Epistemic Modality and the Dynamics of Discourse

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1 Introduction

Consider the following conversation in ordinary language:

(1) Context: Mark is running late but hasn't been able to find his keys in his pants, his bag, or on his nightstand. His partner, Sue, is also getting ready.

  a) M: I think I've lost my keys.
  b) S: They might be on the kitchen table.
  c) M: Good point; I'll go look.

Major problem:

- provide a semantics for 'might' which, when coupled with a plausible pragmatic story, explains the information flow in (1) and other puzzling phenomena

Semantics-Focused Approaches:

- Descriptivism: Kratzer, DeRose, von Fintel and Gillies, Stalnaker
- Expressivism: Yalcin, dynamic semanticists (Veltman, Willer)

Our approach:

- Start directly from pragmatics and immediate intuitions about the dynamics of conversation

Our general claims:

- Assertion primarily functions to coordinate doxastic states
- 'Might' primarily functions to express *lack* of belief (abelief)

Our specific proposals:

- A very intuitive model of assertability
- Derivation of doxastic state expressed by a given assertion
- The update operation(s) performed when an assertion is accepted

This combination of tools solves all of the problems. Tentative conclusion: the informational dynamics of assertion provides the right level of explanation for these phenomena. Moreover, our explanation is compatible with (versions of) all of the leading semantic theories.

2 Proposal: 'Might' as Abelief Coordinator

Our model of doxastic states: a set of worlds W with a plausibility order $\succeq$.

- Belief that p: truth throughout most plausible worlds

Consider a 'factual' version of (1):

(2) Context: as in (1)

  a) M: I think I've lost my keys.
  b) S: They are on the kitchen table.
  c) M: Thanks!

Sue, in (2b):

- *expresses* that she believes that the keys are on the table
- *invites* Mark to modify his doxastic state so as to acquire that belief

Let t, b, n, and p be the propositions that the keys are on the table, in his bag, on his nightstand, or in his pocket, respectively. We can model Mark's doxastic state with 5 worlds: $W = \{t, b, n, p, L\}$. In our abused notation, the worlds t, b, etc. are worlds in which only the corresponding proposition is true. L is a world in which the keys are lost, i.e. a world in which none of t, b, n, p are true.
What Mark's doxastic state looks like before:

(3) $L \succ b, n, p \succ t$

What Mark's doxastic state looks like after acceptance:

(4) $t \succ L \succ b, n, p$

What about (1)? Sue's 'might' assertion in (1b):

- *expresses* that she abelieves that $\neg t$
- *invites* Mark to modify his doxastic state so as to acquire that abelief

What Mark's doxastic state looks like after acceptance:

(5) $t, L \succ b, n, p$

In general, acceptance of:

- B-assertion triggers *conservative revision*:
  $\uparrow p\,(\succeq)$ is just like $\succeq$ with the best p-worlds made more plausible than all others
- A-assertion triggers *conservative expansion*:
  $\upharpoonright p\,(\succeq)$ is just like $\succeq$ with the best p-worlds merged with the previous best worlds

3 Two Problems for Mixed Assertions

What about an assertion like: $(p \wedge \Diamond q) \vee \Diamond(s \wedge (\Diamond t \wedge \neg q))$?

- Problem 1: what doxastic state is expressed?
- Problem 2: what update operation is performed on acceptance?

3.1 Language

We work with a standard logical language containing: atomic proposition letters $(p, q, r, \dots)$, boolean operators $\neg, \vee, \wedge$, $\Diamond\varphi$ ("φ might be the case"), and $B\varphi$ ("the agent believes that φ").

3.2 Assertability Logic

Let $\mathbf{s}$ be an information set (a set of possible worlds). We will define what it means for a formula to be *assertable* relative to an information set.

Definition 1 (General Assertability Conditions). *Given a set of worlds W, an information state $\mathbf{s} \subseteq W$, and a valuation V:*

- $\mathbf{s} \Vdash p$ *iff:* $\forall w \in \mathbf{s}$*:* $w \in V(p)$
- $\mathbf{s} \Vdash \neg\varphi$ *iff:* $\forall w \in \mathbf{s}$*:* $\{w\} \nVdash \varphi$
- $\mathbf{s} \Vdash \varphi \wedge \psi$ *iff:* $\mathbf{s} \Vdash \varphi$ *and* $\mathbf{s} \Vdash \psi$
- $\mathbf{s} \Vdash \varphi \vee \psi$ *iff:* $\exists \mathbf{s}_1, \mathbf{s}_2$*:* $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$ *and* $\mathbf{s}_1 \Vdash \varphi$ *and* $\mathbf{s}_2 \Vdash \psi$
- $\mathbf{s} \Vdash \Diamond\varphi$ *iff:* $\mathbf{s} \nVdash \neg\varphi$

Quick consequences:

