Repeated substructure → Repeated Work

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# Dynamic Programming

- Challenge:
  - Repeated substructure → Repeated Work
- Insight:
  - Global parse composed of sub-parses
  - Can record these sub-parses and re-use
- Dynamic programming avoids repeated work by recording the subproblems
  - Here, stores subtrees

# Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic ($n^3$) or less

# Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic ($n^3$) or less
- Several different implementations
  - Cocke-Kasami-Younger (CKY) algorithm
  - Earley algorithm
  - Chart parsing

# Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- **Grammar Equivalence**
- CKY parsing algorithm

# Grammar Equivalence and Form

- *Weak* Equivalence
  - **Accepts** same language
  - May produce **different** structures

- *Strong* Equivalence
  - Accepts same language
  - Produces **same** structures

# Grammar Equivalence and Form

# Grammar Equivalence and Form

- Reason?
  - We can create a weakly-equivalent grammar that allows for greater efficiency
  - This is required by the CKY algorithm

# Chomsky Normal Form (CNF)

- Required by CKY Algorithm

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- All productions are of the form:
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  - $A \rightarrow \mathbf{a}$

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# Chomsky Normal Form (CNF)

- Required by CKY Algorithm
- All productions are of the form:
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- Most of our grammars are not of this form:
  - $S \rightarrow \textit{Wh-NP}\ \textit{Aux}\ \textit{NP}\ \textit{VP}$
- Need a general conversion procedure

# Chomsky Normal Form (CNF)

# Chomsky Normal Form (CNF)

- Weak equivalence: for every CFG $G$, there is a weakly equivalent CNF grammar $G'$.
  - i.e.: there is a grammar in CNF s.t. $L(G) = L(G')$.

# CNF Conversion

Hybrid productions:

$INF\text{-}VP \rightarrow \textbf{to}\ VP$

Unit productions:

$A \rightarrow B$

Long productions:

$A \rightarrow B\ C\ D\ \ldots$

# CNF Conversion: Hybrid Productions

- Hybrid production:
  - Replace all terminals with dummy non-terminal
  - $\textit{INF-VP} \rightarrow \textbf{to}\ \textit{VP}$
    - $\textit{INF-VP} \rightarrow \textit{TO}\ \textit{VP}$
    - $\textit{TO} \rightarrow \textbf{to}$

# CNF Conversion: Unit Productions

- Unit productions:
  - Rewrite RHS with RHS of all derivable, non-unit productions
  - If $A \overset{*}{\Rightarrow} B$ and $B \rightarrow \gamma$, **add** $A \rightarrow \gamma$ [where $\gamma$ is any non-unit RHS]
  - [*$A \overset{*}{\Rightarrow} B$: B is reachable from A by a sequence of unit productions]*
- *Nominal* → *Noun*, *Noun* → **dog**
  - *Nominal* → **dog**
  - *Noun* → **dog**
- NB: this example has $\gamma$ as a single terminal, but the rule applies to all non-unit RHS.

# CNF Conversion: Long Productions

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- Long productions

$$S \rightarrow Aux\ NP\ VP$$

# CNF Conversion: Long Productions

- Long productions

| | |
|---|---|
| $S \rightarrow Aux\ NP\ VP$ | |
| $S \rightarrow \boldsymbol{X1}\ VP$ | $\boldsymbol{X1} \rightarrow Aux\ NP$ |

# CNF Conversion: Long Productions

- Long productions

| $S \rightarrow Aux\ NP\ VP$ | |
|---|---|
| $S \rightarrow \boldsymbol{X1}\ VP$ | $\boldsymbol{X1} \rightarrow Aux\ NP$ |