- Relative to singletons $\{w\}$, this logic is classical
- $\mathbf{s} \Vdash \Diamond\varphi$ iff $\exists w \in \mathbf{s} : \{w\} \Vdash \varphi$
- Relative to singletons, $\Diamond\varphi$ and φ are equivalent

3.3 Doxastic Logic

Definition 2 (Doxastic model). *A* doxastic model *is a tuple* $\mathcal{M} = \langle W, \{\succeq_w\}, V\rangle$ *where:*

- W *is a set of* worlds
- $\succeq_w$*, the* plausibility order *on* W *at* w*, is a* total pre-order *on* W*: a reflexive, transitive, total relation.*
- V *is a* valuation function *assigning a proposition (i.e. a set of worlds) to each atom* p*.*

We will denote by $\mathbf{b}_w$ the set of 'belief worlds' at w, that is the set of worlds maximal in $\succeq_w$.

Definition 3 (Static Semantics)**.**

- $\mathcal{M}, w \vDash p$ *iff:* $w \in V(p)$
- $\mathcal{M}, w \vDash \neg\varphi$ *iff:* $\mathcal{M}, w \nvDash \varphi$
- $\mathcal{M}, w \vDash \varphi \wedge \psi$ *iff:* $\mathcal{M}, w \vDash \varphi$ *and* $\mathcal{M}, w \vDash \psi$
- $\mathcal{M}, w \vDash Best(\varphi)$ *iff: for every* v *such that* $v \succ_w w$*:* $\mathcal{M}, w \vDash \neg\varphi$
- $\mathcal{M}, w \vDash B\varphi$ *iff: for every* $v \in \boldsymbol{b}_w$*,* $\mathcal{M}, v \vDash \varphi$

As a warm-up to our main theorem, note the following:

$$\mathbf{b}_w \Vdash \Diamond\varphi \text{ iff } \mathcal{M}, w \vDash \neg B \neg\varphi$$

Theorem 1 (From assertion to doxastic state expression)**.** *For every sentence* φ *in the assertability language, there exists a sentence* φ^* *with the features:*

(1) φ^* *is of the form:* $B\varphi \wedge \neg B(\neg\psi_1) \wedge \ldots \wedge \neg B(\neg\psi_n)$

(2) φ^* *contains no* $\Diamond$ *operators*

such that for every doxastic model $\mathcal{M}$ *and world* w*:*

$$\boldsymbol{b}_w \Vdash \varphi \textit{ iff } \mathcal{M}, w \vDash \varphi^*$$

3.4 Dynamics

To address the second problem, we enrich the language with expressions of the form $[\Uparrow \varphi]\,\psi$ with intended reading: "after radical revision by φ, ψ holds".

Definition 4 (Radical Revision)**.** *We denote by* $\Uparrow P$ *the* radical revision *operation on doxastic models, where: the model that results from applying* $\Uparrow P$ *to* $\mathcal{M}$ *updates the ordering* $\succeq$ *of* $\mathcal{M}$ *so that all of the* P*-worlds in* $\mathcal{M}$ *are moved to the top of the ordering. Call the resulting model* $\mathcal{M} \Uparrow P$*.*

Definition 5 (Dynamic Semantics)**.** *We can extend the static semantics with the following clause:*

- $\mathcal{M}, w \vDash [\Uparrow \varphi]\psi$ *iff:* $\mathcal{M} \Uparrow \varphi, w \vDash \psi$

Using this framework, we have the resources to define *conservative revision* and *conservative expansion* operations, respectively as follows:

i. $\uparrow \varphi \ ::= \ \Uparrow Best(\varphi)$

ii. $\upharpoonleft \varphi \ ::= \ \Uparrow [Best(\varphi) \vee Best(\top)]$

Now, we can define an operation that tells us how to update on a doxastic state expression.

Definition 6 (Simultaneous Update)**.** *By simultaneous update to believe* φ *and abelieve* $\psi_1, \ldots, \psi_n$*, we mean to perform the operation:*

$$[\Uparrow\!\!\upharpoonleft \varphi, \psi_1, \ldots, \psi_n] ::= [\Uparrow Best(\varphi) \vee (Best(\neg\psi_1) \wedge \varphi) \vee \ldots \vee (Best(\neg\psi_n) \wedge \varphi)]$$

We show that this definition handles our keys cases with aplomb. If φ^* is a doxastic state expression, we will abbreviate the above by $[\Uparrow\!\!\upharpoonleft \varphi^*]$. In the case when φ^* has no conjunct $B\varphi$, replace φ with $\top$. In the case when φ^* has no conjunct $\neg B\psi_i$, set $n = 1$ and $\psi_1 = \bot$.