- Introduce unique nonterminals, and spread over rules

# CNF Conversion

Convert terminals in hybrid rules to dummy non-terminals

Convert unit productions

Binarize long production rules

| $\mathcal{L}_1$ Grammar | $\mathcal{L}_1$ in CNF |
|---|---|
| $S \rightarrow NP\ VP$ | $S \rightarrow NP\ VP$ |
| $S \rightarrow Aux\ NP\ VP$ | $S \rightarrow X1\ VP$ |
| | $X1 \rightarrow Aux\ NP$ |
| $S \rightarrow VP$ | $S \rightarrow book \mid include \mid prefer$ |
| | $S \rightarrow Verb\ NP$ |
| | $S \rightarrow X2\ PP$ |
| | $S \rightarrow Verb\ PP$ |
| | $S \rightarrow VP\ PP$ |
| $NP \rightarrow Pronoun$ | $NP \rightarrow I \mid she \mid me$ |
| $NP \rightarrow Proper\text{-}Noun$ | $NP \rightarrow TWA \mid Houston$ |
| $NP \rightarrow Det\ Nominal$ | $NP \rightarrow Det\ Nominal$ |
| $Nominal \rightarrow Noun$ | $Nominal \rightarrow book \mid flight \mid meal \mid money$ |
| $Nominal \rightarrow Nominal\ Noun$ | $Nominal \rightarrow Nominal\ Noun$ |
| $Nominal \rightarrow Nominal\ PP$ | $Nominal \rightarrow Nominal\ PP$ |
| $VP \rightarrow Verb$ | $VP \rightarrow book \mid include \mid prefer$ |
| $VP \rightarrow Verb\ NP$ | $VP \rightarrow Verb\ NP$ |
| $VP \rightarrow Verb\ NP\ PP$ | $VP \rightarrow X2\ PP$ |
| | $X2 \rightarrow Verb\ NP$ |
| $VP \rightarrow Verb\ PP$ | $VP \rightarrow Verb\ PP$ |
| $VP \rightarrow VP\ PP$ | $VP \rightarrow VP\ PP$ |
| $PP \rightarrow Preposition\ NP$ | $PP \rightarrow Preposition\ NP$ |

| $\mathscr{L}_1$ **Grammar** | $\mathscr{L}_1$ **in CNF** |
|---|---|
| $S \rightarrow NP\ VP$ | $S \rightarrow NP\ VP$ |
| $S \rightarrow Aux\ NP\ VP$ | $S \rightarrow X1\ VP$ |
| | $X1 \rightarrow Aux\ NP$ |
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| | $S \rightarrow X2\ PP$ |
| | $S \rightarrow Verb\ PP$ |
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| $VP \rightarrow Verb\ NP\ PP$ | $VP \rightarrow X2\ PP$ |
| | $X2 \rightarrow Verb\ NP$ |
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| $\mathcal{L}_1$ Grammar | $\mathcal{L}_1$ in CNF |
|---|---|
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| $S \rightarrow Aux\ NP\ VP$ | $S \rightarrow X1\ VP$ |
| | $X1 \rightarrow Aux\ NP$ |
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| $Nominal \rightarrow Nominal\ Noun$ | $Nominal \rightarrow Nominal\ Noun$ |
| $Nominal \rightarrow Nominal\ PP$ | $Nominal \rightarrow Nominal\ PP$ |
| $VP \rightarrow Verb$ | $VP \rightarrow book \mid include \mid prefer$ |
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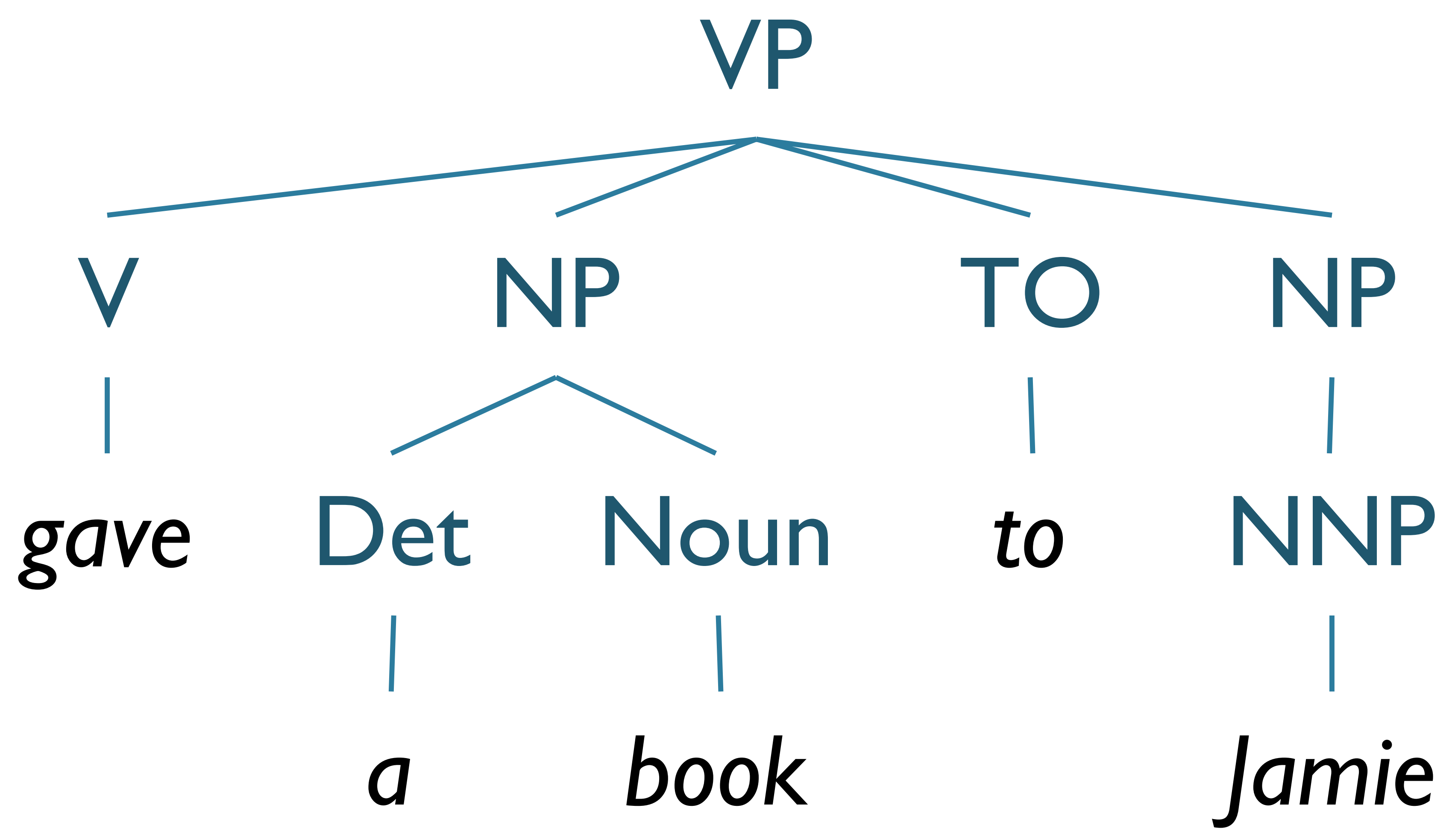
| $VP \rightarrow Verb\ PP$ | $VP \rightarrow Verb\ PP$ |
| $VP \rightarrow VP\ PP$ | $VP \rightarrow VP\ PP$ |
| $PP \rightarrow Preposition\ NP$ | $PP \rightarrow Preposition\ NP$ |