Proposition 1. *Let* φ *be a sentence in the assertability language. Then:*

i. If φ *expresses no abeliefs, then*

$$[\Uparrow\!\!\upharpoonleft \varphi^*] = [\Uparrow \varphi]$$

ii. If φ *expresses a single abelief, then*

$$[\Uparrow\!\!\upharpoonleft \varphi^*] = [\upharpoonleft \psi_1]$$

Q: is every sentence in the language with dynamic operators equivalent to some sentence in the static 'base' language? Yes!

Proposition 2. *The following recursion axioms are valid for the class of doxastic models:*

$$\begin{array}{lcl}
[\Uparrow \varphi]\, p & \leftrightarrow & p \\
[\Uparrow \varphi]\, \neg\psi & \leftrightarrow & \neg\, [\Uparrow \varphi]\, \psi \\
[\Uparrow \varphi]\, \psi \wedge \chi & \leftrightarrow & [\Uparrow \varphi]\, \psi \wedge [\Uparrow \varphi]\, \chi \\
[\Uparrow \varphi]\, B\psi & \leftrightarrow & (E\varphi \wedge U\,(\varphi \to [\Uparrow \varphi]\, \psi)) \vee (\neg E\varphi \wedge B\, [\Uparrow \varphi]\, \psi)
\end{array}$$

Proposition 3. *The following recursion axioms are valid for the class of doxastic models: the ones for atoms, negations, and conjunctions above but with* $\upharpoonleft \varphi$ *and*

$$[\upharpoonleft \varphi]\, B\psi \quad \leftrightarrow \quad B\, [\upharpoonleft \varphi]\, \psi \wedge B^{\varphi}\, [\upharpoonleft \varphi]\, \psi$$

Theorem 2. *The following recursion axioms are valid for the class of doxastic models:*

$$\begin{array}{lcl}
[\Uparrow \varphi]\, B^{\chi}\psi & \leftrightarrow & \left(\neg E\,(\varphi \wedge [\Uparrow \varphi]\, \chi) \wedge B^{[\Uparrow \varphi]\chi}\, [\Uparrow \varphi]\, \psi\right) \vee \\
 & & (E\,(\varphi \wedge [\Uparrow \varphi]\, \chi) \wedge U\,(\varphi \wedge [\Uparrow \varphi]\, \chi \to [\Uparrow \varphi]\, \psi)) \\
[\upharpoonleft \varphi]\, B^{\chi}\psi & \leftrightarrow & \left(B^{\varphi}\neg\, [\upharpoonleft \varphi]\, \chi \wedge B^{[\upharpoonleft \varphi]\chi}\, [\upharpoonleft \varphi]\, \psi\right) \vee \\
 & & \left(\neg B^{\varphi}\neg\, [\upharpoonleft \varphi]\, \chi \wedge B^{\varphi \wedge [\upharpoonleft \varphi]\chi}\, [\upharpoonleft \varphi]\, \psi \wedge \left(\neg B \neg\, [\upharpoonleft \varphi]\, \chi \to B^{[\upharpoonleft \varphi]\chi}\, [\upharpoonleft \varphi]\, \psi\right)\right)
\end{array}$$

4 Welcome Consequences

- Epistemic contradictions

$$\mathbf{s} \nVdash p \wedge \Diamond \neg p$$

- Disagreement
  - Possibility of disagreement
  - Nature / content of disagreement
- Interactions with conjunction and disjunction

$$\mathbf{s} \Vdash \Diamond p \wedge \Diamond q \text{ iff } \mathbf{s} \Vdash \Diamond p \vee \Diamond q$$

- Abelief in explicit reasoning
  - Broome: natural language is insufficient for active reasoning because it cannot express abelief. But it can, with 'might'!

5 Competitors

- 'Might' as B-assertion (cf. Kratzer, Papafragou, von Fintel and Gillies)
- 'Might' as test (cf. Veltman, Willer)
- 'Might' as credence expresser (cf. Moss, Swanson)
- 'Might' as awareness-raiser (cf. Lyons, Swanson, Yalcin)
- 'Might' as retraction (cf. Portner, Yablo)
- 'Might' as update on context set (cf. Stalnaker)

6 Semantic Neutrality

A principle linking assertability and semantic conditions:

(AH) φ is assertible with respect to $\mathbf{s}$ iff: φ 'holds' throughout $\mathbf{s}$

view	priority	'holds'	(AH)?	extra conditions
expressivism	$\Leftarrow$	$\mathcal{M}, w, \mathbf{s} \vDash \varphi$	✓	none
contextualism	$\Leftarrow$	$\mathcal{M}, w, \mathbf{s}_w \vDash \varphi$	✓	$\forall v \in \mathbf{s}_w : \mathbf{s}_v = \mathbf{s}_w$
force modifier	$\Rightarrow$	$\{w\} \Vdash \varphi$	X	N/A

7 Conclusion

Summary: theorizing about 'might' at the level of discourse dynamics – as an abelief expresser and coordinater – has proven to be a very fruitful strategy. Absent an alternative as successful, it's the best strategy.