# Variation in CNF: Binarization

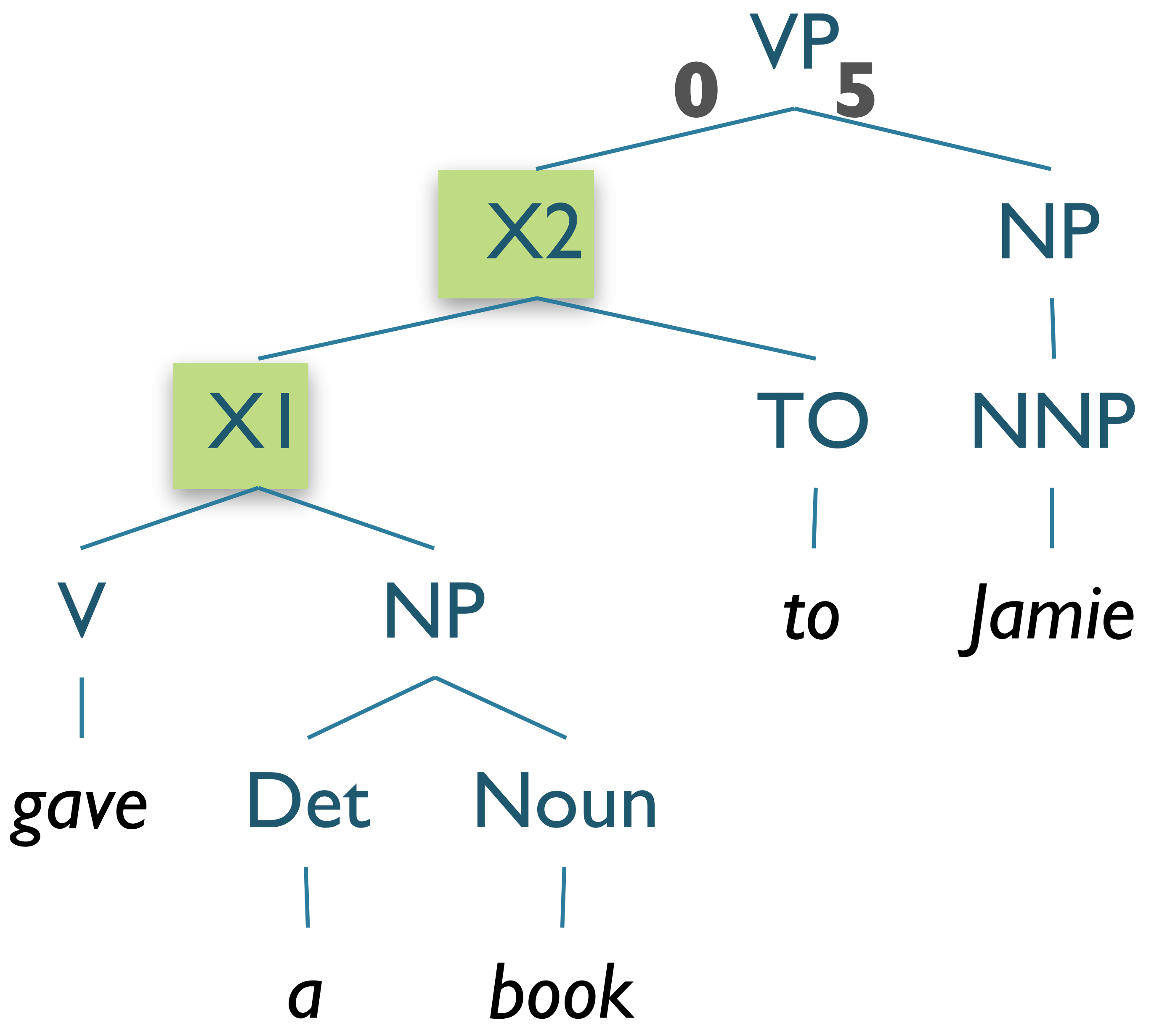
**Original Rule**

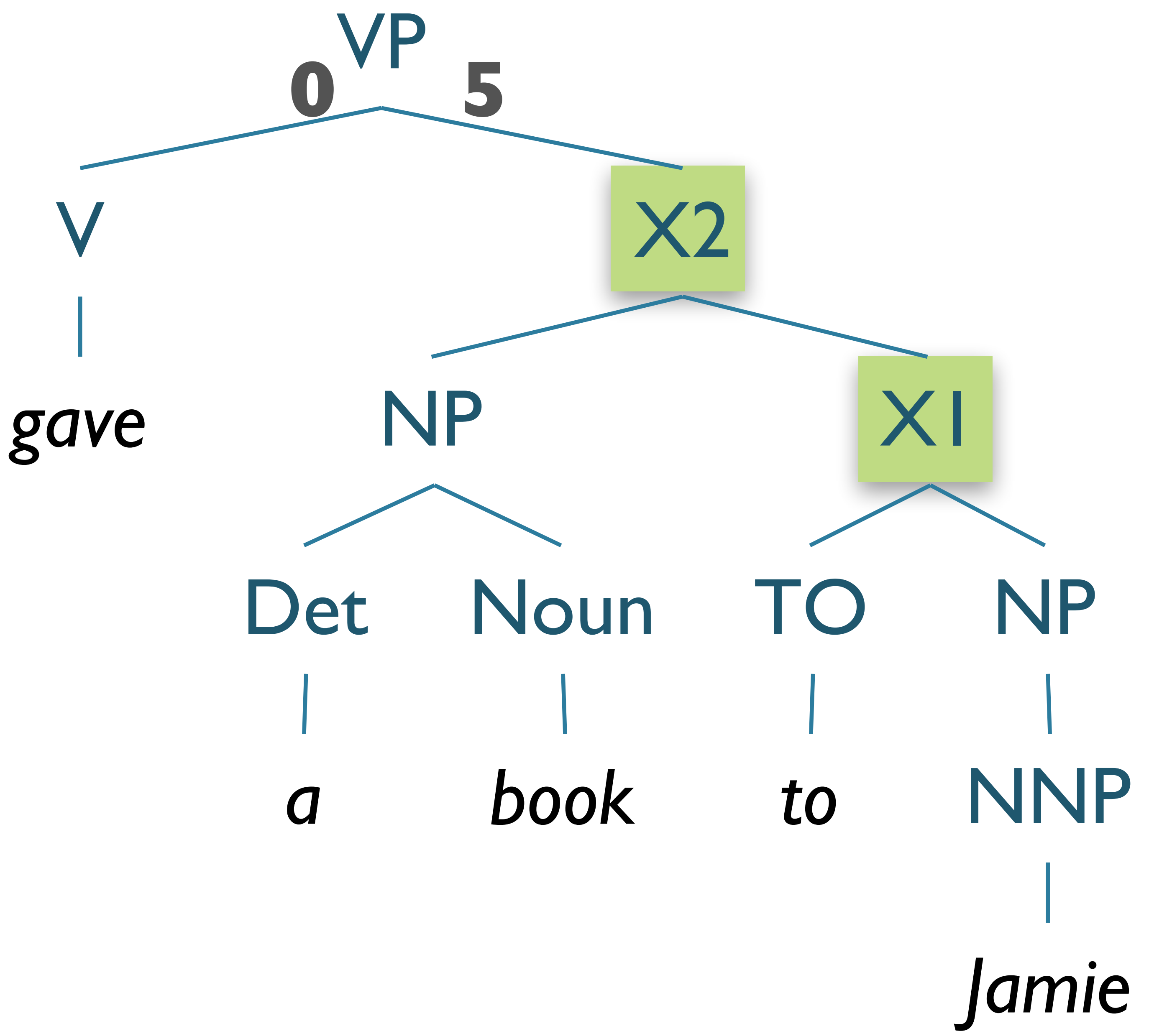
$VP \rightarrow V\ NP\ TO\ NP$



# Variation in CNF: Binarization

| Original Rule | | | |
|---|---|---|---|
| $VP \rightarrow V\ NP\ TO\ NP$ | | | |
| **Left to Right Reduction** | | **Right to Left Reduction** | |
| $VP \rightarrow X1\ TO\ NP$ | $X1 \rightarrow V\ NP$ | $VP \rightarrow V\ NP\ X1$ | $X1 \rightarrow TO\ NP$ |
| $VP \rightarrow X2\ NP$ | $X2 \rightarrow X1\ TO$ | $VP \rightarrow V\ X2$ | $X2 \rightarrow NP\ X1$ |



# Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- **CKY parsing algorithm**

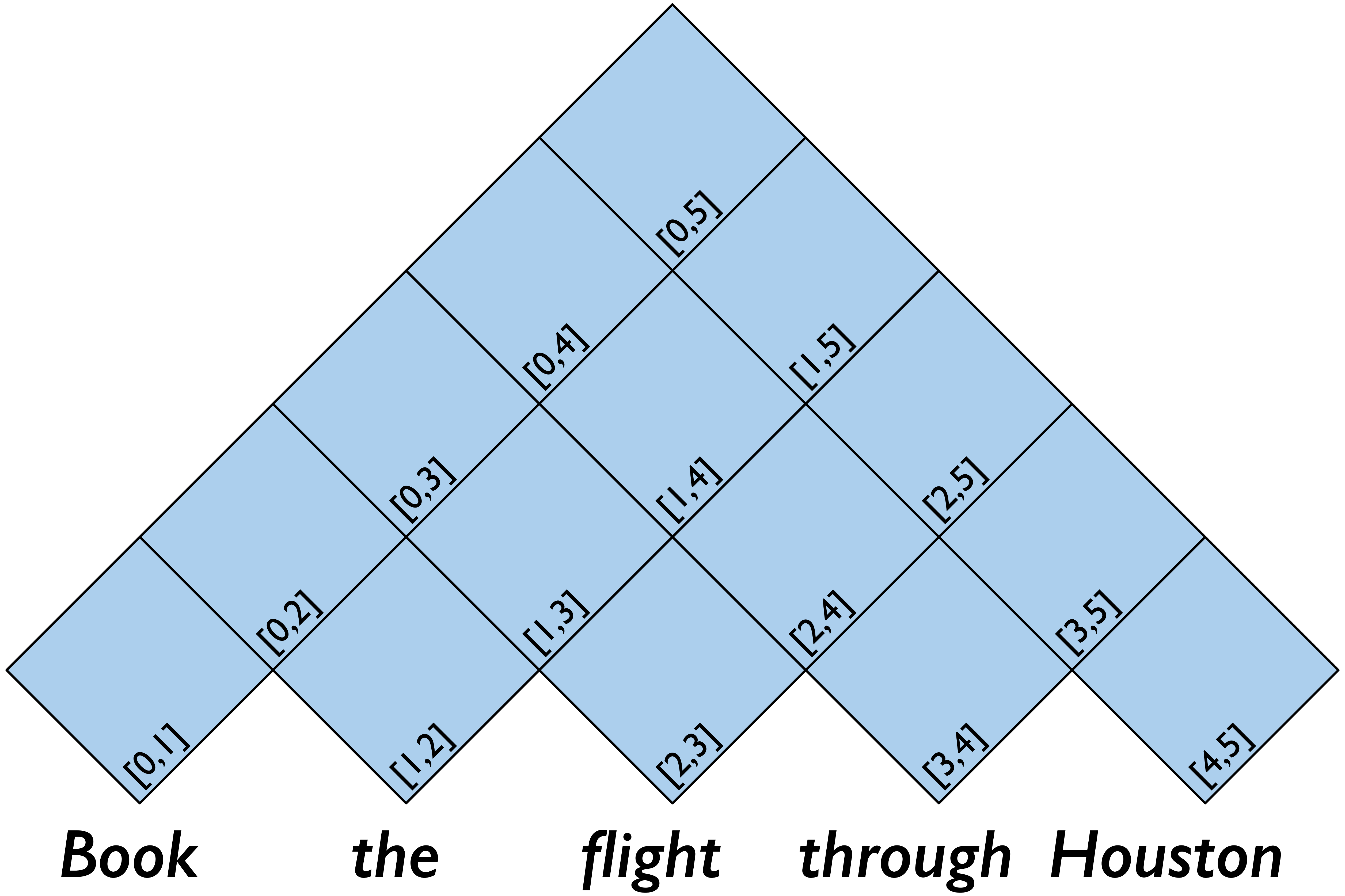
# CKY Parsing

- (Relatively) efficient parsing algorithm
- Based on tabulating substring parses to avoid repeat work
- Approach:
  - Use CNF Grammar
  - Build an $(n + 1) \times (n + 1)$ matrix to store subtrees
    - Upper triangular portion
  - Incrementally build parse spanning whole input string

# CKY Matrix

| *Book* | *the* | *flight* | *through* | *Houston* |
|---|---|---|---|---|
| [0,1] | [0,2] | [0,3] | [0,4] | [0,5] |
| | [1,2] | [1,3] | [1,4] | [1,5] |
| | | [2,3] | [2,4] | [2,5] |
| | | | [3,4] | [3,5] |
| | | | | [4,5] |

# CKY Matrix



[0,5]
[0,4]
[1,5]
[0,3]
[1,4]
[2,5]
[0,2]
[1,3]
[2,4]
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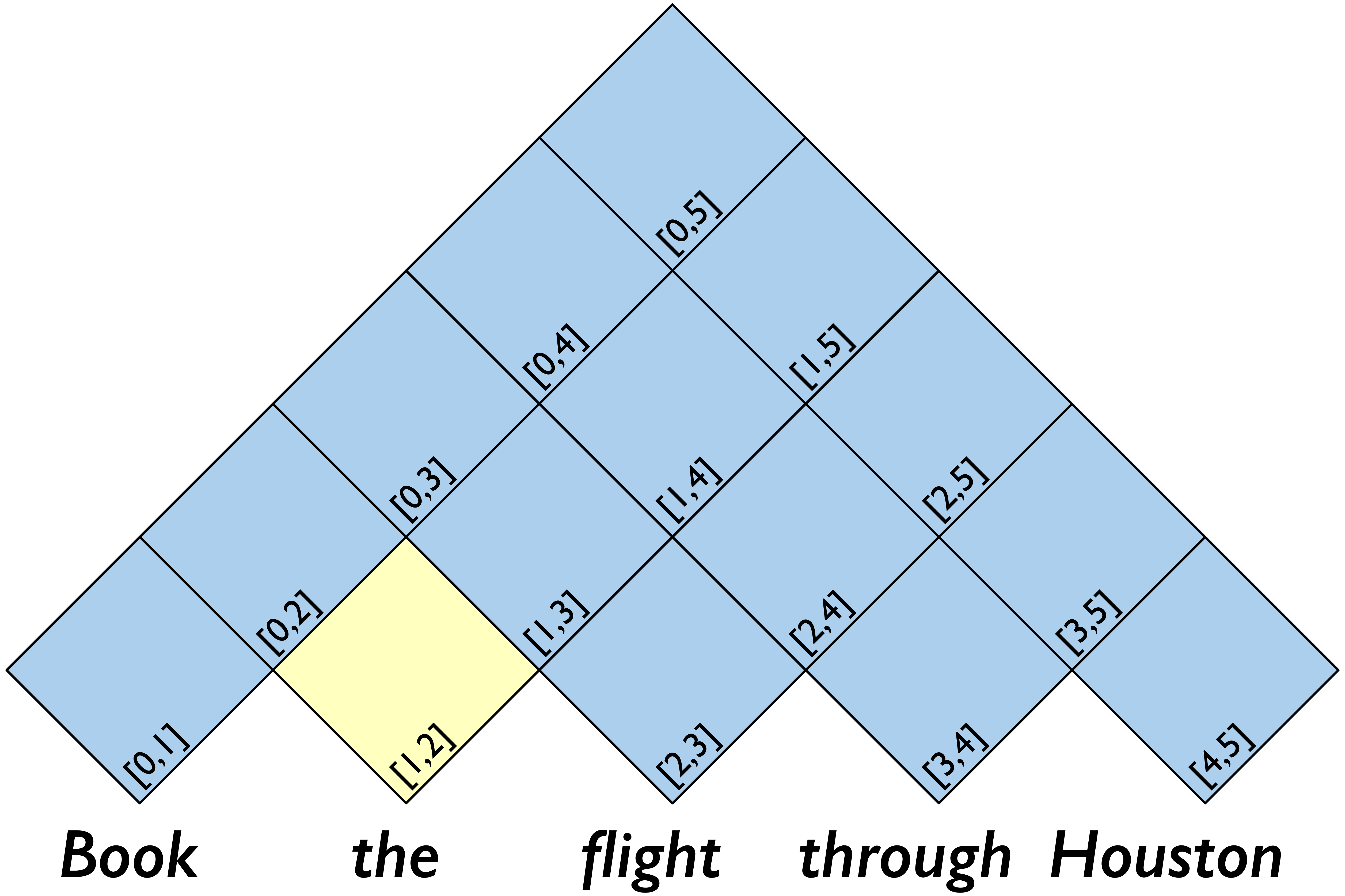
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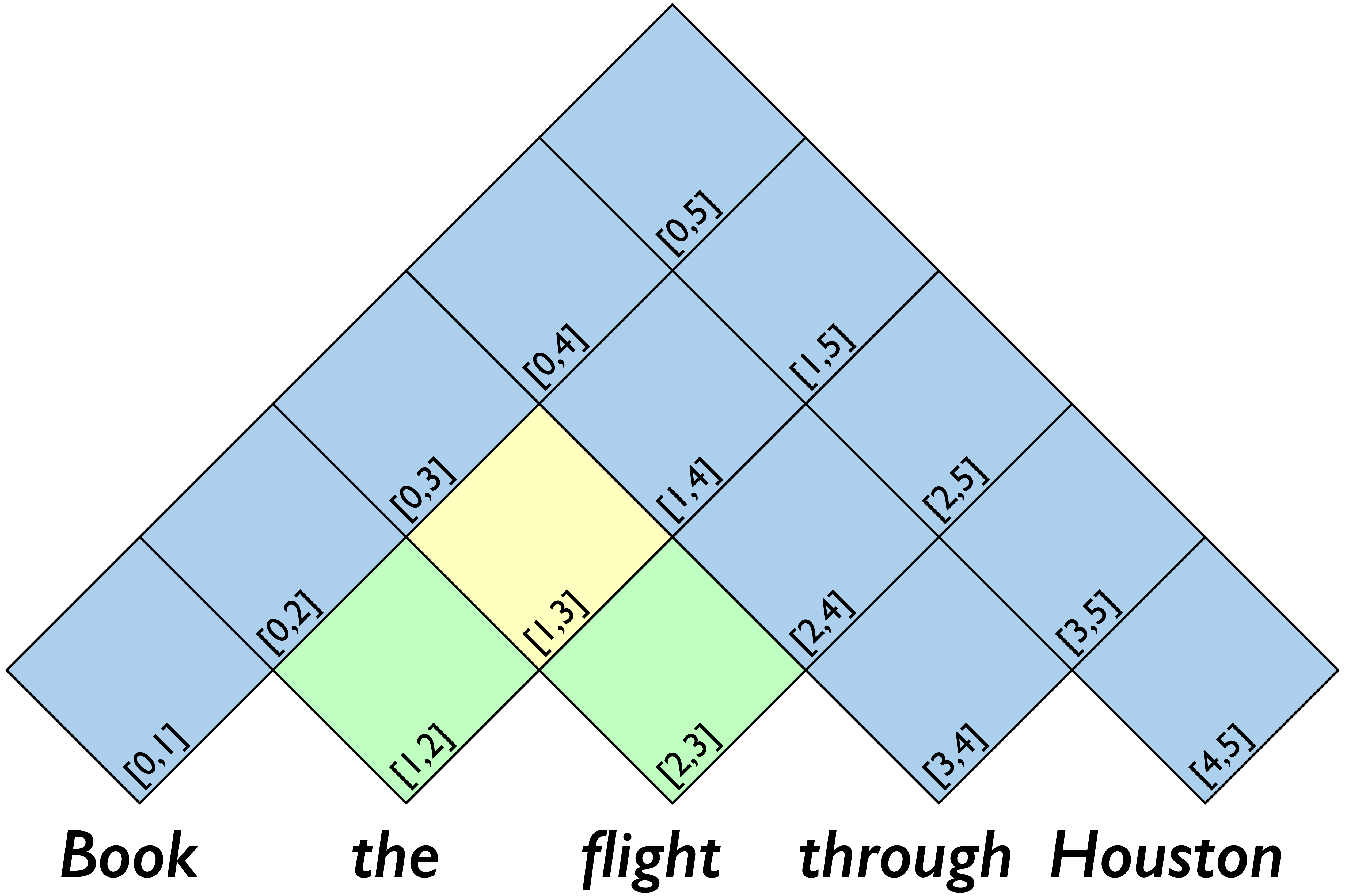
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[2,3]
[3,4]
[4,5]
*Book*
*the*
*flight*
*through*
*Houston*

# Dynamic Programming in CKY

- Key idea:
  - for $i < k < j$
  - …and a parse spanning substring [ $i$, $j$ ]
  - $\text{There is a } k$ such that there are parses spanning [ $i$, $k$ ] and [ $k$, $j$ ]
  - We can construct parses for whole sentences by building from these partial parses
- So to have a rule $A \rightarrow B\ C$ in [ $i$, $j$ ]
  - Must have $B$ in [ $i$, $k$ ] and $C$ in [ $k$, $j$ ] for some $i < k < j$
  - CNF forces this for all $j > i + 1$